Optimal Monetary and Exchange Rate Policies in Crisis-Prone Small Open Economies*

Joydeep Bhattacharya¹, Rajesh Singh²

¹ Iowa State University; e-mail: joydeep@iastate.edu
² Iowa State University; e-mail: rsingh@iastate.edu

January 31, 2005 (Very Preliminary Draft)

**Abstract** Banking crises are fairly ubiquitous events with large adverse consequences. This paper studies the potential for banking crises and their effects on exchange rate policies in a microfounded general equilibrium dynamic model in the tradition of Diamond and Dybvig (1983) and Chang and Velasco (2000). More specifically, we produce a small open economy pure exchange overlapping generations model with random relocation along the lines of Smith (2002). The combination of random relocation and the assumed role of currency in interlocation trade creates random location - and country - specific liquidity needs. Banks naturally arise to provide much-desired insurance against these liquidity shocks. In this setting, if withdrawal demand for the bank’s deposits is high enough, the bank will exhaust all its cash reserves and a banking crisis will occur. We provide a complete characterization of optimal interest rate policies in this setting. We find that nominal interest rates that are desirable from a welfare perspective may be associated with a high probability of banking crises. We go on to study the classic issue of the relative desirability of fixed versus flexible exchange rate regimes by introducing time-varying random endowments into the above structure. This makes the banks' portfolio allocations dependent on the exchange rate regime. Under a fixed exchange rate regime, by interest rate parity, the banks’ portfolio choice is deterministic and constant over time and this is supported by the injection/removal of nominal balances by the central bank. Under a flexible exchange rate regime, the money supply stays constant but a rate-of-return uncertainty emerges that is in addition to the income uncertainty common to both regimes. Our results show that a flexible exchange rate regime is superior in an ex-ante welfare sense relative to the fixed exchange rate regime; however, the ordering is reversed when it comes to the likelihood of banking and currency crises.

* Very preliminary draft prepared for submission to conferences. The latest version can be downloaded from http://www.econ.iastate.edu/faculty/bhattacharya/work.htm
1 Introduction

Banking crises, or more generally, substantial banking sector woes, are by no means rare. Lindgren et al. (1996) have reported that over the 1980–96 period at least two-thirds of IMF member countries experienced significant banking sector problems. Some like Goldstein and Turner (1996) have additionally argued that the incidence of banking crises in the 1980s and 1990s has been significantly higher than in the 1970s, and much higher than in the 1950s and 1960s. The increased frequency and seriousness of these crises deserve attention for at least two reasons. First, as is well-known, they are often accompanied by huge resolution costs, current and future output losses, and inevitable implications for the future conduct and scope of fiscal and monetary policy. Perhaps more importantly, as international financial markets become more integrated, the fallout on other countries, especially close trading partners, is becoming nontrivial.

This paper studies the potential for banking crises and their concomitant domestic and international repercussions. The principal foci, as in Chang and Velasco (2000), are on the liquidity-provision and maturity transformation functions of banks. As is well-known, at least since Diamond and Dybvig (1983), banking involves juggling assets and liabilities with different maturity structures. A bank’s liabilities are typically short-term deposits, while most of its assets are usually held in long-term, less liquid investments. While the maturity transformation activity produces major social benefits, it also exposes banks to the possibility of a liquidity shortage. In our setting, banks face uncertain liquidity demand from depositors. When such demand is “sufficiently high”, banks run out of cash reserves, and a crisis (liquidity shortage) occurs. In an small open economy setting with free financial flows, changes in domestic as well as world interest rates will determine the opportunity cost of liquidity, thereby influencing the possibility of a banking crisis.

More specifically, we produce a single good, pure exchange overlapping generations, small open economy model, that is a marriage of Smith (2002) with Betts and Smith (1997). In our set up, there are two spatially separated locations in the domestic economy. Agents are initially assigned to a location with a country. They receive a fixed non-stochastic endowment only when young and care only about their consumption when old. Near the end of a period, a fixed (random) fraction of these agents must move to the other locations within the country (to the rest of the world). The only real asset is a storage technology which, unlike in Smith (2002), cannot be prematurely liquidated. The only asset that may be transported across locations is currency (goods are immobile). The domestic central bank issues its own fiat currency whereas the foreign currency can be obtained from the rest of the world. Each currency is dominated in its return by storage. The combination of random relocation and the assumed function of currency in interlocation exchange creates random location – and country – specific liquidity needs. Banks
naturally arise to provide much-desired insurance against the possibility of relocation. At the start of any period, they take in deposits and divide their portfolio between domestic/foreign currency and storage. Once the “relocation shock” is realized, they provide payments to depositors that are contingent on their relocation status.

As originally studied by Champ, Smith, and Williamson (1996) and more recently by Smith (2002), such a setting is conducive to the occurrence of banking crises. Indeed, if withdrawal demand for the bank’s deposits is high enough, the bank will exhaust all its cash reserves; such suspension of cash payments to depositors is what characterizes a banking crisis according to Noyes (1909). It is here that the international dimension added to Smith (2002) starts to show some action. Notice that the choice of the optimal portfolio for banks depends crucially on the return to domestic and foreign currencies, which, in turn, are determined by the respective countries’ money growth rates.

To fix ideas, let us fix our attention on the case that we study extensively below, one where a fixed known fraction of agents in the home and foreign country move within locations in that country but a random fraction of agents in the home country move to the foreign country; no one from the foreign country moves to the home country. In this setting, when the stationary nominal interest in the home country \( I \) is higher than that in the foreign country \( I^* \), i.e., \( I > I^* \) (when domestic funds have a higher opportunity cost than foreign funds), we can show that home banks use up all domestic currency reserves to pay the domestic movers. Non-movers get a higher return than movers within the home country. When the realized fraction of movers to the foreign nation is below an endogenous threshold, it is not efficient to exhaust all reserves of the foreign currency on those few movers. Instead, it makes sense for the bank in the home country to equalize the ex-post returns to foreign movers and non-movers which they can achieve by selling some of its reserves of foreign currency and paying the non-movers in goods in the next period. Above the aforementioned threshold, all the foreign reserves of the home banks are used up to pay the foreign movers. The latter get a lower return than the non-movers. As long as the ex-post rate of return is equalized between Foreign movers and non-movers, both obtain a higher rate of return relative to home-movers. However, as the number of foreign movers increases the rate of return to foreign movers is lowered while that of non-movers is higher. If too many foreign movers are realized then their rate of return may even fall below Home movers\]. As discussed in Smith (2002) and Champ, Smith, and Williamson (1996), this situation can be labelled a “banking crisis”/currency crisis. Furthermore, assuming a uniform distribution for the foreign relocation probability, we can exactly characterize the portfolio weights for foreign and domestic currency. In particular, we can show that the higher is \( I^* \), the lower is the portfolio weight on foreign currency, and lower is the threshold foreign relocation probability at which
banks exhaust all their foreign cash reserves. In short, the higher the opportunity cost of foreign funds, the higher is the probability of a banking crisis. This result is in line with the empirical evidence on banking crises as presented in Eichengreen and Rose (1998) where they find that a one percent increase in the “Northern” interest rates “is associated with an increase in the probability of Southern banking crises of around three percent.”

When $I \ll J^*$ obtains, matters are substantially different; indeed, there are two endogenous thresholds for the foreign relocation probability that we have to contend with. Below the first threshold, the bank is able to provide complete insurance; non-movers, foreign and domestic movers all receive the same return ex-post. This requires banks to not use up all their home currency reserves to pay the domestic movers. Instead, some domestic currency is used to buy goods for the non-movers. Similarly, not all the foreign reserves of the home banks are used up to pay the foreign movers. Once the first threshold is crossed, the banks can no longer protect the foreign movers even as they end up exhausting all their foreign currency reserves. At this point, the non-movers and the domestic movers continue to earn the same return, a return that is higher than what the banks can offer the foreign movers. After the second threshold has been reached, the return to the foreign movers gets lower and lower; the return to non-movers will keep increasing as the same amount of storage is now being divided among lesser and lesser number of non-movers. In turn now, it may not be necessary to transfer domestic currency cash reserves to non-movers in the same amount as before. Banks exhaust all domestic currency reserves to pay the domestic movers whose return is now lower than what the non-movers get.

