Borrowing constraints, human capital accumulation, and growth

José De Gregorio

Ministry of Finance, Teatinos 120, Santiago, Chile
Research Department, International Monetary Fund, Washington DC 20431, USA
Center for Applied Economics, Department of Industrial Engineering, Universidad de Chile, Santiago, Chile

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Abstract

Borrowing constraints increase aggregate savings, and therefore may increase growth. This paper argues, however, that by reducing human capital accumulation, borrowing constraints also have negative effects on growth. These effects are discussed in an overlapping-generations model with endogenous growth. Empirical evidence for OECD and developing countries lend support to the main predictions of the model.

Key words: Borrowing constraints; Growth; Human capital; Savings

JEL classification: E21; O16; O41

1. Introduction

In recent years, as the result of developments in the theory of economic growth, renewed interest has emerged in understanding the determinants of human capital accumulation. The purpose of this paper is to study the effects of...
borrowing constraints on human capital accumulation and growth. I construct a life-cycle model with endogenous growth where individuals face borrowing constraints and have to decide during their youth how much time to devote to education.

This consideration is often disregarded, with most emphasis placed on the effects of borrowing constraints on physical capital accumulation. For example, according to Modigliani (1986):

'Imperfections in the credit markets as well as the uncertainty of future income prospects may, to some extent, prevent consumers from borrowing as much as would be required to carry out the unconstrained optimum consumption plan. Such a constraint will have the general effect of postponing consumption and increase \( w \) [aggregate wealth-income ratio] and \( s \) [savings rate].'

The application of this result to a framework where savings is positively associated with economic growth, in the transition and in the steady state, has led Jappelli and Pagano (1994) to conclude that borrowing constraints increase growth. In this paper I argue, in contrast, that borrowing constraints have also negative effects on growth. This occurs because the inability of individuals to borrow against future income reduces the incentives for human capital accumulation.

This paper is related to recent literature on the effects of credit market imperfections on human capital accumulation. In independent work Christou (1993) develops a neoclassical growth model with borrowing constraints with results, obtained by simulating the model, similar to those of this paper. Buiter and Kletzer (1992) argue that the inability to borrow may reduce human capital accumulation in a model where individuals must self-finance their training costs. In this model I assume that education is free and focus on the trade-off between working and studying. Finally, Barro, Mankiw, and Sala-i-Martin (1995) discuss the implications of borrowing constraints to finance education for convergence of income across countries.

The framework is an overlapping-generations model of a small open economy where individuals live for three periods and are endowed with one unit of nonleisure time in each of the first two periods. They can increase skills by devoting time to education in their youth, and thus they increase labor supply in efficiency units, in their middle age. However, individuals need resources to consume while they are acquiring education. I assume that individuals face a borrowing constraint, which implies that education involves an opportunity cost from foregone labor income. This cost will induce young individuals to reduce time

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2The open economy assumption simplifies the presentation by allowing to assume a fixed real interest rate and to obtain analytic solutions. Later in the paper I also discuss the implications in the closed economy.
devoted to education and to increase time devoted to work. In the model, growth is sustained by the accumulation of physical and human capital, and borrowing constraints affect growth by changing the composition of capital.

In the basic model I show that borrowing constraints reduce human capital accumulation and growth. I also show, consistent with Modigliani's life-cycle, that for a given rate of output growth borrowing constraints increase savings. However, once growth effects are incorporated, I show that the savings rate could decline.

