Class Problem on Overlapping Generations Model of Money

1. Consider an overlapping generations model where all agents except the initial old live for two time periods. They all have an endowment of \( y > 0 \) units of perishable corn when young and nothing when old. At each date \( t \), \( N_t \) young agents are born where \( N_t = (1 + n)N_{t-1} \). Assume corn is perishable and cannot be saved. Furthermore, they all have the following utility function:

\[
U(c_1, c_2) = \beta \log c_1 + (1 - \beta) \log c_2
\]

where \( 1 > \beta > 0 \). Do not assume particular values for \( \beta, n, \) and \( y \) to solve this problem. Use the specific utility function given.

(a) Write down a condition on \( \beta \) under which a typical agent prefers to consume the bundle \( c_1 = 5, c_2 = 5 \) over the bundle \( c_1 = 10, c_2 = 1 \).

(b) Write down the per-period budget constraints facing a typical young agent if there are no savings instruments. Draw the intertemporal budget constraint. Label the axes.

(c) Write down the problem faced by a young agent at date \( t \). Write down the problem faced by an old agent at date \( t \). (Hint: is there any possibility of trade between agents here?)

(d) Compute the solution to the agent’s problem in (d).

(e) Write down the constraint faced by a social planner at date \( t \). Draw the intertemporal constraint facing the planner.

(f) What is the problem faced by a social planner?

(g) Compute the choices of \((c_1, c_2)\) that the social planner would pick for this economy. Call this \((c_{1sp}, c_{2sp})\).

(h) Are all young agents better off under the solution in (g) than in (d)? Explain.

2. Consider the same economy as above except now the initial old have printed $ \( M \). These are intrinsically worthless green pieces of paper with pictures of presidents on them. Let \( p_t \) denote the price level at \( t \). Let \( z_t \) denote the demand for money for a single young agent at \( t \).

(a) In plain english, what is meant by the statement: \( p_t \) is the price level at \( t \).

(b) Write down the per period budget constraints facing a typical agent. Draw the intertemporal budget constraint.

(c) In terms of \( v \) or \( p \), what is the expression for the real return on holding $1 between \( t \) and \( t + 1 \)?

(d) Compute the real return to holding $1.

(e) What does the agent’s intertemporal budget constraint look like? Do you notice any similarities with 1(e) above?

(f) Compute the optimal choices of \((c_1, c_2)\) that the agent would choose. Call them \((c_{1m}, c_{2m})\). Do they differ from the ones in 1(g)? Explain.

(g) Will money have value in this economy?