Growth

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Motivation

- Goal: to understand factors that affect long-term performance of an economy.
- long-term → usually 20-30 years. All factors including capital stock etc. are variable. Trend vs blips.
GDP per capita: The dollar value of a country’s final output of goods and services in a year (its GDP), divided by its population.
Long run

- Why should we care about the long-run?
- “Rule of 72”: 72 divided by the rate of growth of an economy will approximately give the number of years it takes for an economy to double in size.
# Growth facts

## TABLE 1.1  STATISTICS ON GROWTH AND DEVELOPMENT

<table>
<thead>
<tr>
<th></th>
<th>GDP per capita, 1997</th>
<th>GDP per worker, 1997</th>
<th>Labor force participation rate, 1997</th>
<th>Average annual growth rate, 1960–97</th>
<th>Years to double</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>“Rich” countries</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.A.</td>
<td>$20,049</td>
<td>$40,834</td>
<td>0.49</td>
<td>1.4</td>
<td>50</td>
</tr>
<tr>
<td>Japan</td>
<td>16,003</td>
<td>25,264</td>
<td>0.63</td>
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<td>14,650</td>
<td>31,986</td>
<td>0.46</td>
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<tr>
<td>U.K.</td>
<td>14,472</td>
<td>29,295</td>
<td>0.49</td>
<td>1.9</td>
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<td>Spain</td>
<td>10,685</td>
<td>29,396</td>
<td>0.36</td>
<td>3.5</td>
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<tr>
<td><strong>“Poor” countries</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>China</td>
<td>2,387</td>
<td>3,946</td>
<td>0.60</td>
<td>3.5</td>
<td>20</td>
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<tr>
<td>India</td>
<td>1,624</td>
<td>4,156</td>
<td>0.39</td>
<td>2.3</td>
<td>30</td>
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<tr>
<td>Zimbabwe</td>
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<td>2,561</td>
<td>0.49</td>
<td>0.4</td>
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<td>697</td>
<td>1,437</td>
<td>0.49</td>
<td>0.5</td>
<td>146</td>
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<tr>
<td><strong>“Growth miracles”</strong></td>
<td></td>
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<tr>
<td>Hong Kong</td>
<td>18,811</td>
<td>28,918</td>
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<td>5.2</td>
<td>13</td>
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<td>Taiwan</td>
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<td>26,779</td>
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<td>South Korea</td>
<td>10,131</td>
<td>24,325</td>
<td>0.42</td>
<td>5.9</td>
<td>12</td>
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<td><strong>“Growth disasters”</strong></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Venezuela</td>
<td>6,760</td>
<td>19,455</td>
<td>0.35</td>
<td>−0.1</td>
<td>−517</td>
</tr>
<tr>
<td>Madagascar</td>
<td>577</td>
<td>1,334</td>
<td>0.43</td>
<td>−1.5</td>
<td>−46</td>
</tr>
</tbody>
</table>
**Huge effects from tiny differences**

<table>
<thead>
<tr>
<th>annual growth rate of income per capita</th>
<th>percentage increase in standard of living after...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>...25 years</td>
</tr>
<tr>
<td>2.0%</td>
<td>64.0%</td>
</tr>
<tr>
<td>2.5%</td>
<td>85.4%</td>
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</tbody>
</table>
Cross section differences

large disparities in per capita income across countries

### International Differences in the Standard of Living: 1999

<table>
<thead>
<tr>
<th>Country</th>
<th>Income per Person (in U.S. dollars)</th>
<th>Country</th>
<th>Income per Person (in U.S. dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>$31,910</td>
<td>China</td>
<td>3,550</td>
</tr>
<tr>
<td>Japan</td>
<td>25,170</td>
<td>Indonesia</td>
<td>2,660</td>
</tr>
<tr>
<td>Germany</td>
<td>23,510</td>
<td>India</td>
<td>2,230</td>
</tr>
<tr>
<td>Mexico</td>
<td>8,070</td>
<td>Pakistan</td>
<td>1,860</td>
</tr>
<tr>
<td>Russia</td>
<td>6,990</td>
<td>Bangladesh</td>
<td>1,530</td>
</tr>
<tr>
<td>Brazil</td>
<td>6,840</td>
<td>Nigeria</td>
<td>770</td>
</tr>
</tbody>
</table>

*Source: World Bank.*

- Nigeria’s per capita income is about 5% that of the US!
Not always like this.