We find that the portfolio weight attached to domestic (foreign) currency falls (rises) with the domestic nominal interest rate. Interestingly, the portfolio weight attached to domestic currency is higher than the known fraction of home movers implying that banks optimally hold more of the domestic currency than they need solely for the purposes of paying the domestic movers. We can also show that the probability of a banking crisis is increasing in the domestic nominal interest rate. However, ex ante welfare, in general equilibrium, increases as the domestic nominal interest rate increases. This result underscores a tension between welfare and crisis probabilities first recognized in Smith (2002): while monetary policy can reduce the crisis probability by reducing the nominal interest rate (and approach the Friedman rule), such policies may hurt aggregate welfare.

We go on to study the classic issue of the relative desirability of fixed versus flexible exchange rate regimes. To that end, we introduce time-varying random endowments into the above structure. This makes the banks’ portfolio allocations dependent on the exchange rate regime. Under a fixed exchange rate regime, by interest rate parity, the banks’ portfolio choice is deterministic and constant over time and is supported by the central bank injecting or withdrawing nominal
balances. This in turn generates a novel inter-generational income redistribution. Under a flexible exchange rate regime, the money supply stays constant (hence no income redistribution) but a rate-of-return uncertainty emerges that is in addition to the income uncertainty common to both regimes. Our results show that a flexible exchange rate regime is welfare superior relative to fixed exchange rate. However, the ordering is reversed when it comes to banking and currency crisis.

To get an intuitive albeit informal sense of this, we start by noting that under the flexible rate regime, the exchange rate adjustments and consequent interest rate changes induce banks to tilt their portfolios towards the highest paying assets. Suppose for instance that markets anticipate a low devaluation in any period. Banks will then tilt their portfolio towards domestic reserves, which in turn will induce price level to fall. As a result, the expected devaluation will now be lower. Thus, in equilibrium, exchange rate adjustments will be muted due to changes in portfolio demand. Thus, even though the flexible exchange rate regime suffers from the rate-of-return uncertainly, the price mechanism ensures that the range of fluctuations is diminished.

Fixed exchange rate regime, on the other hand, eliminates rate-of-return uncertainty, but the inefficiency that stems from income redistribution makes it worse. Essentially, the exchange-rate adjustment mechanism along with portfolio choice implies that banks will be allocating their assets in the most efficient way. However, note that, despite these adjustments the domestic assets sometimes may become too attractive, and banks will put less weight on foreign currency reserves under such cases. As a result, the probability of crisis under flexible exchange rates is higher than under fixed exchange rates.

Our work is one in a line of papers that follow the original modeling insights of Diamond and Dybvig (1983) and create microfounded general equilibrium structures in which banks arise “naturally” and are susceptible to crises that look real. The paper that is closest in spirit to our work is Chang and Velasco (2000). There the authors study a small open Diamond-Dybvig economy with three periods where a subset of agents (the “patient”) derive direct utility from holding the domestic currency. We make progress by “generalizing [their] results to truly dynamic settings and to more satisfactory specifications of money demand.” (Chang and Velasco, 2000). Additionally, a major advantage of our analysis is that we can quantify the exact probability of a banking crisis; Chang and Velasco (2000) are only able to show conditions under which a banking crisis will occur. A point of contrast with their results is the following. Chang and Velasco find that “a flexible rate system implements the social optimum and eliminates runs” while we find that the probability of crisis under flexible exchange rates is not zero and indeed is higher than under fixed exchange rates.
2 The Model

2.1 The environment

Consider a pure exchange single good world with two countries named Home and Foreign, similar in many respects to that studied in Betts and Smith (1997). Each country has two symmetric spatially separated locations. Each economy (and each location) is the home of an infinite sequence of two-period lived overlapping generations. At each date \( t = 1, 2, 3, \ldots \), a continuum of ex ante identical young agents of unit mass is born. A young agent in either country receives an endowment of \( w \); old agents receive no endowment. Agents care only about their second period consumption \( (c) \) ordered by \( \ln(c) \). In what follows, the variables with (without) asterisk denote Foreign (Home) variables.

Goods are assumed to be immobile between locations and countries but young agents may move in a manner described below. At the start of each date, newly born agents are allotted to a country and a location within it. Towards the end of the period, after asset markets shut, some young agents will move across locations within a country and some with have to travel across countries. Specifically, let \( \pi (\pi^*) \) be the probability of within-country relocation faced by a young agent and \( \lambda (\lambda^*) \) denote the fraction of young agents who have to move abroad. The rest stay put (the “non-movers”). To keep analytics simple, we posit that \( \pi, \pi^*, \) and \( \lambda^* \) are known and constant. However, \( \lambda \) is a random variable with a known distribution \( f(\lambda) \); specifically, in much of what we present below, we assume that \( \lambda \) is uniformly distributed over \([0, \bar{\lambda}]\) where \( \bar{\lambda} \leq 1 - \pi \).

Following Smith (2002), the only real asset is a commonly available (in both countries) storage technology: \( \mathcal{F}(\kappa) = \rho \kappa \), i.e., 1 unit invested in this technology yields a sure gross real return of \( \rho \) the following period. Recall relocated agents can not transport goods to their new homes. Within a country, agents can carry the domestic currency to the other location and subsequently exchange for consumption goods. Cross-country relocation requires agents to carry with them the currency of the country they will be relocating to. Unlike Smith (2002), we disallow investments in storage to be liquidated (scrapped) even at a cost. Before proceeding further, some definitions and notation are in order:

\[
 m_t \equiv \frac{M_t}{p_t}, \quad m_t^* \equiv \frac{M_t^*}{p_t^*}, \quad I_t \equiv \frac{p_{t+1}}{p_t} \rho, \quad I_t^* \equiv \frac{p_{t+1}^*}{p_t^*} \rho, \quad (1)
\]

where \( M \) and \( p \) denote nominal balances and price level, respectively; \( e \) is the nominal exchange rate denominated in units of Home currency per unit of Foreign currency. We assume that Banks can exchange foreign currency with goods in the Foreign country and can transport it to Home.
with no cost. As a result \( p = e p^* \), i.e., the real exchange rate \( x = \frac{e p^*}{p} = 1 \), for all \( t \). Then,

\[
I_t = I_t^* \frac{p_{t+1}}{p_{t+1}} = I_t^* \frac{e_{t+1}}{e_t} = I_t^* (1 + \varepsilon_{t+1})
\] (2)

where \( \varepsilon_{t+1} \) denotes the rate of exchange rate devaluation between \( t \) and \( t + 1 \). Henceforth, we will focus on a situation where \( I_t \geq 1 \) and \( I_t^* \geq 1 \) hold, implying that storage dominates the real return to holding either currency.

The role of banks is motivated as follows. At the start of a period, young agents at any location receive their endowment at which point asset markets open. Agents make decisions on how much currency and how much storage to hold. Asset markets close at this point. The relocation shock is subsequently revealed. At this point, agents who are relocated within and across nations will need to acquire the appropriate currency. Only the currency of their destination country is valuable to them; all their other holdings of other assets, for lack of an open market for trade, is worthless to them. In short, agents in this economy face considerable risks associated with relocation. We assume competitive banks arise to supply insurance to these risk averse agents. It is will evident that all young agents will choose to deposit their entire endowment with a bank. As Nash competitors on both the deposit and the asset sides, and aided by the law of large numbers, banks will make portfolio choices about storage and the two currencies so that these choices maximize ex-ante expected utility of a representative depositor subject to balance sheet constraints described below.

2.2 The Home bank’s problem

We start by studying the problem of portfolio selection faced by home banks. As all agents are ex-ante identical, they deposit their endowment at banks who in turn make portfolio allocations between home currency, foreign currency, and storage as given by the per depositor resource constraint

\[
w + \tau_t \geq m_{ht} + m_{ft} + s_t;
\] (3)

here \( \tau_t = \frac{M_t - M_{t-1}}{p_t} \) denotes the net government transfer (seigniorage) to the young agents in the home country, \( m_h \) and \( m_f \) denote the real value of home and foreign cash reserves respectively, and \( s \) denotes storage. No transfers are made to the old. Also notice that the government transfer goes only to the domestic agents even though part of the government’s seigniorage was collected from foreigners holding the home currency. We will have more to say on this later in the paper.

For future reference, define

\[
\gamma_h \equiv \frac{m_h}{w + \tau}, \quad \gamma_f \equiv \frac{m_f}{w + \tau}.
\] (4)
Then $\gamma_h$ ($\gamma_f$) represents the domestic (foreign) currency reserve to deposit ratio of the home banks. Notice that $1 - \gamma_f - \gamma_h = \frac{s}{w+\tau}$ must hold (this is the portion invested in real storage).

