Econ 302

Math Review

Function: relationship among two or more variables expressed in mathematical language

\[ Y = F(X, Z, W) \quad Y = F(X) \]

\[ Y = 5X \]

How to plot this?

(0,0)

\[ X = 2 \quad Y = 10 \]

Are (2,5) (3,4) (1,3) (5,25) on this line?

![Graph showing a linear relationship between X and Y]

10

y

1 2

x
Linear functions: straight lines

General form of a linear function:

\[ y = f(x) = mx + c \]

- \( m \) is slope
- \( c \) is intercept

By how much does \( y \) go up when \( x \) goes up by 1 unit?

Positive and negative sloped lines

- Tiredness vs exercise
- Interest rates vs time
- Room temp on really cold day in Iowa vs # of hours the central heat runs
\[ y = \text{happiness} \]

\[ x = \text{hours spent playing video games} \]

Slope = \( \frac{2}{4} = 0.5 \)

Increase in hours spent playing video games increases happiness but less than proportionately

Equation depicting this relationship: \( y = 0.5 \ x \)

What if the relationship was proportional?

\[ y = x \]

More than proportional?
\[ y = 2x + 5 \]

Compare to \( y = mx + c \)

Slope = 2

Intercept = 5

Two points is enough:

\( (x = 0, y = 5) \) \( (x = 1, y = 7) \)

Shift versus movement along a line

\[ M = 2p + 2I \]

\( M, p \) : variables

Start with \( I = 1 \)

\( (p = 0, M = 2), (p = 3, M = 8) \)

Movement along a line

When \( p \) goes down from 3 to 2, \( M \) goes down from 8 to 6

When \( I \) goes up from 1 to 2

\( (p = 0, M = 4), (p = 3, M = 10) \) shift
**Trickier Example**

\[ p = -5 + 100q \quad \text{and} \quad p = 100q - 5 \]

\[
\text{Slope} = 100 \\
\text{Intercept} = -5
\]

Compare to \( y = m x + c \)

What if we wanted to draw \( p \) on the horizontal axis?

\[ p = -5 + 100q \quad \text{Rewrite as} \quad 100q = p + 5 \]

\[ \text{Or } q = \frac{p + 5}{100} \]

\[ q = \frac{p}{100} + \frac{5}{100} \quad \text{Slope} = \frac{1}{100} \]

\[ \text{Intercept} = \frac{5}{100} \]
$y = \text{happiness}$

$x = \text{hours spent playing video games}$

Non-linear functions

Pleasure from backrubs vs. time

$y = x^2$

$y = 5x^{0.5}$

$y = 2 - x^2$

$y = 3 + x^{25}$

Always a power different from 1; compare to $y = 5x + 12$

Slope keeps changing

Two points no longer enough
Pints of beer vs drunkenness

- # of days with in-laws and my irritability

Effect on health vs hours of exercise in a day

- Hours of exercise in a day
- Effect on health

Effect on grades vs # of hrs spent studying in a night

- Effect on grades
- # of hrs spent studying

Your height vs your age

- Your height
- Your age
\[ y = a + b \, x^\alpha \quad \text{a, b are constants} \]

\[ \alpha \text{ is a exponent} \]

Formula for slope (derivative)

\[ \frac{dy}{dx} = b \alpha \, x^{\alpha - 1} \]

\[ y = x^2 \]
\[ y = a + b \, x^\alpha \]
\[ a = 0 \quad b = 1 \]
\[ \alpha = 2 \]
\[ \frac{dy}{dx} = 2x \]
\[ x = 0, \, y = 0 \]
\[ x = 1, \, y = 1 \]
\[ x = 2, \, y = 4 \]
\[ x = 3, \, y = 9 \]

Solving equations in 1 unknown variable:

\[ y = 2x - 5 \]
\[ 2x = y + 5 \]
\[ x = (y + 5)/2 \]
\[ y = 25 - 3 \, x^2 \]
\[ 3x^2 = 25 - y \]
\[ x^2 = (25 - y)/3 \]
\[ x = \sqrt{\left(\frac{25 - y}{3}\right)} \]
Solving two equations in two unknowns

\[ 3y + 5x = 15 \]
\[ 2y + 3x = 10 \]

\[ 5x = 15 - 3y \]
\[ x = \frac{15 - 3y}{5} \]
\[ x = 0 \]

Exponents and powers

\[ 2 \times 2 = 2^2 \]
\[ a \times a = a^2 \]

\[ \frac{1}{2} = 2^{-1} \]
\[ \frac{a}{b} = a b^{-1} \]
\[ \frac{2x}{3y^2} = \frac{2}{3} (xy^{-2}) \]
\[ \sqrt{2} = 2^{0.5} = 2^{\frac{1}{2}} \]
\[ \sqrt{\frac{w y^2}{2q}} = \left( \frac{w y^2}{2q} \right)^{0.5} = \left( 0.5 w y^2 q^{-1} \right)^{0.5} \]
Solving non-linear equations in 1 variable

\(3x^2 = 5 \quad \rightarrow \quad x^2 = \frac{5}{3} \quad \text{Raise both to power 1/2} \quad \rightarrow \quad x = \sqrt{\frac{5}{3}}\)

\(4q^2 = 5k \quad \rightarrow \quad q^2 = \frac{5}{4}k \quad \rightarrow \quad q = \sqrt{\frac{5}{4}k}\)

\(3k^{0.3} = 5 \quad \rightarrow \quad k^{0.3} = \frac{5}{3} \quad \text{Raise both to power 1/0.3} \quad \rightarrow \quad k = \left(\frac{5}{3}\right)^{1/0.3}\)

Example:

\[
2 \times k^\alpha = 3k
\]

\[
\frac{k^\alpha}{k} = \frac{3}{2}
\]

\[
k^{\alpha - 1} = \frac{3}{2}
\]

\[
(k^{\alpha - 1})^{\frac{1}{\alpha - 1}} = \left(\frac{\frac{3}{2}}{\alpha - 1}\right)^{\frac{1}{\alpha - 1}}
\]

\[
k = \left(\frac{\frac{3}{2}}{\alpha - 1}\right)^{\frac{1}{\alpha - 1}}
\]