The Baumol-Tobin Model

- A transactions theory of money demand.
- We assume that the consumer’s wealth is divided between cash on hand and savings account deposits.
- Benefit of holding money: convenience (not having to go to the bank/ATM each time you needed money); cash pays no interest.
- Cost: foregone interest in savings accounts.
- Suppose a person plans to spend $Y gradually (uniformly) over the course of a year.

Notation

- Notation:
  - $Y =$ total spending, done gradually (uniformly) over the year
  - $i =$ interest rate on savings account
  - $N =$ number of trips consumer makes to the bank to withdraw money from savings account
  - $F =$ cost of a trip to the bank (e.g., if a trip takes 15 minutes and consumer’s wage = $12/hour, then $F = $3)

Money holdings over the year

For $N = 1$:
- Money holdings
- Average = $Y/2$

For $N = 2$:
- Money holdings
- Average = $Y/4$
Money holdings over the year

<table>
<thead>
<tr>
<th>Time</th>
<th>Money holdings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td>Y/3</td>
</tr>
<tr>
<td>2/3</td>
<td>Y/6</td>
</tr>
<tr>
<td>1</td>
<td></td>
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</tbody>
</table>

\[ N = 3 \]

The cost of holding money

- In general, average money holdings = \( Y/2N \)
- Foregone interest = \( i \times (Y/2N) \)
- Cost of \( N \) trips to bank = \( F \times N \)
- Thus,
  \[
  \text{total cost} = i \times \frac{Y}{2N} + F \times N
  \]

Given \( Y, i, \) and \( F, \) consumer chooses \( N \) to minimize total cost

Finding the cost-minimizing \( N \)

- Take the derivative of total cost with respect to \( N \), then set it equal to zero:
  \[
  - \frac{iY}{2N^2} + F = 0
  \]
- Solve for the cost-minimizing \( N^* \)
  \[
  N^* = \sqrt{\frac{iY}{2F}}
  \]
The money demand function

- The cost-minimizing value of $N$: $N^* = \sqrt{\frac{iY}{2F}}$
- To obtain the money demand function, plug $N^*$ into the expression for average money holdings:

  \[
  \text{average money holding} = \sqrt{\frac{FY}{2i}}
  \]

- Money demand depends positively on $Y$ and $F$, and negatively on $i$.

Example

- Suppose a person wishes to spend $10,000 in a year. His hourly wage is $40 per hour and he spends half an hour on each of his visits to the bank. The nominal interest rate on his savings account at the bank is 10%.
- How often should he go to the bank?
- On average, how much money should he carry on him?

The money demand function

- The Baumol-Tobin money demand function:

  \[
  \left(\frac{M}{P}\right)^d = \sqrt{\frac{FY}{2i}} = L(i,Y,F)
  \]

  - automatic teller machines
  - internet banking
  - wages (higher wages increase the opportunity cost of time spent visiting the bank)
  - bank or brokerage fees

- Ignore $F$: Then money demand rises with $Y$ and falls with $i$.

Aggregate money demand

- For the whole economy, we summarize the demand for money by a function $L$ called the money demand function

  \[
  \frac{M}{P} = L(Y,r)
  \]

  - $M/P = \text{aggregate demand for money (nominal in \$)}$
  - $P = \text{the price level}$
  - $Y = \text{real GDP}$
  - $i = \text{real interest rate earned by other assets}$
  - $L = \text{a function relating } Y \text{ and } r \text{ to money demand}$
The money demand function

\[
\left( \frac{M}{P} \right)^d = L(r, Y)
\]

\(\left( \frac{M}{P} \right)^d\) = real money demand, depends
  - negatively on \(r\)
    - \(r\) is the real opportunity cost of holding money
      - \(r = i - \pi\) where \(r\) = real interest rate,
      - \(i\) = nominal interest rate, \(\pi\) = inflation rate
  - positively on \(Y\)
    - higher \(Y\) ⇒ more spending
      - ⇒ so, need more money

Equilibrium in the money market

\[
\frac{M^s}{P} = \left( \frac{M}{P} \right)^d = L(r, Y)
\]

Equilibrium in the money market gives us the other equation connecting \(r\) and \(Y\).

LM curve

- Fix money supply at \(M^s = M\) and fix prices at \(P\).
- Then \(\pi = 0\)
- Then \(M/P = L(Y, r) = hY - q r\)
- where \(M, P\) are constants, \(h\) and \(q\) are parameters, \(r\) & \(Y\) are only variables

LM curve

- \(r = (h/q)Y - (M/P)/q\)
- LM curve; positively sloped