Banks, at the start of a period, and upon receiving deposits decide what their $\gamma_h$ and $\gamma_f$ should be. At this point, asset markets close and towards the end of the period, the relocation shock is revealed. At that point, the banks will have to pay domestic (foreign) movers in domestic (foreign) currency. Much will depend on the exact realization of that shock. As such, in this environment, unlike in Betts and Smith (1997), banks announce a state-contingent schedule of returns for the three types of agents. Let $r_h(\lambda)$ and $r_f(\lambda)$ denote the gross real returns promised to agents relocated domestically and abroad, respectively; let $r(\lambda)$ denote the gross real return offered to domestic depositors who are not relocated (hereafter, the non-movers). Finally let $\alpha(\lambda)$ denote the state-contingent fraction of foreign currency reserves (per depositor) that is used to pay foreign movers; similarly, let $\beta(\lambda)$ denote the state-contingent fraction of domestic currency reserves (per depositor) that is used to pay domestic movers. State-contingent returns must satisfy (see Appendix ??)

$$
\lambda_t r_f(\lambda_t) \leq \alpha(\lambda_t) \gamma_f \left( \frac{p_t}{p_{t+1}} \frac{e_{t+1}}{e_t} \right), \quad (5a)
$$

$$
\pi r_h(\lambda_t) \leq \beta(\lambda_t) \gamma_h \frac{p_t}{p_{t+1}}, \quad \text{and} \quad (5b)
$$

$$
(1 - \pi - \lambda_t) r(\lambda_t) \leq (1 - \alpha(\lambda_t)) \gamma_f \left( \frac{p_t}{p_{t+1}} \frac{e_{t+1}}{e_t} \right) + (1 - \beta(\lambda_t)) \gamma_h \frac{p_t}{p_{t+1}} + (1 - \gamma_f - \gamma_h) \rho. \quad (5c)
$$

These constraints have standard interpretations. Consider as an example, constraint (5a). Once $\lambda$ is realized, the banks becomes aware that it has to pay $r_f(\lambda_t)$ to each of these foreign movers. It uses up a fraction $\alpha$ of its per depositor foreign currency reserves ($\gamma_f$) for this purpose. Foreign movers each receive $\alpha(\lambda_t) \gamma_f$ units of foreign currency which when brought to their final destination in the following period yields $\frac{p_t}{p_{t+1}} \frac{e_{t+1}}{e_t}$ per unit in the form of goods. The constraint (5b) is simpler. Banks know $\pi$ upfront and so they know in advance that a fraction $\pi$ of their clientele will move domestically and will have to be given $r_h(\lambda_t)$. They can finance this payout by (partially) using their reserves of domestic currency, $\beta(\lambda_t) \gamma_h$. In the new location, this cash is worth $\beta(\lambda_t) \gamma_h \frac{p_t}{p_{t+1}}$ in goods.

As noted earlier, banks are Nash competitors in the deposit market, which implies that they maximize the expected utility of a young agent. The banks’ problem can be stated as

$$
\max_{\gamma_f, \gamma_h, \alpha, \beta} \int_0^1 \left[ \pi \ln r_h(\lambda) + \lambda \ln r_f(\lambda) + (1 - \pi - \lambda) \ln r(\lambda) \right] f(\lambda) d\lambda \quad (6)
$$

subject to (3) - (5c) and non-negativity of $\gamma_f, \gamma_h, \alpha, \text{and } \beta$. Moreover, none of these choices can exceed unity.
Recall that the portfolio allocations \( \{\gamma_h, \gamma_f\} \) are made ex-ante. Thus, for stationary government policies the variables defined in (1) will have a stationary path. Then, \( \gamma_h \) and \( \gamma_f \) will be time-invariant and, as these are chosen ex-ante, will also be independent of the realization of \( \lambda \). Thus, in what follows, we drop time subscripts to simplify notation. Then using (1) and (4), constraints (5a) - (5c) can be rewritten more compactly as

\[
\begin{align*}
    r_f (\lambda) &= \frac{\alpha (\lambda)}{\lambda} \frac{\rho}{I}, \\
    r_h (\lambda) &= \frac{\beta (\lambda)}{\pi} \frac{\rho}{I_f}, \\
    r (\lambda) &= \frac{\rho}{1 - \pi - \lambda} \left[ (1 - \alpha (\lambda)) \frac{\gamma_f}{I_s} + (1 - \beta (\lambda)) \frac{\gamma_h}{I} + (1 - \gamma_f - \gamma_h) \right],
\end{align*}
\]

where, after invoking optimality, it is assumed that (5a)-(5c) will hold with equality. To reiterate, banks maximize (33) subject to (3) and (7a) - (7c).

It is convenient to conceptualize the bank’s problem as a two stage problem and work backwards: 1) in the second stage, given \( \gamma_h \) and \( \gamma_f \), the banks choose \( \alpha (\lambda) \) and \( \beta (\lambda) \) for each \( \lambda \), and 2) given the schedules \( \alpha (\lambda) \) and \( \beta (\lambda) \), they choose \( \gamma_h \) and \( \gamma_f \) so as to maximize (33). Herein lies the seed of much of what is to come. Banks choose \( \gamma_h \) and \( \gamma_f \) by maximizing ex-ante expected utility. Since agents are risk-averse, the bank chooses \( \gamma_h \) and \( \gamma_f \) to try and provide complete insurance against the upcoming relocation shock. It faces two tensions. On the one hand, since the return on cash is lower than that on storage, the bank will want to economize on cash holdings. However, since storage cannot be liquidated, and insurance provision requires cash, the bank does not want too illiquid a portfolio. Since asset markets close right after these choices have been made, the bank is “stuck” with its choices as it awaits the realization of \( \lambda \). Ex post, the bank may or may not succeed in providing complete insurance. Too high a withdrawal demand will compromise this function and precipitate a crisis.

We start by focusing on the second stage problem. It is easy to verify that the first-order-condition for the choice of \( \alpha (\lambda) \) is:

\[ r (\lambda) \geq r_f (\lambda), \quad \alpha = 1 \text{ if } \alpha (\lambda) < 1 \]  

Equation (8) states that agents relocated abroad will get the same return as those staying put, only if there is sufficient foreign cash reserves in stock for the emigrants. Observe that if the (ex-post) realized \( \lambda \) is low, distributing the entire stock of foreign currency reserves equally among the movers will be inefficient. The best feasible allocation in that situation is one that gives equal consumption to foreign emigrants and the non-movers. On the other hand, if a sufficiently large fraction of population must relocate abroad (i.e., ex-post \( \lambda \) is high), given that the stock of foreign currency reserves is fixed, each mover will get a low return (one lower than non-movers).
Of course, all of the reserves will have to be exhausted and \( \alpha(\lambda) = 1 \) would obtain. For future reference note that using (1), (2), (7a), and (7c), equation (8) can be rewritten as

\[
\frac{\rho}{1 - \pi - \lambda} \left[ \frac{(1 - \alpha(\lambda)) \gamma_f}{I^*} + \frac{(1 - \beta(\lambda)) \gamma_h}{I} + (1 - \gamma_h - \gamma_f) \right] \geq \frac{\alpha(\lambda) \rho \gamma_f}{\lambda I^*}, \quad \text{"} = j \text{ if } \alpha(\lambda) < 1
\]

Similarly, the optimal choice of \( \beta(\lambda) \), i.e., how much of home currency reserves to pay out, yields:

\[
r(\lambda) \geq r_h(\lambda), \quad \text{"} = j \text{ if } \beta(\lambda) < 1
\]

Equation (10) has an interpretation similar to that of (8). Of course, in this case, since \( \pi \) is known, the optimal rule for \( \beta \) is not state contingent. Again, using (1), (2), (7b), (7c), equation (10) can be rewritten as

\[
\frac{\rho}{1 - \pi - \lambda} \left[ \frac{(1 - \alpha(\lambda)) \gamma_f}{I^*} + \frac{(1 - \beta(\lambda)) \gamma_h}{I} + (1 - \gamma_h - \gamma_f) \right] \geq \frac{\beta(\lambda) \rho \gamma_h}{\pi I^*}, \quad \text{"} = j \text{ if } \beta(\lambda) < 1
\]

What does the bank do with any undistributed reserves of currency? It uses them to pay non-movers; it goes to the money market in the following period and sells the currency and gives the non-movers goods in exchange. Limited communication prevents the bank from using these reserves to pay movers. Obviously, the decision to exhaust reserves of any currency (as evident from (9) or (11)) depends on how “good” one currency is relative to the other. For instance, if the home currency is relatively weak, banks would prefer to use its home currency reserves before touching anything else.