Figure 1. The Evolution of Regional Income per Capita 1 - 2000
Korea vs. Philippines

The example of the Philippines and Korea. GDP per-capita $640 in 1975 US$; 28 million people in Philippines versus 25 million in Korea, 27% of Filipinos lived in Manila versus 28% in Seoul; all boys of primary school age were in school in both nations; 1960-1988 → Philippines grew at 1.8% per year while Korea grew at 6.2% per year.
Questions

1. why are some countries poor and others rich? what explains the gap?
2. why do some grow faster than others?
3. will poor countries catch up with the rich over time?
Production

- Crucial to understand what constitutes an economy’s physical capacity to produce goods and services.
- Output is produced by “factors of production”; e.g., capital goods, people, land, energy...more of all these factors translates into more output.
- How effectively are these used?
- production function:

\[ Y_t = A \cdot F(K_t, N_t) \] (1)

where \( Y \) = real output produced at time \( t \), \( A \) = a measure of productivity, \( K_t \) = capital stock at time \( t \), \( N \) = number of workers employed at time \( t \) (assumed to be the size of the adult work force).
Production function

- For the US, the relationship is

\[ Y_t = A K_t^{\frac{1}{3}} N_t^{\frac{2}{3}}. \]  

- \( A \) is called “total factor productivity”.
- Notion of Solow residual.
- The “missing ingredient”
Technical progress: change in $A$

- 1970: 50,000 computers in the world
- 2000: 51% of U.S. households have 1 or more computers

- The real price of computer power has fallen an average of 30% per year over the past three decades.

- The average car built in 1996 contained more computer processing power than the first lunar landing craft in 1969.

- Modems are 22 times faster today than two decades ago.

- Since 1980, semiconductor usage per unit of GDP has increased by a factor of 3500.

- 1981: 213 computers connected to the Internet
- 2000: 60 million computers connected to the Internet
The production function

- What does (2) look like?
- To draw a picture showing how $Y$ changes with change in both $K$ and $N$ would require a 3-D picture. Hold $N$ (and $A$) fixed and see how $Y$ changes with $K$.
- Fix $N = \bar{N}$ workers; and $A = \bar{A}$. Then

$$Y_t = \left[ \bar{A} \left( \frac{\bar{N}}{3} \right)^{\frac{2}{3}} \right] K_t^{\frac{1}{3}}.$$

- Two important features:
  1. Slopes upward from left to the right
  2. Slope becomes flatter from left to right.
Marginal product of capital

- Notion of marginal product of capital $MPK$.
- $MPK = \text{increase in output } (\Delta Y) \text{ resulting from a one-unit increase in the capital stock (holding labor and productivity fixed)}$
- $MPK \text{ at a point } = \left( \frac{\Delta Y}{\Delta K} \right) \text{ at that point } = \text{slope of the production function at that point } = \text{derivative of the function } F \text{ with respect to } K \text{ at that point.}$
- Diminishing marginal productivity of capital; example
Marginal product of capital

- \( MPK = \left( \frac{\Delta Y}{\Delta K} \right) \Rightarrow \Delta Y = MPK \times \Delta K \)

- Example: suppose labor is fixed and \( MPK = 1/5 \). If we increase \( K \) by 10 units, i.e. \( \Delta K = 10 \), we can calculate \( \Delta Y \) by

\[
\Delta Y = MPK \times \Delta K = \frac{1}{5} \times 10 = 2
\]

- We use the \( MPK \) to convert changes in \( K \) to changes in output.
Marginal product of labor

