Practice Questions (hints)

1. Suppose the production function in Ghana is given by

$$Y = AK^{\alpha}L^{1-\alpha}$$

and $\alpha = 0.3$.

(a) Show that ceteris paribus, a 10% increase in the labor force leads to about a 6.9% increase in GDP.
`initially we have`

$$Y_{\text{before}} = AK^{\alpha}L^{1-\alpha}$$

`after the increase we have`

$$Y_{\text{after}} = AK^{\alpha}(1.1L)^{1-\alpha}$$

`so`

$$\frac{Y_{\text{after}}}{Y_{\text{before}}} = \frac{AK^{\alpha}(1.1L)^{1-\alpha}}{AK^{\alpha}L^{1-\alpha}} = (1.1)^{1-\alpha} = (1.1)^{0.7} = 1.069$$

(b) Show that ceteris paribus, a 10% increase in the capital stock leads to about a 2% increase in GDP.
`initially we have`

$$Y_{\text{before}} = AK^{\alpha}L^{1-\alpha}$$

`after the increase we have`

$$Y_{\text{after}} = A(1.1K)^{\alpha}L^{1-\alpha}$$

`so`

$$\frac{Y_{\text{after}}}{Y_{\text{before}}} = (1.1)^{\alpha} = (1.1)^{0.3} = 1.029$$

(c) Show that ceteris paribus, a 10% increase in TFP leads to a 10% increase in GDP.

(d) What do you conclude from your answers to (a), (b), and (c)?
`Because of diminishing marginal productivity of labor and capital, increases in labor and capital do not lead to proportionate increases in output. The effect of an increase in the labor force is more than that of capital. TFP increases have proportionate effects.`

2. Suppose the production function in Thailand is given by

$$Y = AK^{\alpha}L^{1-\alpha}$$

and $\alpha = 0.3$.

(a) Ceteris paribus, suppose L went up from 1 to 2. By how much does Y go up by?
initially we have

$$Y_{\text{before}} = AK^{\alpha}(1)^{1-\alpha}$$

after the increase we have

$$Y_{\text{after}} = AK^{\alpha}(2)^{1-\alpha}$$

so

$$\frac{Y_{\text{after}}}{Y_{\text{before}}} = \frac{AK^{\alpha}(2)^{1-\alpha}}{AK^{\alpha}(1)^{1-\alpha}} = (2)^{1-\alpha} = (2)^{0.7} = 1.62$$

so output goes up by 1.62

(b) Ceteris paribus, suppose L went up from 2 to 3. By how much does Y go up by?
initially we have

$$Y_{\text{before}} = AK^{\alpha}(2)^{1-\alpha}$$

after the increase we have

$$Y_{\text{after}} = AK^{\alpha}(3)^{1-\alpha}$$

so

$$\frac{Y_{\text{after}}}{Y_{\text{before}}} = \frac{AK^{\alpha}(3)^{1-\alpha}}{AK^{\alpha}(2)^{1-\alpha}} = \left(\frac{3}{2}\right)^{0.7} = 1.32$$

so output goes up by 1.32; so rate of increase in output has fallen from 6.2% to 3.2%.

(c) Ceteris paribus, suppose L went up from 3 to 4. By how much does Y go up by?
initially we have

$$Y_{\text{before}} = AK^{\alpha}(3)^{1-\alpha}$$

after the increase we have

$$Y_{\text{after}} = AK^{\alpha}(4)^{1-\alpha}$$

so

$$\frac{Y_{\text{after}}}{Y_{\text{before}}} = \frac{AK^{\alpha}(4)^{1-\alpha}}{AK^{\alpha}(3)^{1-\alpha}} = \left(\frac{4}{3}\right)^{0.7} = 1.22$$

so output goes up by 1.22; so rate of increase in output has fallen from 3.2% to 2.2%.

(d) What do you conclude from your answers to (a), (b), and (c)? Also plot Y against L holding A and K fixed.

3. An economy has the production function

$$Y = 0.2(K + \sqrt{N})$$

In the current period, $K = 100$ and $N = 100$.

(a) Graph the relationship between Y and K holding L fixed at its current value. What is the MPK? Does MPK diminish?

(b) Graph the relationship between Y and L holding K fixed at its current value. Find the MPN when L goes up from 100 to 110. Compare this result with the MPN for an increase in L from 110 to 120. Does MPN diminish?