the MPN for an increase of labor from 100 to 110. Compare this result with the MPN for an increase in labor from 110 to 120. Does the marginal productivity of labor diminish?

3. Acme Widget, Inc., has the following production function.

<table>
<thead>
<tr>
<th>Number of Workers</th>
<th>Widgets Produced</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>33</td>
</tr>
</tbody>
</table>

a. Find the MPN for each level of employment.
b. Acme can get $5 for each widget it produces. How many workers will it hire if the nominal wage is $38? If it is $27? If it is $22?
c. Graph the relationship between Acme’s labor demand and the nominal wage. How does this graph differ from a labor demand curve? Graph Acme’s labor demand curve.
d. With the nominal wage fixed at $38, the price of widgets doubles from $5 each to $10 each. What happens to Acme’s labor demand and production?
e. With the nominal wage fixed at $38 and the price of widgets fixed at $5, the introduction of a new automatic widget maker doubles the number of widgets that the same number of workers can produce. What happens to labor demand and production?
f. What is the relationship between your answers to part (d) and part (e)? Explain.

4. The marginal product of labor (measured in units of output) for a certain firm is

$$MPN = A(100 - N),$$

where $A$ measures productivity and $N$ is the number of labor hours used in production. The price of output is $2.00 per unit.
a. If $A = 1.0$, what will be the demand for labor if the nominal wage is $10? If it is $20? Graph the demand curve for labor. What is the equilibrium real wage if the supply of labor is fixed at 95?
b. Repeat part (a) for $A = 2.0$.

5. For an economy the MPN is

$$MPN = 500 - 0.5N,$$

where $N$ is aggregate employment. The aggregate quantity of labor supplied is $400 + 8(1 - t)w$, where $w$ is the real wage and $t$ is the income tax rate. The expression for labor supply reflects the fact that workers care about their after-tax real wage, $(1 - t)w$, rather than their before-tax real wage, $w$. Employers aren't taxed.
a. Find the before-tax real wage and employment when $t = 0.50$.
b. Find the before-tax real wage and employment when $t = 0.25$. Compare the result with that in part (a). Explain why the cut in the income tax rate has the effect that it does on the labor market.

6. In this problem you are to find the effects of a legal minimum wage on the labor income of unskilled workers. Assume that the marginal product of labor for unskilled labor is

$$MPN = 100 - 0.2N.$$

The supply of unskilled labor is $80 + 2w$, where $w$ is the real wage received by unskilled labor.
a. If there is no minimum wage, find the equilibrium values of the real wage, employment, and labor income for unskilled workers.
b. Now suppose that a minimum wage that sets the real wage at 70 is instituted. What are the new levels of employment and total labor income for unskilled workers?
c. Repeat parts (a) and (b) for $MPN = 100 - 0.9N$.

How does the imposition of a minimum wage of 70 affect labor income now? How does the impact of a minimum wage on labor income depend on the sensitivity of labor demand to the real wage?

7. Consider an economy with 500 people in the labor force. At the beginning of every month 5 people lose their jobs and remain unemployed for exactly one month; one month later, they find new jobs and become employed. In addition, on January 1 of each year 20 people lose their jobs and remain unemployed for six months before finding new jobs. Finally, on July 1 of each year 20 people lose their jobs and remain unemployed for six months before finding new jobs.
a. What is the unemployment rate in this economy in a typical month?
b. What fraction of unemployment spells lasts for one month? What fraction lasts for six months?
c. What is the average duration of an unemployment spell?
d. On any particular date, what fraction of the unemployed are suffering a long spell (six months) of unemployment?

8. Use the data in Fig. 3.14 to calculate how many people become unemployed during a typical month. How many become employed? How many leave the labor force?
4. \( MPN = A(100 - N) \)
   a. \( A = 1 \), \( MPN = 100 - N \).
      (1) \( W = $10 \). \( w = W / P = $10 / $2 = 5 \).
       Setting \( w = MPN, 5 = 100 - N \), so \( N = 95 \).
      (2) \( W = $20 \). \( w = W / P = $20 / $2 = 10 \).
       Setting \( w = MPN, 10 = 100 - N \), so \( N = 90 \).
       These two points are plotted as line \( ND^a \) in Fig. 3.12. If labor supply = 95, then the equilibrium real wage is 5.
   b. \( A = 2 \), \( MPN = 2(100 - N) \).
      (1) \( W = $10 \). \( w = W / P = $10 / $2 = 5 \).
       Setting \( w = MPN, 5 = 2(100 - N) \), so \( 2N = 195 \), so \( N = 97.5 \).
      (2) \( W = $20 \). \( w = W / P = $20 / $2 = 10 \).
       Setting \( w = MPN, 10 = 2(100 - N) \), so \( 2N = 190 \), so \( N = 95 \).
       These two points are plotted as line \( ND^b \) in Fig. 3.12. If labor supply = 95, then the equilibrium real wage is 10.