The above discussion makes clear that the ex-post (and therefore, ex-ante) allocation rules will depend on the relative magnitudes of \( I \) and \( I^* \). We first show that when \( I > I^* \), the allocations to agents relocated domestically are invariant to any other parameter.

### 2.3 The case of \( I > I^* \)

When \( I > I^* \) obtains, the home currency is dominated by both the foreign currency as well as storage. Banks will therefore economize on its holdings of the home currency. Since \( \pi \), the fraction of domestic movers, is known, we conjecture that banks will hold exactly only that amount of the home currency as is needed to pay the domestic movers. Indeed our conjecture is that banks will find it in their best interest to exhaust its holdings of the dominated home currency before “liquidating” other assets. We start by computing the equilibrium (partial) function for \( \alpha \) that describes the optimal use of the bank’s foreign currency reserves.
Lemma 1 Suppose $I > I^*$. Then

$$\alpha(\lambda) = \begin{cases} \frac{\lambda}{1-\pi} \left[ 1 + \frac{(1-\gamma_h-\gamma_f)}{\gamma_f} I^* \right] & \text{if } \lambda \leq \hat{\lambda}, \\ 1, & \text{if } \lambda > \hat{\lambda}, \end{cases} \tag{12}$$

where

$$\hat{\lambda} \equiv \frac{(1-\pi)\gamma_f}{\gamma_f + (1-\gamma_h-\gamma_f) I^*} \tag{13}$$

As discussed above, efficiency considerations require the bank to equalize the payouts to the emigrants and the non-movers. When $\lambda$ is small, there are too few people relocating abroad and the bank is holding “too much” of foreign currency reserves (and arguably “too little” storage). Clearly, it is not efficient to distribute the entire stock of these reserves to the few emigrants. As such, the bank holds onto $(1-\alpha)$ of its reserves which it converts to goods and pays non-movers the following period. As $\lambda$ rises, the bank needs more and more of its reserves to pay the foreign movers the same as the non-movers. Eventually an endogenous threshold level of foreign currency liquidity demand given by $\hat{\lambda}$ is reached. Beyond this, the bank exhausts all its foreign currency reserves in paying out the foreign movers who now get a smaller payout than the non-movers will (“too many” movers, not enough reserves to pay each of them what a non-mover will get).

For future reference, note that as $\hat{\lambda}$ falls, the probability of a banking crisis (bank exhausting its currency reserves) increases.

We can compute the state-contingent returns offered by the banks. Using (12) in (7a) yields

$$r_f(\lambda) = \begin{cases} \frac{\gamma_f I^*}{\lambda} \rho, & \text{if } \lambda \leq \hat{\lambda}, \\ \frac{1}{\lambda} \rho \gamma_f I^* & \text{if } \lambda > \hat{\lambda}, \end{cases} \tag{14}$$

Further, using (12) in (7c) yields

$$r(\lambda) = \begin{cases} \frac{\rho \gamma_f}{\lambda}, & \text{if } \lambda \leq \hat{\lambda}, \\ \frac{1}{\lambda} (1-\gamma_h-\gamma_f) \rho, & \text{if } \lambda > \hat{\lambda}. \tag{15} \end{cases}$$

Of course, using Lemma 1, it is clear from (7b) that

$$r_h(\lambda) = \frac{\rho}{\pi \gamma_h}, \tag{16}$$

which is not state dependent. Then, using (16), (14), and (15) in (33), along with some rearrangement yields

$$W = \int_0^\lambda (1-\pi) \ln \left( \frac{1}{\lambda} \frac{\rho \gamma_f}{I^*} \right) f(\lambda) d\lambda + \int_\lambda^\hat{\lambda} \lambda \ln \left( \frac{1}{\lambda} \frac{\rho \gamma f}{I^*} \right) f(\lambda) d\lambda$$

$$+ \int_\hat{\lambda}^\lambda (1-\pi - \lambda) \ln \left( \frac{1-\gamma_h-\gamma_f}{1-\pi - \lambda} \rho \right) f(\lambda) d\lambda + \int_\lambda^\hat{\lambda} \pi \ln \left( \frac{\rho \gamma_h}{I^*} \right) f(\lambda) d\lambda \tag{17}$$
The ex-ante choice of $\gamma_f$ is derived by maximizing the expression for stationary welfare in (17). Some algebra yields

$$\frac{\hat{\lambda}(1 - \pi)}{\gamma_f^2 + 1 - \gamma_h - \gamma_f} \left( \frac{1}{I^*} - 1 \right) + \frac{\hat{\lambda}^2 - \hat{\lambda}^2}{2\gamma_f} - \frac{1}{1 - \gamma_h - \gamma_f} \left( \hat{\lambda} - \hat{\lambda} \right) \left( 1 - \pi - \frac{\hat{\lambda} + \hat{\lambda}}{2} \right) = 0 \quad (18)$$

which simplifies to

$$\frac{\hat{\lambda}^2 + \hat{\lambda}^2}{2\gamma_f} = \frac{\hat{\lambda}^2}{\gamma_f I^*} + \frac{\hat{\lambda} - \hat{\lambda}}{(1 - \gamma_f - \gamma_h)} \left[ 1 - \pi - \frac{1}{2} \left( \hat{\lambda} + \hat{\lambda} \right) \right]. \quad (19)$$

Also, $\gamma_h$ is similarly computed from

$$-\frac{\hat{\lambda}(1 - \pi)}{\gamma_f^2 + 1 - \gamma_h - \gamma_f} - \frac{1}{1 - \gamma_h - \gamma_f} \left( \hat{\lambda} - \hat{\lambda} \right) \left( 1 - \pi - \frac{\hat{\lambda} + \hat{\lambda}}{2} \right) + \frac{\hat{\lambda} \pi}{\gamma_h} = 0 \quad (20)$$

which further simplifies to

$$\frac{\hat{\lambda} \gamma_h}{\gamma_f} = \frac{\hat{\lambda}^2}{\gamma_f I^*} + \frac{\hat{\lambda} - \hat{\lambda}}{(1 - \gamma_f - \gamma_h)} \left[ 1 - \pi - \frac{1}{2} \left( \hat{\lambda} + \hat{\lambda} \right) \right] \quad (21)$$

Recall from (13) that $\hat{\lambda}$ is itself a function of $\gamma_f$ and $\gamma_h$. It turns out that combining (19) and (21) produces a quartic equation. Simultaneous solution of (19) and (21) yields the optimal $\gamma_f$ and $\gamma_h$. We collect some information about these in the following lemma.

**Lemma 2**

a) $\gamma_h = \pi$,

b) $\hat{\lambda} < \hat{\lambda}$ and $\gamma_f > \hat{\lambda}$

and,

c) $\gamma_f$ is computed from

$$(1 - \pi)^2 \gamma_f^2 - \left( 2\hat{\lambda} \gamma_f - \hat{\lambda}^2 \right) \left[ \gamma_f + (1 - \pi - \gamma_f) I^* \right]^2 = 0.$$
Finally, our conjecture that $\beta(\lambda) = 1$ for all $\lambda$ is indeed true for all $I > I^*$. To see this, note from (9) and (11) that $\beta(\lambda) = 1$ for all $\lambda$ if and only if

$$\frac{\alpha(\lambda) \gamma_f}{I^*} \geq \frac{1}{\pi} \gamma_h$$

(22)

Further, using (12) and part (a) of Lemma 2, (22) can be rewritten as

$$\gamma_f \geq \frac{\lambda I^*}{T}$$

(23)

That is, $\beta(\lambda) = 1$ for all $\lambda$ if and only if (23) holds. Using part (b) of Lemma 2, it follows that (23) trivially holds for all $I \geq I^*$.