- Notion of marginal product of labor $MPN$
- $MPN = \text{increase in output (}\Delta Y\text{)}$ resulting from a one-unit increase in labor (holding capital fixed)
- Diminishing marginal productivity of labor
Changes in productivity

\[ Y_t = \left[ \bar{A} \left( \bar{N} \right)^{\frac{2}{3}} \right] K_t^{\frac{1}{3}} \]

what happens when \( \bar{A} \) rises?
Growth Accounting

- Suppose we allow both $N$ and $K$ to change:

$$\Delta Y = (MPN \times \Delta N) + (MPK \times \Delta K)$$

$$\Rightarrow \frac{\Delta Y}{Y} = \frac{(MPN \times \Delta N)}{Y} + \frac{(MPK \times \Delta K)}{Y}$$

$$\Rightarrow \frac{\Delta Y}{Y} = \frac{(MPN \times N)}{Y} \frac{\Delta N}{N} + \frac{(MPK \times K)}{Y} \frac{\Delta K}{K}$$
\[
\frac{\Delta Y}{Y} = \frac{(MPN \times N)}{Y} \frac{\Delta N}{N} + \frac{(MPK \times K)}{Y} \frac{\Delta K}{K}
\]

- \(\frac{\Delta N}{N}\) = growth rate of labor
- \(\frac{\Delta K}{K}\) = growth rate of capital
- \(\frac{(MPN \times N)}{Y} = \left(\frac{N \Delta Y}{\Delta N}\right) = \left(\frac{\Delta Y}{\Delta N}\right)\) = percentage change in output resulting from a 1% change in labor use = \(a_N\)
- Similarly, \(\frac{(MPK \times K)}{Y} = a_K\)
Growth Accounting Equation

Now allowing $A$ to change too:

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + a_N \frac{\Delta N}{N} + a_K \frac{\Delta K}{K}$$

Called “growth accounting equation”. Breaks up output growth into its components. Answer questions of the form: what fraction of GDP growth can be attributed to growth in productivity, what fraction to capital accumulation, etc.

For the US, $a_K = 0.3$, $a_N = 0.7$. 
\[
\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + a_N \frac{\Delta N}{N} + a_K \frac{\Delta K}{K}
\]

- Example: Suppose \( \frac{\Delta N}{N} = \frac{\Delta K}{K} = 0 \) and \( \frac{\Delta A}{A} = 10\% \). Then (3) says that \( \frac{\Delta Y}{Y} = 10\% \).

- Example: Suppose \( \frac{\Delta A}{A} = \frac{\Delta K}{K} = 0 \) and \( \frac{\Delta N}{N} = 10\% \). Then (3) says that \( \frac{\Delta Y}{Y} = (a_N) \times 10\% = (0.7) \times 10\% = 7\% \).

- Thus, a 10\% increase in labor use, with capital and productivity unchanged, leads to only a 7\% increase in output. Why?
## Accounting for Economic Growth in the United States

<table>
<thead>
<tr>
<th>Years</th>
<th>Output Growth $\Delta Y/Y$</th>
<th>Capital $\alpha \Delta K/K$</th>
<th>Labor $(1 - \alpha) \Delta K/K$</th>
<th>Total Factor Productivity $\Delta A/A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950–1999</td>
<td>3.6</td>
<td>1.2</td>
<td>1.3</td>
<td>1.1</td>
</tr>
<tr>
<td>1950–1960</td>
<td>3.3</td>
<td>1.0</td>
<td>1.0</td>
<td>1.3</td>
</tr>
<tr>
<td>1960–1970</td>
<td>4.4</td>
<td>1.4</td>
<td>1.2</td>
<td>1.8</td>
</tr>
<tr>
<td>1970–1980</td>
<td>3.6</td>
<td>1.4</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td>1980–1990</td>
<td>3.4</td>
<td>1.2</td>
<td>1.6</td>
<td>0.6</td>
</tr>
<tr>
<td>1990–1999</td>
<td>3.7</td>
<td>1.2</td>
<td>1.6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

(average percentage increase per year)