The paper presents empirical evidence on the effects of borrowing constraints on human capital accumulation and growth in two samples of countries. The first one is a sample of OECD countries, for which good indicators of the extent of borrowing constraints faced by consumers have been constructed by Jappelli and Pagano (1994). Those indicators are the ratio of consumer credit to national income and the maximum loan-to-value ratio for the purchase of a house. The other sample is a group of developing countries, where I use a more standard measure of credit market development: the ratio of total credit from the banking sector to the nonfinancial private sector to GDP. The empirical evidence is broadly supportive of the predictions of the model. First, it is found that countries with tight borrowing constraints have lower human capital accumulation. In particular, borrowing constraints have negative effects on secondary school enrollment ratios. Regarding growth, the evidence shows that in general tighter borrowing constraints result in lower growth. After controlling for human capital accumulation, borrowing constraints are still negatively correlated with growth, which may reflect the existence of other channels through which borrowing constraints affect growth, or, alternatively, because borrowing constraints may be most prevalent in countries which pursue other policies that hinder growth.

The paper is organized in five sections. Section 2 presents the basic model. Section 3 characterizes the equilibrium and shows the main results on the effects of borrowing constraints on human capital accumulation, savings, and growth. Section 4 presents the empirical evidence, and finally, Section 5 concludes.

2. The model

In this section I present an endogenous growth model of a small open economy inhabited by three nongrowing overlapping generations.

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3 One could think also that if individuals are unable to save in financial assets, they may have incentives to increase education in order to transfer resources to the future. In a model with heterogeneous agents and both borrowing and lending constraints De Gregorio and Kim (1995) show that the net effect of those constraints on human capital accumulation is negative.
2.1. Technology

The economy produces a single good according to the following technology:

\[ Y_t = a K_t^{\phi} (H_t \ell_t)^{1-\phi}, \]

where \( K \) is capital, \( H \) is the level of skills of a unit of labor inputs (average level of human capital), and \( \ell \) represents total (raw) labor inputs. The model assumes that all individuals in the same generation have the same level of skills. The usual solution to the firms' optimization problem sets factor costs equal to their marginal productivity:

\[ w_t = a(1 - \phi)(\sigma_t / \ell_t)^{\phi}, \quad (1) \]

\[ 1 + r_t = a\phi(\sigma_t / \ell_t)^{(1-\phi)}, \quad (2) \]

where \( w \) is the real wage per efficiency unit of labor, \( r \) is the real interest rate given by international capital markets and assumed to be constant over time, and \( \sigma \) is the ratio between physical and human capital \((K_t/H_t)\).

In Lucas (1988) human capital is the engine of growth, since it is produced by a linear technology where the only input is human capital. In contrast, I want to focus on an economy where both human and physical capital are the engines of growth. Hence, I allow both types of capital to be inputs in the production of human capital (Rebelo, 1991). Specifically, an individual who has initially a level of skills equal to \( H_t \) and spends \( h_t \) units of time in education during \( t \) will increase \( H \) according to

\[ H_{t+1} = \delta H_t + \theta h_t K_t^2 H_t^{1-a} \]

\[ = (\delta + \theta h_t a_t^2) H_t, \quad (3) \]

where \( \theta \) is a parameter that measures the efficiency of the process of human capital accumulation. The parameter \( \delta \) is a positive number and could be less than one if human capital depreciates, or it could be greater than one if there is a minimum level of growth of skills (e.g., exogenous technological progress) for which no education is required.\(^4\) Eq. (3) represents also the aggregate law of motion of human capital.

It is important to note that the stock of capital relevant in Eq. (3) is the stock of capital used in the production of final goods. Thus, this is equivalent to assuming that physical capital exerts an externality in the process of human capital accumulation, which implies that education can be provided free of charge.

\(^4\)As discussed later, on the young generation inherits its human capital from the middle-aged generation. So, at any point in time, all members of working generations have the same human capital. Therefore, it is not necessary to keep track of \( H \) for individuals in different generations.
2.2. Households

The economy is populated by three overlapping generations. Population is constant and the size of each generation is normalized to one. Individuals maximize a time-separable logarithmic utility function. Individuals born at $t$ inherit the level of skills of their parents ($H_t$). In the first two periods of their life, individuals are endowed with one unit of nonleisure time. In the third period they are retired.\(^5\) When young, they can allocate their time to work or to study. By allocating a fraction $h$ of their endowment to education they are able to increase their level of skills in the middle age to $H_{t+1}$ according to (3).\(^6\) Therefore, education plays the role of transferring labor supply (in efficiency units) to the future. Although education is provided free of charge, individuals face the opportunity cost stemming from foregone labor income.