2.4 The case of $I << I^*$

However, for $I << I^*$, i.e., when the domestic currency yields a relatively superior return, (23) may be violated. In that case, banks may not always want to exhaust their reserves of domestic currency. Consequently, matters are substantially different from the case where $I > I^*$. Indeed, there are two endogenous thresholds for the foreign relocation probability that become relevant. Below the first threshold, the bank is able to provide complete insurance; non-movers, foreign and domestic movers all receive the same return ex-post. This requires banks to not use up all their home currency reserves to pay the domestic movers. Instead, some domestic currency is kept aside and used to buy goods for the non-movers. Similarly, not all the foreign reserves of the home banks are used up to pay the foreign movers. Once the first threshold is crossed, the banks can no longer protect the foreign movers even as they end up exhausting all their foreign currency (the weaker currency) reserves. At this point, the non-movers and the domestic movers continue to earn the same return, a return that is higher than what the banks can offer the foreign movers. After the second threshold has been reached, the return to the foreign movers gets lower and lower; the return to non-movers will keep increasing as the same amount of storage is now being divided among lesser and lesser number of non-movers. In turn now, it may not be necessary to transfer domestic currency cash reserves to non-movers in the same amount as before. Banks exhaust all domestic currency reserves to pay the domestic movers whose return is now lower than what the non-movers get.

Formally, for all $\lambda \in [0, \hat{\lambda}]$, (9) and (11) will hold with equality. Manipulating them, one can rewrite

$$\lambda \left[ \gamma_f + (1 - \beta(\lambda)) \gamma_h \frac{I^*}{T} + (1 - \gamma) I^* \right] \geq (1 - \pi) \alpha(\lambda) \gamma_f, \quad "=" \quad \text{if } \alpha(\lambda) < 1$$

(24a)

$$\pi \left[ (1 - \alpha(\lambda)) \gamma_f \frac{I}{T} + \gamma_h + (1 - \gamma) I \right] \geq (1 - \lambda) \beta(\lambda) \gamma_h, \quad "=" \quad \text{if } \beta(\lambda) < 1$$

(24b)

The optimal rules for use of currency reserves are collected in the next lemma.
**Lemma 3** When $I \ll I^*$,

$$
\alpha = \begin{cases} 
\frac{\lambda}{\hat{\lambda}} & \text{if } \lambda \leq \hat{\lambda}, \\
1 & \text{if } \lambda > \hat{\lambda}, 
\end{cases} \quad (24y)
$$

and

$$
\beta = \begin{cases} 
\tilde{\beta} & \text{if } \lambda \leq \tilde{\lambda} \\
\frac{1 - \tilde{\lambda}}{\lambda} & \text{if } \lambda \in [\tilde{\lambda}, \hat{\lambda}] \\
1 & \text{if } \lambda \geq \hat{\lambda}, 
\end{cases} \quad (24z)
$$

where

$$
\tilde{\lambda} \equiv (1 - \pi) - \frac{\pi}{\gamma_h} (1 - \gamma_f - \gamma_h) I 
$$

$$
\hat{\lambda} \equiv \frac{\gamma_f \frac{f_f}{f}}{\gamma_f \frac{f_f}{f} + \gamma_h + (1 - \gamma_f - \gamma_h) I} 
$$

$$
\tilde{\beta} \equiv \pi \frac{\gamma_f \frac{f_f}{f} + \gamma_h + (1 - \gamma_f - \gamma_h) I}{\gamma_h} 
$$

Figure 1 exhibits equations (24y) and (24z) graphically.

![Fraction of currencies utilized in relocations (I < I')]
For future reference note,

\[
rf = rh = r = \frac{\rho \gamma_f}{\lambda I^*} = \frac{\beta \rho \gamma_h}{\pi I}, \text{ for } \lambda \leq \hat{\lambda}
\]

\[
rf = \frac{1}{\lambda} \rho \gamma_f < rh = r = \frac{1 - \hat{\lambda} \rho \gamma_h}{1 - \lambda \pi}, \text{ for } \lambda \in \left[\hat{\lambda}, \tilde{\lambda}\right]
\]

\[
rf = \frac{1}{\lambda} \rho \gamma_f < rh = \rho \gamma_h < r = \frac{1 - \rho}{1 - \pi - \lambda \rho}, \text{ for } \lambda \in \left[\tilde{\lambda}, \bar{\lambda}\right]
\]

When the realized \(\lambda\) is fairly low (\(\lambda \leq \hat{\lambda}\)), the bank successfully equalizes returns across the three agent types without exhausting its currency reserves. Unused currency reserves help pay the non-movers (in a sense, since \(\lambda\) is low, too much currency and hence too little storage is being held; the bank readjusts and uses excess currency reserves to buy goods for the non-movers). Beyond that immediate threshold, the bank exhausts all its foreign currency (the worse asset) reserves; but there are too many foreign movers and “too little” reserves to pay each of them the same as everyone else. So the foreign movers get a lower payment. Excess reserves of domestic currency go to the non-movers. At the next threshold, the bank runs out of its stock of domestic currency at which point the home movers start to get a payout lower than what the non-movers get.

We are now in a position to start investigating the optimal reserve holdings of the two currencies. Using (39), (33) can be rewritten as

\[
W = \int_0^{\hat{\lambda}} \ln \left(\frac{1}{\lambda} \rho \gamma_f \right) f(\lambda) d\lambda + \int_{\hat{\lambda}}^{\tilde{\lambda}} (1 - \lambda) \ln \left(\frac{1 - \hat{\lambda} \rho \gamma_h}{1 - \lambda \pi}\right) f(\lambda) d\lambda
\]

\[
+ \int_{\tilde{\lambda}}^{\bar{\lambda}} (1 - \pi - \lambda) \ln \left(\frac{1 - \gamma_h - \gamma_f}{1 - \pi - \lambda \rho}\right) f(\lambda) d\lambda + \int_{\tilde{\lambda}}^{\bar{\lambda}} \pi \ln \left(\frac{\rho \gamma_h}{I \pi}\right) f(\lambda) d\lambda
\]

\[
+ \int_{\lambda}^{\bar{\lambda}} \ln \left(\frac{1}{\lambda} \rho \gamma_f \right) f(\lambda) d\lambda
\]

It can be checked that the ex-ante optimal choice of \(\{\gamma_f, \gamma_h\}\) obtained by maximizing \(W\) above solves

\[
\gamma_f : \frac{\hat{\lambda}}{\gamma_f} + \frac{\pi}{\gamma_f} + \frac{1}{1 - \gamma_h - \gamma_f} \left(\frac{I^* - 1}{I} - \frac{1}{\gamma_f} + (1 - \gamma_h - \gamma_f) \left(\hat{\lambda} - \hat{\lambda}\right) \left(1 - \frac{\hat{\lambda} + \lambda}{2}\right) \right) = 0
\]

\[
- \frac{1}{(1 - \gamma_h - \gamma_f)} \left(\hat{\lambda} - \hat{\lambda}\right) \left(1 - \pi - \frac{\hat{\lambda} + \lambda}{2}\right) + \frac{\hat{\lambda}^2 - \hat{\lambda}^2}{2 \gamma_f} = 0,
\]
and
\[
\gamma_h : \frac{\lambda}{\pi + \frac{\lambda}{2} + (1 - \gamma_h - \gamma_f) \left( \frac{1}{2} - 1 \right)} + \frac{\frac{1}{2} - 1}{\gamma_h - \gamma_f} \left( 1 - \lambda \right) \left( 1 - \left( \frac{\lambda + \lambda}{2} \right) \right) +
\]

\[
- \frac{1}{(1 - \gamma_h - \gamma_f)} \left( \lambda - \lambda \right) \left( 1 - \pi - \left( \frac{\lambda + \lambda}{2} \right) \right) + \left( \lambda - \lambda \right) \frac{\pi}{\gamma_h} = 0
\]

(32)

We proceed to study how the optimal reserve holdings respond to the domestic nominal interest rate, holding the foreign interest rate fixed. Suppose we choose the following parametric specification: \( \pi = 0.3, \lambda = 0.6 < 1 - \pi \), and \( I^* = 1.3 \) and let \( I \) vary between 1.01 and 1.1. Figure 2 plots \( \gamma_h \) and \( \gamma_f \) as \( I \) varies. Several items deserve mention. First, not surprisingly, \( \gamma_h \) and \( \gamma_f \) respond in exactly opposite ways; while \( \gamma_h \) falls with \( I \), \( \gamma_f \) increases with it. As \( I \) increases with \( I^* \) held fixed, and \( I \) stays below \( I^* \), banks decrease (increase) their domestic (foreign) currency reserves. As \( I \) increases, banks want to economize on currency holdings; since the domestic opportunity cost is relatively increasing with \( I \), the bank reduces its domestic currency holdings. Second, \( \gamma_h > \pi \) for this range of \( I \) and \( I^* \) implying that banks optimally hold more of the domestic currency than they need solely for the purposes of paying the domestic movers. Finally, as a comparative static exercise, if \( I^* \) is raised to 1.35, the \( \gamma_h (\gamma_f) \) locus shifts up (down).

\[
\text{Fraction of Home and Foreign currencies held (} l < l^* \text{)}
\]

\[ \gamma_h \] and [\( \gamma_f \)]

Figure 2
How does \( \lambda \) behave as a function of \( I \), i.e., how does the probability of a banking crisis change with the domestic nominal interest rate? For \( \pi = 0.2, \lambda = 0.6 < 1 - \pi, I^* = 1.1 \), and \( I \in [1.01, 1.2] \), Figure 3 presents \( \hat{\lambda}(I) \). Recall the higher the value of \( \hat{\lambda} \), the lower the probability of a crisis. Notice that domestic interest rate can affect \( \hat{\lambda} \) only when it is below \( I^* \). In fact, the threshold value of \( I \) that violates (??) lies somewhere between 1.05 and 1.075.