5. \( MPN = 500 - 0.5N \). \( NS = 400 + 8(1 - t)w \). Setting \( NS = N \) and \( w = MPN \) gives \( w = MPN = 500 - 0.5N = 500 - 0.5[400 + 8(1 - t)w] = 300 - 4(1-t)w \). Solving the equation \( w = 300 - 4(1-t)w \) for \( w \) gives \( w = 300 / (5 - 4t) \).
   a. \( t = 0.5 \). \( w = 300 / (5 - 4t) = 300 / (5 - 2) = 100 \). \( N = 400 + 8(1 - t)w = 400 + 8(1 - 0.5)100 = 800 \).
   b. \( t = 0.25 \). \( w = 300 / (5 - 4t) = 300 / (5 - 1) = 75 \). \( N = 400 + 8(1 - t)w = 400 + 8(1 - 0.25)75 = 850 \).
   The reduction in the marginal tax rate increases the after-tax real wage, \( (1 - t)w \). This leads to an increase in labor supply. The increased labor supply causes the before-tax real wage to decline, so labor demand increases. In equilibrium there is a higher after-tax real wage, a lower before-tax real wage, and both labor supply and labor demand are higher.

6. a. Labor demand: \( w = 100 - 0.2N \). Labor supply: \( N = 80 + 2w \). Plug labor supply into labor demand to solve for \( w \): \( w = 100 - 0.2N = 100 - 0.2(80 + 2w) \). Simplify this expression to get \( 1.4w = 84 \), or \( w = 60 \). Then \( N = 80 + 2w = 80 + 2(60) = 200 \). Labor income = \( wN = 60 \times 200 = 12,000 \).
   b. The minimum wage is effective in this case because the real wage in the absence of a minimum wage is 60, but the minimum wage is 70. The effect of the minimum wage is to raise the real wage, thus reducing labor demand but raising labor supply. Labor demand is found by solving the labor demand equation at the minimum wage level of 70: \( w = 100 - 0.2N \), so \( 70 = 100 - 0.2N \), which has the solution \( N = 150 \). Labor supply is \( N = 80 + 2w = 80 + 2(70) = 220 \). Since labor demand is less than labor supply, employment equals labor demand of 150, with 70 unemployed. Labor income is \( wN = 70 \times 150 = 10,500 \).
   c. Labor demand: \( w = 100 - 0.9N \). Labor supply: \( N = 80 + 2w \). Plug labor supply into labor demand to solve for \( w \): \( w = 100 - 0.9N = 100 - 0.9(80 + 2w) \). Simplify this expression to get \( 2.8w = 28 \), or \( w = 10 \). Then \( N = 80 + 2w = 80 + 2(10) = 100 \). Labor income = \( wN = 10 \times 100 = 1000 \).
The minimum wage is also effective in this case because the real wage in the absence of a minimum wage is 10, but the minimum wage is 70. Labor demand is found by solving: \( w = 100 - 0.9N \), so 70 = 100 - 0.9N, which has the solution N = 33 1/3. Labor supply is \( N = 80 + 2w = 80 + (2 \times 70) = 220 \). Again, since labor demand is less than labor supply, employment equals labor demand of 33 1/3, with 186 2/3 unemployed. Labor income is \( wN = 70 \times 33 1/3 = 2333 1/3 \).

The elasticity of labor demand with respect to the real wage is \( (\Delta N / N) / (\Delta w / w) \), as shown in Appendix A. This is equal to \( (\Delta N / \Delta w)(w / N) \). The first part of this is just the slope of the labor demand curve.

In the first case \( MPN = 100 - 0.2N \), labor demand can be solved as a function of the real wage as \( N = 500 - 5w \). The slope of the labor demand curve is -5, \( w = 60 \), and \( N = 200 \). So the elasticity of labor demand is \(-5 \times (60 / 200) = -1.5\).

In the second case \( MPN = 100 - 0.9N \), labor demand is \( N = 111 1/9 - 10 / 9 \) w. The slope of the labor demand curve is \(-10 / 9 \), \( w = 10 \), and \( N = 100 \). So the elasticity of labor demand is \((-10 / 9)(10 / 100) = -1 / 9 = -0.111\).

Since the elasticity of labor demand in the first case is larger in absolute value than in the second case, labor demand in the first case is more sensitive to changes in the real wage than it is in the second case. In general, the more sensitive is labor demand to changes in the real wage, the bigger will be the drop in income due to imposition of a minimum wage. If labor demand is not very sensitive to a change in the minimum wage, labor income may even increase when the minimum wage changes, as it does in part c.

1. At any date, 25 people are unemployed: 5 who have lost their jobs at the start of the month and 20 who have lost their jobs either on January 1 or July 1. The unemployment rate is \( 25 / 500 = 5\% \).

2. Each month, 5 people have one-month spells. Every six months, 20 people have six-month spells. The total number of spells during the year is \( (5 \times 12) + (20 \times 2) = 100 \). Sixty of the spells (60\% of all spells) last one month, while 40 of the spells (40\% of all spells) last six months.

3. The average duration of a spell is \( 0.60 \times 1 \) month + \( 0.40 \times 6 \) months = 3 months.

4. On any given date, there are 25 people unemployed. Twenty of them (80\%) have long spells of unemployment, while 5 of them (20\%) have short spells.

\textbf{Number who become unemployed:}

\textbf{in not in the labor force:} 2\% of 65.2 million = 1.304 million

\textbf{in employed:} 1\% of 119.3 million = 1.193 million

| total | 2.497 million |

\textbf{Number who become employed:}

\textbf{in unemployed:} 22\% of 8.9 million = 1.958 million

\textbf{in not in the labor force:} 3\% of 65.2 million = 1.956 million

| total | 3.914 million |