Individuals are assumed to be borrowing constrained. Specifically, I assume that individuals cannot borrow more than $\Psi - 1$ times their current income, where $\Psi$ is a constant greater than one and reflects the extent of borrowing constraints. In one extreme, $\Psi = 1$ represents the case where no borrowing is allowed. In the other extreme, as $\Psi$ goes to infinity, there are no borrowing constraints. Although this specification of borrowing constraint is not derived from imperfections in credit markets, it seems quite realistic since, usually, the amount of credit is conditioned by current income. One can also think of this constraint as imposed by the government.

Denoting consumption in period $t + j$ of an individual born at time $t$ by $c_{t,t+j}$ and the subjective discount factor by $\beta$, the consumer's maximization problem is

\[
\max_{\{c_{t,j}\}_{j=0}^2} \log(c_{t,t}) + \beta \log(c_{t,t+1}) + \beta^2 \log(c_{t,t+2}),
\]

subject to

\[
c_{t,t} + \frac{c_{t,t+1}}{1 + r} + \frac{c_{t,t+2}}{(1 + r)^2} = w_t H_t (1 - h_t) + \frac{w_{t+1} H_{t+1}}{1 + r}, \tag{4}
\]

\[
c_{t,t} \leq \Psi w_t H_t (1 - h_t), \tag{5}
\]

and

\[
0 \leq h_t \leq 1, \quad \{c_{t,t+j}\}_{j=0}^2 \geq 0. \tag{6}
\]

\(^5\)The results are essentially the same if the third generation is omitted, but that generation is required to have positive savings rate.

\(^6\)Strictly speaking, Eq (3) should be written as $H'_{t+1} = (\delta + h_t \theta \sigma^2) H_t$, where $j$ is an index for individuals, but to simplify the presentation, and using the fact that the solution will be symmetric, the superscript $j$ is omitted.
Eq. (4) is the intertemporal budget constraint and Eq. (5) is the borrowing constraint.\footnote{Note that only the borrowing constraints for the young generation are included, since this is the only generation with incentives to borrow. One could include the constraint $c_{t+1} \leq \Psi w_{t+1} H_{t+1}$, but it would not be binding.}

Individuals know that by spending $h_t$ units of time in education, skills grow according to (3), which substituted into (4) yields

$$c_{t,t} + \frac{c_{t,t+1}}{1 + r} + \frac{c_{t,t+2}}{(1 + r)^2} = H_t \left[ w_t(1 - h_t) + w_{t+1} \frac{\delta + h_t \theta \sigma_t^2}{1 + r} \right].$$

(7)

Before analyzing the optimal solution of the individual’s problem, it is instructive to analyze the case of no borrowing constraints. Since $h_t$ has no direct effect on individuals’ utility, the optimization can be separated into two stages: in the first stage $h_t$ is chosen so as to maximize the present discounted value of labor income, and then, in the second stage, the optimal consumption path is chosen. In addition, as a result of the assumption of linearity in the returns to education, the optimal solution will be a corner one, either $h_t = 0$ or $h_t = 1$.\footnote{The linearity assumption only facilitates the algebra and discussion, but is not essential to obtain the main results.}

The current cost of each unit of time devoted to education is $w_t h_t$, while the present value of additional income (earned next period) is $w_{t+1} H_t \theta \sigma_t^2 / (1 + r)$. Therefore, in the absence of borrowing constraints the optimal choice will be $h_t = 1$ [0] if $w_{t+1} \theta \sigma_t^2 / w_t (1 + r) > [<] 1$. Then, $h_t = 1$ [0] if the return of one unit of effective labor devoted to education is greater [less] than the return of one unit of effective labor devoted to savings.