![Figure 3](image.png)

3 General Equilibrium, welfare, and policy

Both governments follow a constant nominal money growth policy given by

\[
M_t^* = \mu M_{t-1}^*; M_t^* = \mu^* M_{t-1}^*
\]

Thus, in a stationary equilibrium

\[
\frac{p_{t+1}}{p_t} = \mu; \quad \frac{p_{t+1}^*}{p_t^*} = \mu^*; \quad I = \mu \rho, \text{ and } I^* = \mu^* \rho
\]

Symmetrically, let

\[
w = w^*, \quad \pi = \pi^*
\]

Further, let

\[
\lambda^* = \frac{\bar{\lambda}}{2}
\]

Then,

\[
\gamma_h^* = \frac{\bar{\lambda}}{2}, \gamma_f^* = \pi
\]
We can show that $\gamma_f > \lambda > \frac{1}{2}$. Thus, Home is paying larger seigniorage to Foreign by holding a larger fraction of Foreign money. Finally, we assume that seigniorage is paid as lump sum transfers to domestic young women, which implies

$$\tau = \frac{\mu - 1}{\mu} m = \left(1 - \frac{\rho}{I}\right) (m_h + m_h') = i \left(\gamma_h (w + \tau) + \gamma_h' (w + \tau^*)\right)$$

$$\tau^* = \frac{\mu^* - 1}{\mu^*} m^* = \left(1 - \frac{\rho}{I^*}\right) (m_f + m_f') = i^* \left(\gamma_f (w + \tau) + \gamma_f' (w + \tau^*)\right)$$

where we define $i \equiv 1 - \frac{\tau}{I}$ and $i^* \equiv 1 - \frac{\tau^*}{I^*}$. These two together yield

$$\begin{bmatrix}
1 - i\gamma_h & -i\frac{\lambda}{2} \\
-i\gamma_f & 1 - i^*\pi
\end{bmatrix}
\begin{bmatrix}
\tau \\
\tau^*
\end{bmatrix} = w
\begin{bmatrix}
i \left(\gamma_h + \frac{\lambda}{2}\right) \\
i^* \left(\pi + \gamma_f\right)
\end{bmatrix}$$

Then

$$\tau = \frac{i \left(\gamma_h + \frac{\lambda}{2}\right) (1 - i^*\pi) + i^* \left(\pi + \gamma_f\right) i\frac{\lambda}{2}}{(1 - i^*\pi)(1 - i\gamma_h) - i\frac{\lambda}{2} i^*\gamma_f}$$

$$\tau^* = \frac{i^* \left(\pi + \gamma_f\right) (1 - i\gamma_h) + i \left(\gamma_h + \frac{\lambda}{2}\right) i^*\gamma_f}{(1 - i^*\pi)(1 - i\gamma_h) - i\frac{\lambda}{2} i^*\gamma_f}$$

Recall, however, that banks take $\tau$ as given and therefore their equilibrium allocations $\{\gamma_f, \gamma_h\}$ are independent of the fact that equilibrium $\tau$ is a function of $\{\gamma_f, \gamma_h\}$.

Fix $I^* = 1.3$, $\rho = 1.1$, i.e., $\mu^* = 1.3/1.1$, $\pi = 0.2$, $\lambda = 0.8$. The following Figure 4 shows how Home interest rate affects Home and Foreign welfare.

![Figure 4](image)

4 Endowment uncertainty and alternative exchange rate regimes

Now we propose that the endowment $w$ is random, i.i.d., and uniform over support $[\bar{w}, \bar{w}]$. It is easy to see that banks’ portfolio allocations will now be state-contingent, i.e., $\gamma_h$ and $\gamma_f$ will
be functions of \( w \). Furthermore, uncertain endowment will induce a stochastic distribution of real balances both in domestic and foreign currency. In what follows, we look for a stationary distribution of real balances which is consistent with a rational expectations equilibrium.

The banks’ problem can now be stated as

\[
\max_{\{\gamma_f, \gamma_h, \alpha(\lambda), \beta(\lambda)\}} \int_{w'} \left\{ \int_{\lambda} \left[ \pi \ln r_f(\lambda) + \lambda \ln r_f(\lambda) + (1 - \pi - \lambda) \ln r(\lambda) \right] f(\lambda) \, d\lambda \right\} f(w') \, dw' \quad (33)
\]

subject to

\[
\begin{align*}
  r_f(\lambda) &= \frac{\alpha(\lambda)}{\lambda} \gamma_f I^* \\
r_h(\lambda) &= \frac{\beta(\lambda)}{\pi} \gamma_h I^* \\
r(\lambda) &= \frac{\rho}{1 - \pi - \lambda} \left[ (1 - \alpha(\lambda)) \frac{\gamma_f}{I^*} + (1 - \beta(\lambda)) \frac{\gamma_h}{I} + (1 - \gamma_f - \gamma_h) \right],
\end{align*}
\]

where \( w' \) denotes next period’s endowment and \( f(w') \) is its density function. In addition, \( \gamma_f, \gamma_h \geq 0, \gamma_f + \gamma_h \leq 1 \), and \( \alpha, \beta \in [0, 1] \). As before, the problem is solved in two stages. In the second stage, after \( \lambda \) is realized, \( \alpha(\lambda) \) and \( \beta(\lambda) \) are optimally chosen given \( \gamma_f \) and \( \gamma_h \). The first stage takes these optimal \( \lambda \)-contingent rules into account and determines \( \gamma_h \) and \( \gamma_f \). For simplicity, we retain the notation \( \gamma_h \) and \( \gamma_f \), although now they are \( w \)-contingent.

We assume that the world nominal rate of interest \( I^* = \rho (1 + \pi^*) \) is constant. In particular, \( \pi^* = 0 \), which implies that \( I^* = \rho \). Since domestic currency real balances are now \( w \)-contingent, a constant money growth rule implies that domestic price level (and thus the exchange rate) will be stochastically distributed. On the other hand, if the government adheres to an inflation (devaluation) target, money supply will follow a stochastic distribution. Clearly, the equilibria and welfare under the two scenarios will be different. In what follows, we compare allocations under the two rules.

A word on notation will be in order. Note that all ex-ante \( (\gamma_h, \gamma_f) \) and ex-post \( (\alpha, \beta) \) rules will be \( w \)-contingent, and hence will not be constant over time. However, as the analysis below confirms, these rules only depend on the current realizations of \( w \) and expectation of next period’s real balances that, given the \( w \)-contingent allocation rules, follow a stationary distribution. In equilibrium, current allocation rules coincide with future rules. Hence, given a realized value of \( w \), each period’s problem is identical. Essentially, the solution entails finding a fixed point of ex-ante allocation functions \( \gamma_h(w) \), and \( \gamma_f(w) \). Again, for convenience, we retain our earlier notation \( \gamma_h \) and \( \gamma_f \) instead of \( \gamma_h(w) \) and \( \gamma_f(w) \).
4.1 Flexible exchange rate regime

Here we assume that the money supply is fixed, i.e., $\mu = 1$ for all $t$. With constant money supply, $\tau_t = \frac{M_t - M_{t+1}}{p_t} = 0$. Hence, $m = \gamma_h w$. Then $I_t = \rho \frac{\alpha_t}{p_t} = \rho \frac{m_t}{m_t}$. Then, banks’ rate-of-return constraints can be rewritten as

$$r_f (\lambda) = \frac{\alpha (\lambda)}{\lambda} \frac{p}{T}$$

$$r_h (\lambda) = \frac{\beta (\lambda) \gamma_h m'}{\pi} = \frac{\beta (\lambda) \gamma_h (w') w'}{w}$$

$$r (\lambda) = \frac{\rho}{1 - \pi - \lambda} \left[ (1 - \alpha (\lambda)) \frac{\gamma_f}{I} + (1 - \beta (\lambda)) \gamma_h \frac{m'}{m} + (1 - \gamma_f - \gamma_h) \right]$$

As before, $\gamma_h$ and $\gamma_f$ will be independent of the realization of $\lambda$.