Source: U.S. Department of Commerce, U.S. Department of Labor, and the author’s calculations. The parameter $\alpha$ is set to equal 0.3.
<table>
<thead>
<tr>
<th>Country</th>
<th>Per.</th>
<th>$g_Y$</th>
<th>$\alpha$</th>
<th>$\alpha g_K$</th>
<th>$(1 - \alpha)g_L$</th>
<th>$g_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>60-90</td>
<td>3.2</td>
<td>0.4</td>
<td>59%</td>
<td>−8%</td>
<td>49%</td>
</tr>
<tr>
<td>Italy</td>
<td>60-90</td>
<td>4.1</td>
<td>0.38</td>
<td>49%</td>
<td>3%</td>
<td>48%</td>
</tr>
<tr>
<td>UK</td>
<td>60-90</td>
<td>2.5</td>
<td>0.39</td>
<td>52%</td>
<td>−4%</td>
<td>52%</td>
</tr>
<tr>
<td>Argentina</td>
<td>40-80</td>
<td>3.6</td>
<td>0.54</td>
<td>43%</td>
<td>26%</td>
<td>31%</td>
</tr>
<tr>
<td>Brazil</td>
<td>40-80</td>
<td>6.4</td>
<td>0.45</td>
<td>51%</td>
<td>20%</td>
<td>29%</td>
</tr>
<tr>
<td>Chile</td>
<td>40-80</td>
<td>3.8</td>
<td>0.52</td>
<td>34%</td>
<td>26%</td>
<td>40%</td>
</tr>
<tr>
<td>Mexico</td>
<td>40-80</td>
<td>6.3</td>
<td>0.63</td>
<td>41%</td>
<td>23%</td>
<td>36%</td>
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<tr>
<td>Japan</td>
<td>60-90</td>
<td>6.8</td>
<td>0.42</td>
<td>57%</td>
<td>14%</td>
<td>29%</td>
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<tr>
<td>Hong Kong</td>
<td>66-90</td>
<td>7.3</td>
<td>0.37</td>
<td>42%</td>
<td>28%</td>
<td>30%</td>
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<tr>
<td>Singapore</td>
<td>66-90</td>
<td>8.5</td>
<td>0.53</td>
<td>73%</td>
<td>31%</td>
<td>−4%</td>
</tr>
<tr>
<td>South Korea</td>
<td>66-90</td>
<td>10.3</td>
<td>0.32</td>
<td>46%</td>
<td>42%</td>
<td>12%</td>
</tr>
<tr>
<td>Taiwan</td>
<td>66-90</td>
<td>9.1</td>
<td>0.29</td>
<td>40%</td>
<td>40%</td>
<td>20%</td>
</tr>
</tbody>
</table>
Supply side of Solow model

- One good, say corn produced using labor and capital. Set $A = 1$ for now.
- Extensive form:
  \[ Y = F(K, N) \]
- Define $y \equiv \frac{Y}{N}$ output per worker and $k \equiv \frac{K}{N}$ capital per worker; then $y = f(k)$ is in intensive form
Extensive to intensive form

Example: suppose \( F(K, N) = K^{1/2}N^{1/2} \)

\[
Y = K^{1/2}N^{1/2}
\]

Since \( y = \frac{Y}{N} \) and \( k = \frac{K}{N} \), then,

\[
y = \frac{Y}{N} = k^{\frac{1}{2}}
\]

Therefore, the per-worker production function is

\[
y_t = f(k_t) = k_t^{\frac{1}{2}}
\]

(intensive form)
Output per worker, $y$

Output, $f(k)$

Capital per worker, $k$

$MPK$
Demand side of Solow model

- Corn has two uses: can consume it or can leave some aside as seedcorn for the next year (investment).

\[ y_t = c_t + i_t. \]

- Solow assumed that investment per worker is a fixed fraction \( s \) of output per worker, i.e.,

\[ i_t = s \, y_t = s \, f(k_t) \]

- \( s = \) saving rate, \( 0 < s < 1 \).