Since I want to focus in the case where borrowing constraints are binding, I will define the following expression:

$$\Omega_t = \frac{w_{t+1} \theta \sigma_t^2}{w_t (1 + r)} - 1 > 0,$$

(8)

and assume that the primitive parameters are such that $\Omega_t$ is always greater than zero. This implies that in the absence of borrowing constraints individuals would choose $h_t = 1$.

Denoting by $\mu_t$ and $\lambda_t$ the Lagrange multiplier associated with the borrowing constraint (5) and the intertemporal budget constraint (7), respectively, the first-order conditions of the household’s problem are

$$1/c_{t,t} = \lambda_t + \mu_t,$$

(9)

$$\beta^2/c_{t,t+1} = \lambda_t / (1 + r),$$

(10)

$$\beta^2/c_{t,t+2} = \lambda_t / (1 + r)^2,$$

(11)

$$\mu_t = \lambda_t \Omega_t / \Psi.$$

(12)
These conditions are very similar to the ones of a three-period optimization without borrowing constraints, except for the additional term \( \mu_t \) in Eq. (9). The shadow value of consumption in the first period of life is greater than the marginal utility of wealth, since it also includes the cost of the borrowing constraint. Then, consumption in the first period is relatively more expensive. Even when \( \beta = 1 + r \) — which in the absence of borrowing constraints would imply a flat consumption path — consumption in the middle age is greater than in the youth. Finally, according to (10) and (11), the path of consumption in the second and third period of life is not affected by borrowing constraints, since they do not bind after youth.

Eq. (12) shows that \( \mu_t \) is always positive, and hence, the borrowing constraint is always binding, except in the limiting case of \( \Psi \to \infty \) where \( \mu_t = 0 \).9

The key difference between a borrowing-constrained economy and a non-constrained one is that in the latter individuals always prefer to devote their youth to education in order to maximize the present discounted value of income, which in turn is required to maximize lifetime utility. In contrast, in the borrowing-constrained economy individuals will choose not to maximize their human wealth in order to maximize utility.

Combining the above equations and using the budget constraint it can be verified that the optimal choice of time devoted to education is given by

\[
h_t = \frac{\Psi \Delta_t - 1 - \delta/(1 + r)}{\Psi \Delta_t + \Omega_t}, \tag{13}
\]

where

\[
\Delta_t = 1 + \beta(1 + \beta)(1 + \Omega_t/\Psi).
\tag{14}
\]

Inspection of Eq. (13) reveals that for a given value of \( \sigma_t \) (and hence \( \Omega \) and \( \Delta \)) and \( r \), the optimal choice of \( h_t \) is increasing when borrowing constraints are relaxed \((dh/d\Psi > 0)\), and it converges to one (the unrestricted optimum) when \( \Psi \) goes to infinity. The lower the value of \( \Psi \), the greater are the incentives to work when young.10

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9 \( \Psi \) needs to go to infinity for \( h \) to approach 1, because when \( \Psi \) goes to infinity and \( h \) to 1, labor income of the young is approaching 0.

10 Note that the parameter values could be such that the solution to (13) is negative. In this case the optimal solution is to set \( h = 0 \). This case shows that despite the no-constrained optimal is \( h = 1 \), borrowing constraint could induce no human capital accumulation at all. To rule out the possibility that \( h = 0 \) it is necessary to assume that \( \text{th} \sigma^2(\beta + \beta^2) > \delta \), which in order to simplify the discussion I assume to hold.
3. The effects of borrowing constraints

In this section I characterize the equilibrium growth path and discuss the effects of borrowing constraints on the rate of human capital accumulation, savings, and growth. It is shown that both, growth and human capital accumulation, increase as borrowing constraints are relaxed.