Thus, banks maximize (33) subject to (3) and (7a) - (7c). It’s convenient to think of it as a two stage problem and work backwards: 1) in the second stage, given $\gamma_h$ and $\gamma_f$, choose $\alpha (\lambda)$ and $\beta (\lambda)$ for each $\lambda$, and 2) given the schedules $\alpha (\lambda)$ and $\beta (\lambda)$, choose $\gamma_h$ and $\gamma_f$ that maximizes (33).

Accordingly, the first-order-condition for the choice of $\alpha (\lambda)$ (for the second stage) is:

$$(1 - \pi - \lambda) E \left\{ \frac{1}{(1 - \alpha (\lambda)) \frac{\gamma_f}{I} + (1 - \beta (\lambda)) \gamma_h \frac{m'}{m} + (1 - \gamma)} \right\} \leq \frac{\gamma_f}{\alpha (\lambda) \gamma_f}, \quad a = j \text{ if } \alpha (\lambda) < 1$$

$$(1 - \pi - \lambda) E \left\{ \frac{m_{t+1}}{(1 - \alpha (\lambda)) \frac{\gamma_f}{I} + (1 - \beta (\lambda)) \gamma_h \frac{m_{t+1}}{m_t} + (1 - \gamma)} \right\} \leq \frac{\pi I}{\beta (\lambda) \gamma_h}, \quad a = j \text{ if } \beta (\lambda) < 1$$

4.1.1 Ex-post $\beta = 1$ for all $\lambda$  
Then, from (36) and (37) it easy to see that ex-post rate-of-returns will follow the same pattern as in Section 2.3. However, we now replace $I$ by $I^c$, where $I^c = \frac{I}{E_t \left( \frac{m_{t+1}}{m_t} \right)}$.

4.1.2 Ex-post $\beta (\lambda) < 1$ for all $\lambda$  
However, for $I^c < I^*$ this may be violated. To show this note that (23) requires that

$$\gamma_f (1 - I^*) \geq (1 - \pi) I^* \left( \frac{1}{I^c} - 1 \right)$$

Formally, for all $\lambda \in [0, \hat{\lambda}]$, (9) and (11) will hold with equality. Manipulating them, one can rewrite

$$(1 - \pi - \lambda) E \left\{ \frac{1}{(1 - \alpha (\lambda)) \frac{\gamma_f}{I} + (1 - \beta (\lambda)) \gamma_h \frac{m_{t+1}}{m_t} + (1 - \gamma)} \right\} \leq \frac{\gamma_f}{\alpha (\lambda) \gamma_f}, \quad a = j \text{ if } \alpha (\lambda) < 1$$

$$(1 - \pi - \lambda) E \left\{ \frac{m_{t+1}}{(1 - \alpha (\lambda)) \frac{\gamma_f}{I} + (1 - \beta (\lambda)) \gamma_h \frac{m_{t+1}}{m_t} + (1 - \gamma)} \right\} \leq \frac{\pi I}{\beta (\lambda) \gamma_h}, \quad a = j \text{ if } \beta (\lambda) < 1$$
\( \hat{\lambda} \) is now determined by

\[
\hat{\lambda} = (1 - \pi) \frac{E \left\{ \frac{1}{(1-\beta(\lambda))\frac{\gamma_f}{\gamma_f} T^{m_{t+1}} + (1-\gamma) \frac{T}{\gamma_f}} \right\}}{1 + E \left\{ \frac{1}{(1-\beta(\lambda))\frac{\gamma_f}{\gamma_f} T^{m_{t+1}} + (1-\gamma) \frac{T}{\gamma_f}} \right\} }, \text{ where}
\]

\[
\frac{1}{\beta(\lambda)} = \frac{1 - \pi - \lambda}{\pi} E \left\{ \frac{1}{(1 - \alpha(\lambda)) + (1 - \beta(\lambda)) \frac{\gamma_f}{\gamma_f} T^{m_{t+1}} + (1 - \gamma) \frac{T}{\gamma_f}} \right\}
\]

For all \( \lambda \leq \hat{\lambda} \), \( \alpha(\lambda) \) and \( \beta(\lambda) \) are now implicitly determined from

\[
\frac{1}{\alpha(\lambda)} = \frac{1 - \pi - \lambda}{\lambda} E \left\{ \frac{1}{(1 - \alpha(\lambda)) + (1 - \beta(\lambda)) \frac{\gamma_f}{\gamma_f} T^{m_{t+1}} + (1 - \gamma) \frac{T}{\gamma_f}} \right\}
\]

Finally, \( \beta(\lambda) = 1 \). Between \( \hat{\lambda} \) and \( \lambda \), \( \beta(\lambda) \) is determined by the following

\[
\frac{1}{\beta(\lambda)} = \frac{1 - \pi - \lambda}{\pi} E \left\{ \frac{\gamma_f T^{m_{t+1}}}{(1 - \beta(\lambda)) \frac{\gamma_f}{\gamma_f} T^{m_{t+1}} + (1 - \gamma) \frac{T}{\gamma_f}} \right\}
\]

And, \( \hat{\lambda} \) is given by

\[
\frac{1 - \pi - \hat{\lambda}}{\pi} E \left\{ \frac{\gamma_f T^{m_{t+1}}}{1 - \gamma} \right\} = 1
\]

\[
1 - \pi - \hat{\lambda} = \pi \frac{I^c}{\gamma_h} (1 - \gamma) \Rightarrow \hat{\lambda} = (1 - \pi) - \pi \frac{I^c}{\gamma_h} (1 - \gamma)
\]

Thus,

\[
r_f = \frac{\alpha(\lambda) \rho \gamma_f}{\lambda T^*}, \quad r = \frac{\rho}{1 - \pi - \lambda} \left[ \frac{(1 - \alpha(\lambda)) \gamma_f}{T^*} + \frac{(1 - \beta(\lambda)) \gamma_h m_{t+1}}{I m_t} + (1 - \gamma) \right], \quad \frac{\pi}{\gamma_h} T^* \frac{I^c}{(1 - \gamma)}
\]

and \( r_h = \frac{\beta(\lambda) \rho \gamma_h m_{t+1}}{\pi T^* m_t} \), for \( \lambda \leq \hat{\lambda} \)

\[
r_f = \frac{1}{\lambda} \frac{\rho \gamma_f}{T^*}, \quad r_h = \frac{\beta(\lambda) \rho \gamma_h m_{t+1}}{\pi m_t}, \text{ and } r = \frac{\rho}{1 - \pi - \lambda} \left[ \frac{(1 - \beta(\lambda)) \gamma_h m_{t+1}}{I m_t} + (1 - \gamma) \right], \quad \frac{\pi}{\gamma_h} T^* \frac{I^c}{(1 - \gamma)}
\]

Finally, it’s worth noting that for \( w = \tilde{w} \), \( \hat{\lambda} = (1 - \pi) - \pi \frac{I^c(\tilde{w})}{\gamma_h(\tilde{w})} (1 - \gamma) \). Thus, \( \tilde{w} \) is implicitly determined from the above equation. For all \( w < \tilde{w} \) it is implied that \( \beta(\hat{\lambda}) < 1 \). Furthermore, for all \( w < \tilde{w} \), where \( w \) solves \( \gamma_h(I^c(w)) = 1 - \gamma_f(I^c(w)) \), where \( \gamma_h \) and \( \gamma_f \) as before maximize the expected utility.
4.1.3 Welfare under flexible exchange rates  

Now

\[
\frac{\rho}{I_t} = \frac{p_t}{p_{t+1}} = \frac{\rho}{I_t} \frac{m_{t+1}}{m_t}
\]

where \( I = \rho \mu \). Clearly, \( \gamma_h \) will be time variant and a function of \( \mu \) and \( m_t : \gamma_h (\mu, m_t) \). Mapping the rules in the deterministic case to this extension will be provided in the appendix. In equilibrium

\[
m_t = \gamma_h (\mu, m_t) (w_t + \tau_t)
\]

Further, substitution \( \tau_t = \left(1 - \frac{1}{\rho}\right) m_t \) in the above obtains

\[
m_t = \frac{\gamma_h (\mu, m_t)}{1 - \gamma_h (\mu, m_t) \left(1 - \frac{1}{\rho}\right)} w_t;
\]

\[m_t = \phi (w_t)\]

Thus, given that \( w \) has a stationary distribution, \( m \) has a stationary distribution. Then, computing \( E_t (m_{t+1}) \) is straightforward. But, characterizing the fixed point analytically is not possible. Hence, we employ numerical techniques.

