Problem Set # 5

1. Prof. Yue believes that high price levels are caused by a high stock of money in the economy. To test her theory, she estimates an equation of the form

\[ p_t = a + bm_t + \varepsilon_t \]

where \( \varepsilon_t \) is a random error term with a known distribution, \( p \) is the price level, and \( m \) is the stock of money. She is then interested in the estimated parameter, \( \hat{b} \). She believes that \( \hat{b} \) represents a good description of the way prices would respond to a change in the stock of money. Prof. Yue also knows that the Central bank follows a money supply rule of the form

\[ m_{t+1} = \bar{m} + \lambda m_t + \epsilon_{t+1} \]

where \( \epsilon_{t+1} \) is a random error term with a known distribution and \( \lambda \) is a policy tool available only to the Central bank. Under what conditions will the Lucas critique not apply to Prof. Yue’s work? (Hint: there is a one line answer to this question)

2. (a) Suppose a government can sell bonds and print money to finance a constant real net-of-interest deficit of \( g > 0 \) in a stationary state. Let \( b \) represent real bonds, \( m \), the real per capita money demand, \( \sigma \), the money growth rate, and \( \rho \), the gross real return on bonds. Derive the government’s budget constraint. Write down an expression for the inflation tax.

(b) Consider an OLG economy where agents have preferences \( U(c_t, c_{t+1}) = \ln c_t + \ln c_{t+1} \) and endowments \((w_1, w_2) > 0 \). The government finances a constant level of purchases, denoted by \( g \) per young person, by printing money. Denote by \( R \), the gross real interest rate on savings in this economy. Derive the steady state equation for the Laffer curve for this economy.

3. Consider the discrete-time, two-period, pure-exchange overlapping generations model where all members of a generation are identical and have the additively-separable utility function \( u(c_1, c_2) = \ln c_1 + \beta \ln c_2 \). The young and old period endowments are given by \( \omega_1 \) and \( \omega_2 \) respectively, \( \omega_1 > 0 \) and \( \omega_2 > 0 \). The population remains constant at \( N > 1 \). Initial agents (those old at \( t = 1 \)) are endowed with \( \$M \) per person, and the money supply is subsequently held constant by the government. Let \( p_t \) denote the time \( t \) price level. Let \( z_t \equiv \frac{M}{Np_t} \). Let \( r_t \) denote the gross rate of return on savings between \( t \) and \( t + 1 \). Assume \( \omega_2 < \beta \omega_1 \).

(a) Carefully write down the maximization problem faced by an young agent.
(b) Derive the optimal savings function $S_t^* = f(\omega_1, \omega_2, r_t)$.

(c) Carefully argue why $r_t = \frac{p_t}{p_{t+1}}$ should hold in equilibrium.

(d) What are the steady state equilibrium values for $z$ and $r$?

(e) Write down the equilibrium law of motion for per young person real balances in this economy. Draw the phase diagram.

(f) Which steady states are locally stable? Prove your assertion mathematically.

Now suppose that the government levies a real lump-sum tax of $\delta > 0$ on all young agents and uses the proceeds to give each old agent a real lump-sum payment of $\delta$.

(g) Carefully write down the maximization problem faced by an young agent.

(h) Derive the optimal savings function $S_t^\delta = f(\omega_1, \omega_2, r_t, \delta)$.

(i) Write down the equilibrium law of motion for per young person real balances in this economy. Draw the phase diagram.

(j) How do increases in $\delta$ affect the positive steady state value of $z$ in this case?

(k) What do you infer about the effects of a social security program on the price level for this economy? Explain.