3.1. Equilibrium growth

A convenient feature of the small open economy with free (physical) capital mobility is that there is no transitional dynamics and the economy is always in its unique steady state growth path. This is the result of perfect capital mobility, since physical capital will instantly flow to keep constant the real interest rate at its international level. Hence, for given \( h \) and \( \sigma \) the evolution of the economy will be governed by the difference equation (3), which implies that \( Y, K, \) and \( H \) will all grow at the same rate, denoted as \( \gamma \) and equal to \( \delta + \sigma \alpha^2 \theta - 1 \). Finally, by aggregating the budget constraints it can be verified that aggregate consumption will also grow at \( \gamma \).

From (2), the equilibrium ratio between physical and human capital is given by

\[
\sigma_t = (2 - h_t)((1 + r)/a\phi)^{-1/(1 - \phi)},
\]

where \( 2 - h \) is total (raw) employment. Real wages per efficiency units will be given by Eq. (1) after replacing \( \sigma \) by (15), and they will be constant over time:

\[
w_t = a(1 - \phi)((1 + r)/a\phi)^{-\phi/(1 - \phi)}.
\]

Finally, the rate of growth can be obtained from Eq. (3):

\[
1 + \gamma_t = \delta + h_t(2 - h_t)\alpha^2 \theta \left( \frac{1 + r}{a\phi} \right)^{-\sigma/(1 - \phi)}.
\]

To show that the economy always grows at a constant rate it is enough to show that the solution to \( h \) implicit in (13) is unique, and therefore, \( \sigma \) and \( \gamma \) are time-invariant, and we can drop the time indices on \( h, \sigma, \) and \( \gamma \).

Lemma 1. Assume that

\[
(2 - h)^{x - 1} \frac{\theta(a\phi)^{\sigma/(1 - \phi)}}{(1 + r)^{y(1 - \phi + x)/(1 - \phi)}} < 1;
\]

then, the solution for \( h(\in [0, 1]) \) in (13) is unique.

Proof. See Appendix A. \( \square \)
Now, the system of three nonlinear equations, given by (13), (15), and (17), can be used to analyze the effects of borrowing constraints on human capital accumulation, growth, and the ratio between physical and human capital. The main results can be summarized in the following proposition:

**Proposition 1.** Under the assumption of Lemma 1 we have that:

(i) \( \frac{dh}{dΨ} > 0 \), 
(ii) \( h < 1 \) and \( \lim_{Ψ \to \infty} h = 1 \), 
(iii) \( \frac{dσ}{dΨ} < 0 \), 
(iv) \( \frac{dγ}{dΨ} > 0 \).

**Proof.** See Appendix A. □

The first and second part of the proposition show in general equilibrium that borrowing constraints reduce time devoted to education, and as borrowing constraints become less tight \( h \) converges to its unconstrained optimum.

It has to be noted that the condition in Lemma 1 is only a sufficient condition for the results to hold, and insures that the indirect effect of changes in \( h \) on \( Ω \) (through changes in \( σ \)) do not offset the direct effect on the LHS of (13). An equivalent expression for this assumption is: \( 2 - h > α(1 + Ω) \). Simple inspection of the parameters indicates that this condition is likely to hold since \( 2 - h > 1 \), \( α < 1 \), and \( Ω \) is small if the difference between the returns to human and physical capital is small. 11

Borrowing constraints also increase the steady-state ratio of physical capital to human capital and the rate of growth. An increase in \( Ψ \) causes a rise in \( h \) and an offsetting decline in \( σ \). However, Proposition 1 establishes that the latter effect never dominates the positive growth effects of an increase in \( Ψ \), since \( h(2 - h)^2 \) [see (17)] is increasing in \( h \).

3.2. Borrowing constraints and savings

An important implication of life-cycle models is that borrowing constraints increase aggregate savings. In this subsection I show that excluding growth effects this implication still holds, but once growth effects are included this result turns out to be ambiguous. Part of this ambiguity stems from the effects of growth on aggregate savings, since savings by the young would dominate dissaving by the old, and hence, savings would increase with growth. This is the result of aggregation over individuals, and I call it ‘aggregate effect’. As Carroll and Weil (1994) point out, however, the evidence shows that even individual savings increase with growth. The model in this paper can also explain this ‘individual effect’.