- Implies: \( c_t \) is proportional to output (income): as \( y \) increases, \( c \) increases, but since \( (1 - s) < 1 \), the increase is less than the increase in \( y \).

\[ c_t = (1 - s) \, f(k_t) \]
Output per worker gets divided in two parts: a part \((1 - s)y_t\) is consumed, and the rest, \(sy_t\) is saved as seedcorn to be invested next year.
Depreciation

- depreciation: wear and tear; technological obsolescence
- Depreciation: part of the capital stock wears out each period (acid rain!); bricks fall off
- Assume a fixed fraction $\delta$ of the capital stock depreciates every period. Therefore, the amount of the capital stock that depreciates away during period $t$ is $\delta k_t$. 
Capital accumulation in Solow model

- Timeline
- Start with $k_t \rightarrow$ produce $f(k_t) \rightarrow$ consume/save
- Savings at the end of time $t$ is $i_t = s y_t = s f(k_t)$; this is seed corn or investment for time $t + 1$.
- An amount, $\delta k_t$, of the start of period capital will depreciate away by the time $t + 1$ has arrived.

$t + 1$'s start of period capital per worker:

$$k_{t+1} = k_t - \delta k_t + s f(k_t)$$
Steady state of Solow model

- Steady state: when $k_{t+1} - k_t = 0$ i.e., $k_{t+1} = k_t = k$,

\[ sf(k) = \delta k \]  \hspace{1cm} (4)

- The new investment, $sf(k)$, goes entirely towards replacing the capital that is getting depreciated, $\delta k$.

- Solution to (4) is $k^* = 0$, and $k^* > 0$. $k^*$ = the steady state capital per worker. Then, output per worker in steady state is $y^* = f(k^*)$, consumption per worker in steady state is $c^* = (1 - s) y^*$. 

Growth Accounting

The Solow Model

Golden Rule

Investment and depreciation

Depreciation, $\delta k$

Investment, $sf(k)$

$k_1$, $k_2$, Capital per worker, $k$

$i_1$, $i_2$, $i^* = \delta k^*$

$\delta k_1$, $\delta k_2$

Capital stock increases because investment exceeds depreciation.

Steady-state level of capital per worker

Capital stock decreases because depreciation exceeds investment.
Calculating the steady state

- Suppose $f(k) = k^\alpha$; then

$$k^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$$

$$y^* = (k^*)^\alpha = \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

different $s$ implies different levels of $k^*$
Transition

- dynamics: transition to the steady state.

\[ k_{t+1} = k_t + sf(k_t) - \delta k_t \]

growth rate: for \( f(k) = k^\alpha \):

\[ g_t \equiv \frac{k_{t+1} - k_t}{k_t} = \frac{sf(k_t) - \delta k_t}{k_t} = sk_t^{\alpha-1} - \delta \]

Plot \( g_t \) against \( k \); see how speed of convergence is high when \( k \) is small.
Example

- Assume $s = 0.3$, $\delta = 0.1$, and $k_0 = 4$.
- Show that $k_1 = 4.2$. 
### Approaching the Steady State: A Numerical Example

**Assumptions:** \( y = \sqrt{k}; \ s = 0.3; \ \delta = 0.1; \ \text{initial } k = 4.0 \)

<table>
<thead>
<tr>
<th>Year</th>
<th>( k )</th>
<th>( y )</th>
<th>( c )</th>
<th>( i )</th>
<th>( \delta k )</th>
<th>( \Delta k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.000</td>
<td>2.000</td>
<td>1.400</td>
<td>0.600</td>
<td>0.400</td>
<td>0.200</td>
</tr>
<tr>
<td>2</td>
<td>4.200</td>
<td>2.049</td>
<td>1.435</td>
<td>0.615</td>
<td>0.420</td>
<td>0.195</td>
</tr>
<tr>
<td>3</td>
<td>4.395</td>
<td>2.096</td>
<td>1.467</td>
<td>0.629</td>
<td>0.440</td>
<td>0.189</td>
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<tr>
<td>4</td>
<td>4.584</td>
<td>2.141</td>
<td>1.499</td>
<td>0.642</td>
<td>0.458</td>
<td>0.184</td>
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<tr>
<td>5</td>
<td>4.768</td>
<td>2.184</td>
<td>1.529</td>
<td>0.655</td>
<td>0.477</td>
<td>0.178</td>
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<tr>
<td>...</td>
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</tr>
<tr>
<td>10</td>
<td>5.602</td>
<td>2.367</td>
<td>1.657</td>
<td>0.710</td>
<td>0.560</td>
<td>0.150</td>
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<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>25</td>
<td>7.321</td>
<td>2.706</td>
<td>1.894</td>
<td>0.812</td>
<td>0.732</td>
<td>0.080</td>
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<td>...</td>
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</table>
Long run