Finally, welfare is again given by

\[
\int_w \left\{ \int_0^\lambda [\pi \ln r_h (\lambda) + \lambda \ln r_f (\lambda) + (1 - \pi - \lambda) \ln r (\lambda)] \, d\lambda \right\} \, f (w) 
\]

\[\frac{m}{\gamma_h} \right\} \, dw + E \left\{ \ln \frac{m}{\gamma_h} \right\}
\]

4.2 Fixed exchange rates

First note that

\[
\frac{e_{t+1}}{e_t} = \frac{p_{t+1}}{p_t} \frac{p_t^*}{p_{t+1}^*}
\]

Hence,

\[
\frac{p_{t+1}}{p_t} = (1 + \varepsilon) (1 + \pi^*)
\]

Without any loss of generality, let \( \pi^* = 0 \) in what follows. Then,

\[
\frac{p_{t}}{p_{t+1}} = \frac{\rho}{I_t} = \frac{1}{1 + \varepsilon} \frac{1}{1 + \pi^*} = \frac{1}{1 + \varepsilon} \frac{\rho}{I^*}
\]

Under fixed exchange rates, we assume \( \varepsilon = 0 \). Then \( I = I^* \) and then ex-ante portfolio allocation rules and ex-post rate-of-return rules simply follow from Section 2.3. Thus, the equilibrium rules are identical to those in the deterministic case.
Welfare  The welfare under fixed rates however will be now given by

\[ \int_0^\lambda \left[ \pi \ln r_h(\lambda) + \lambda \ln r_f(\lambda) + (1 - \pi - \lambda) \ln r(\lambda) \right] f(\lambda) \, d\lambda + E\left\{ \ln \frac{m}{\gamma_h} \right\} \]

(40)

Note that

\[ \tau_t = \frac{M_t - M_{t-1}}{p_t} = m_t - \frac{m_{t-1}}{1 + \varepsilon} = \gamma_h \left[ w_t + \tau_t - \frac{w_{t-1} + \tau_{t-1}}{1 + \varepsilon} \right] \]

Therefore

\[ m_t = \gamma_h (w_t + \tau_t) = \gamma_h \left( w_t + m_t - \frac{m_{t-1}}{1 + \varepsilon} \right) \]

Hence

\[ m_t = -\gamma_h \frac{1}{1 - \gamma_h \frac{1 + \varepsilon}{1 + \varepsilon}} m_{t-1} + \frac{\gamma_h}{1 - \gamma_h} w_t \]

Assuming \( \frac{\gamma_h}{1 - \gamma_h \frac{1 + \varepsilon}{1 + \varepsilon}} < 1 \),

\[ E\{m\} = \frac{\gamma_h}{1 + \left( \frac{\gamma_h}{1 - \gamma_h \frac{1 + \varepsilon}{1 + \varepsilon}} \right)^2} \bar{w} \]

\[ Var\{m\} = \frac{\left( \frac{\gamma_h}{1 - \gamma_h \frac{1 + \varepsilon}{1 + \varepsilon}} \right)^2 \sigma_w^2}{1 - \left( \frac{\gamma_h}{1 - \gamma_h \frac{1 + \varepsilon}{1 + \varepsilon}} \right)^2} \sigma_w^2 \]

If \( \varepsilon = 0 \), then

\[ E\{m\} = \gamma_h \bar{w} \]

\[ Var\{m\} = \frac{\gamma_h^2 \sigma_w^2}{1 - 2\gamma_h} \sigma_w^2 \]

4.3 Numerical results

To compare the two regimes, we assume the following parameter values: \( \pi = 0.2, \lambda = 0.4, \mu = \mu^* = 1, \rho = 1.04, \bar{w} = 0.9 \) and \( \bar{w} = 1.1 \). The following Table 1 presents the numerical value for utility and crisis probability under the two regimes:

<table>
<thead>
<tr>
<th></th>
<th>Utility</th>
<th>Crisis probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed exchange rates</td>
<td>-0.0248042</td>
<td>0.188305</td>
</tr>
<tr>
<td>Flexible exchange rates</td>
<td>-0.0233484</td>
<td>0.211089</td>
</tr>
</tbody>
</table>
Evidently, flexible exchange rate obtain a higher welfare relative to fixed exchange rates. However, the crisis probability is higher under flexible exchange rates.

To get an intuitive albeit informal sense of this, we start by noting that under the flexible rate regime, the exchange rate adjustments and consequent interest rate changes induce banks to tilt their portfolios towards the highest paying assets. Suppose for instance that markets anticipate a low devaluation in any period. Banks will then tilt their portfolio towards domestic reserves, which in turn will induce price level to fall. As a result, the expected devaluation will now be lower. Thus, in equilibrium, exchange rate adjustments will be muted due to changes in portfolio demand. Thus, even though the flexible exchange rate regime suffers from the rate-of-return uncertainly, the price mechanism ensures that the range of fluctuations is diminished. Fixed exchange rate regime, on the other hand, eliminates rate-of-return uncertainty, but the inefficiency that stems from income redistribution makes it worse. Essentially, the exchange-rate adjustment mechanism along with portfolio choice implies that banks will be allocating their assets in the most efficient way. However, note that, despite these adjustments the domestic assets sometimes may become too attractive, and banks will put less weight on foreign currency reserves under such cases. As a result, the probability of crisis under flexible exchange rates is higher than under fixed exchange rates.

5 Conclusion

Banking crises are fairly frequent events, often accompanied by huge resolution costs as well as current and future output losses. These crises also affect both the future conduct and scope of domestic fiscal and monetary policy as well as the decision of countries to peg or float their currencies. This paper studies the potential for banking crises and their concomitant domestic and international repurcussions in a microfounded general equilibrium dynamic model in the tradition of Diamond and Dybvig (1983) and Chang and Velasco (2000). More specifically, we produce a small open economy pure exchange overlapping generations model with random relocation along the lines of Smith (2002). The combination of random relocation and the assumed role of currency in interlocation trade creates random location - and country - specific liquidity needs. Banks naturally arise to provide much-desired insurance against these liquidity shocks. In this setting, if withdrawal demand for the bank’s deposits is high enough, the bank will exhaust all its cash reserves and a banking crisis will occur. We provide a complete characterization of optimal interest rate policies in this setting. We find that nominal interest rates that are desirable from a welfare perspective may be associated with a high probability of banking crises.

We then study the classic issue of the relative desirability of fixed versus flexible exchange rate regimes. To that end, we introduce time-varying random endowments into the above structure.
This makes the banks’ portfolio allocations dependent on the exchange rate regime. Under a fixed exchange rate regime, by interest rate parity, the banks’ portfolio choice is deterministic and constant over time and this is achieved by the central bank injecting or withdrawing nominal balances. This in turn generates a novel inter-generational income redistribution. Under a flexible exchange rate regime, the money supply stays constant (hence no income redistribution) but a rate-of-return uncertainty emerges that is in addition to the income uncertainty common to both regimes. We go on to study the classic issue of the relative desirability of fixed versus flexible exchange rate regimes by introducing time-varying random endowments into the above structure. This makes the banks’ portfolio allocations dependent on the exchange rate regime. Under a fixed exchange rate regime, by interest rate parity, the banks’ portfolio choice is deterministic and constant over time and this is supported by the injection/removal of nominal balances by the central bank. Under a flexible exchange rate regime, the money supply stays constant but a rate-of-return uncertainty emerges that is in addition to the income uncertainty common to both regimes. Our results show that a flexible exchange rate regime is superior in an ex-ante welfare sense relative to the fixed exchange rate regime; however, the ordering is reversed when it comes to the likelihood of banking and currency crises.
References