11 For details on the exact equations see Appendix A.
Total (domestic) savings is equal to private agents' wealth accumulation. Total wealth, in turn, equals the sum of wealth of young and middle-aged generations, since the old generation has zero net wealth. Young individuals have income equal to $w_tH_t(1 - h)$ and consume $w_tH_t\Psi(1 - h)$. Middle-aged individuals (born at $t - 1$) earn $w_tH_t$ as labor income, pay interest and principal for their debt equal to $(1 + r)(\Psi - 1)(1 - h)w_{t-1}H_{t-1}$, and consume according to (9)–(11). After some manipulations, and aggregating over all cohorts, the following expression for total wealth can be obtained:

$$W_t = w_tH_t\left\{1 - (\Psi - 1)(1 - h)\left[1 + \frac{1 + r}{1 + \gamma}\right] - \frac{\beta(1 + r)(1 - h)(\Psi + \Omega)}{1 + \gamma}\right\}$$

$$\equiv w_tH_t f,$$  \hspace{1cm} (18)

where $f$ represents the ratio between aggregate wealth and average labor income.

Savings is equal to the rate of wealth accumulation, which in steady state equals $W_t\gamma/(1 + \gamma)$. Finally, the savings rate, denoted by $s$ and equal to $W_t\gamma/[(1 + \gamma)Y_t]$, can be expressed as

$$s = \frac{\gamma(1 - \phi)}{(1 + \gamma)(2 - h)} f \equiv gf.$$  \hspace{1cm} (19)

Consider first the case that $h$ is not affected by changes in the extent of borrowing constraints. In this case it is easy to show the following result:

**Proposition 2.** If $dh/d\Psi = 0$, a tightening of borrowing constraints (decline in $\Psi$) will increase the savings rate.

**Proof.** See Appendix A. ∎

This is because young individuals dissave (borrow) less when borrowing constraints become more stringent, and middle-aged ones have a smaller financial burden, which allows them to increase savings for retirement.

Nevertheless, once we allow $dh/d\Psi$ and $d\gamma/d\Psi > 0$, the total effect of $\Psi$ on savings is ambiguous, and even at the individual level savings may increase with growth. By examining Eqs. (18) and (19), we can see that an increase in $\gamma$ implies that the rate of growth of wealth also increases (through an increase in $g$). This is the usual aggregate effect. But an increase in $\Psi$ also increases total wealth ($f$), resulting in an increase in individual savings. The reason for this is that, although $W_t\gamma$ unambiguously declines with an increase in $\Psi$,\footnote{It is easy to verify that the expression $\Psi(1 - h)$ is increasing in $\Psi$.} $W_{t-1}\gamma$ may increase. This is the result of higher income during middle age when growth increases (for the same initial income) and the consequent desire to increase consumption during retirement, which is achieved by increasing savings.
3.3. The closed economy

The main difference between the open economy and the closed economy is that in the latter savings decisions determine the rate of physical capital accumulation, and, therefore, borrowing constraints will affect growth through both, the savings rate and the rate of human capital accumulation, with likely opposite effects. In this case, however, the model has no closed-form solution, and since physical capital will not adjust instantly, the closed economy will display nontrivial dynamics.

I have performed some simulations for the steady state of the system (details are available upon request). To solve the model, the stock of capital will be given by equilibrium in the capital markets, that is \( W_t = K_{t+1} \), which after substituting (15), (16), and (18) can be written as

\[
(1 + \gamma)(2 - h)^{\phi} = fa(1 - \phi)\phi^{\phi - 1}. \tag{20}
\]

Now we can use Eqs. (13), (15), (17), and (20) to solve for \( h, \sigma, \gamma, \) and \( r \).