- what happens in the long-run?

\[ k^* = 9. \]

what happens to growth as time passes?

- Post-war Germany grew at 5.7% p.a and Japan at 8.2%. Why?

- poorer countries grow faster!
Population growth

- What does the Solow model predict about the effects of population growth?
- If the capital stock was fixed, this would mean that the *capital per worker* would fall.
- While investment helps raise the capital stock per worker, both depreciation and population growth tend to reduce it.

\[ N_{t+1} = (1 + n) N_t; \]

\[ sf(k) = (n + \delta) k \] (5)
Population growth

Solow model predicts that countries with high rates of population growth have lower levels of capital per worker, and hence, lower levels of output per worker.
Income per person in 1992 (logarithmic scale)

Population growth (percent per year) (average 1960-1992)
optimal capital accumulation

- More capital accumulation comes from more saving, which means less consumption today. Trade-off between less consumption today and more consumption tomorrow.

- What amount of capital accumulation is “best”? 
Intergenerational politics of saving

Suppose the generation alive at time $t$ suddenly decides to consume all the output they had produced. Then $s = 0$; investment $= 0$. That generation consumes an amount $c = f(k)$ the whole output. The capital per worker for next period (time $t+1$ generation) is $k - \delta k$ which is less than what the time $t$ generation had. Hence, their output is lower, and their consumption is lower. So, the saving decision of previous generations has a profound impact on the well-being of future generations.
Golden rule

- Example:

\[ k^* = \left( \frac{s}{\delta} \right) \frac{1}{1-\alpha} \]

higher \( s \) implies higher \( k^* \) and higher \( y^* \); but higher \( s \) for any generation also means lower \( c \) for that generation

- Suppose there was a social planner that could choose the savings rate of every generation. What \( s \) would it choose so as to not hurt one generation at the expense of another?

- Golden rule: Do unto others as you would have others do unto you.
Golden rule

- The planner must stick to choosing among different steady-state levels of $k$; that way, each generation will start off with the same amount of productive resource (capital).
- Any generation uses this to produce output, $f(k)$; then an amount $\delta k$ depreciates. Therefore consumption $c$ for any generation is

\[ c = f(k) - \delta k \]

- The steady state capital per worker that maximizes consumption is $k_{\text{gold}}$. 
Steady-state output, depreciation, and investment per worker

1. To reach the Golden Rule steady state...
2. ...the economy needs the right saving rate.
Golden rule

- $k_{\text{gold}}$ is that value of $k$ where the slope of the function $f(k)$ equals the slope of the line $\delta k$.
- $MPK = \delta$ [recall $MPK$ is just the derivative of $f(k)$ or $f'(k)$]
- The only way to achieve $k_{\text{gold}}$ is to choose the right saving rate $s_{\text{gold}}$.
- How to calculate the golden rule savings rate: $f'(k_{\text{gold}}) = \delta$; For $f(k) = \sqrt{k}$, $f'(k_{\text{gold}}) = \frac{1}{2\sqrt{k}} = 0.1$, or $\frac{1}{\sqrt{k}} = \frac{2}{10}$, or $k_{\text{gold}} = 25$. 
**Golden rule**

### Finding the Golden Rule Steady State: A Numerical Example

**Assumptions:** \( y = \sqrt{k} \); \( \delta = 0.1 \)

<table>
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<tr>
<th>( \delta )</th>
<th>( k^* )</th>
<th>( y^* )</th>
<th>( \delta k^* )</th>
<th>( c^* )</th>
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