The simulations were performed for all values of \( \phi \) and \(\alpha\) between 0.1 and 0.9 at intervals of 0.1. It was found in all simulations that a tightening of borrowing constraints decreased growth and human capital accumulation, but the effects on savings were ambiguous. As \( \alpha \) (the share of physical capital in the production of human capital) increased the negative growth effects of borrowing constraints were increasing, since, as expected, the effects through human capital become more important. For low values of \( \alpha \) it was more likely to find a positive effect of borrowing constraints on savings, offsetting the negative effects on education. Overall, the simulations show that for plausible parameter values the results from Proposition 1 also hold in the closed-economy version of the model.

4. Empirical evidence

This section presents cross-section evidence on the relationship between human capital accumulation, economic growth, and the extent of borrowing constraints. The analysis is performed for two samples of countries. The first one is comprised of 20 OECD countries and the other one of 63 developing countries. The only difference between the two samples is the availability of proxies for borrowing constraints. Appendix B describes the data and lists the countries in each sample.

4.1. Human capital accumulation and borrowing constraints

Human capital accumulation in this paper refers to formal education, and therefore, natural measures are enrollment ratios. I use secondary (SEC) and tertiary (TER) school enrollment ratios from UNESCO.

Jappelli and Pagano (1994) have constructed two proxies for the extent of borrowing constraints in a sample of OECD countries and some developing
countries. The first measure they construct is consumer credit as a fraction of national income in the 1970s \( (CONSC) \) and the 1980s.\textsuperscript{13} I use data for 1970 when available (13 OECD countries) and the value of 1980 when 1970 is not available (4 OECD countries). The second proxy is the maximum loan-to-value ratio \( (LTV) \) for the purchase of a house, which is available for most of the countries in the early 1970s. For the remaining countries, I use the data for 1980.\textsuperscript{14}

The model predicts that the only determinant of human capital accumulation and growth is the extent of borrowing constraints \( (\Psi) \). However, I control for other variables that in a more general specification of the model would appear to be relevant. This also helps to check robustness when additional regressors are included (Levine and Renelt, 1992). I estimate the following regression:

\[
h = \beta_0 + \beta_1 \Psi + \beta_2 Y + \beta_3 E,
\]

where \( \Psi \) is a proxy for the extent of borrowing constraints, \( Y \) is per capita income, and \( E \) is education expenditure over GDP. \( h \) \( (=SEC, TER) \) is measured as the average for 1975, 1980, and 1985. To avoid endogeneity problems, \( \Psi \) and \( Y \) are measured at the beginning of the sample period, that is, in the early 1970s.\textsuperscript{15}

There are no similar data on borrowing constraints for large cross-sections of countries. However, to provide some evidence for developing countries, I use as a proxy the ratio between total credit from the banking system to the nonfinancial private sector (line 32d IFS) and GDP. This variable is called \( CREDIT \).

Although \( CREDIT \) may be a good proxy for the overall degree of financial development, it has some problems when used to proxy the ability of households to borrow. First, \( CREDIT \) not only includes households credit, but also credit to firms. And second, consumer credit is not only intermediated through the banking system. Indeed, the correlations between \( CREDIT \) and their proxies for borrowing constraints among OECD countries are low. However, as reported in De Gregorio and Guidotti (1995), \( CREDIT \) is also not a very good indicator of financial development in OECD countries, because their financial system develops

\textsuperscript{13}Most of these data are measured at the beginning of each decade and they are highly persistent across time.

\textsuperscript{14}Four OECD countries are excluded from the analysis. Iceland, Luxembourg, and Switzerland because of the lack of data, and Sweden because is an outlier. Sweden’s consumer credit is the highest, being twice the one of the following countries and more than four times the average. Its maximum \( LTV \) is also the lowest. The results, however, do not change when Sweden is included, but the precision of the estimates decreases.

\textsuperscript{15}The regressions reported below do not change significantly when contemporaneous, rather than initial, values of the independent variables are used.
to a large extent outside the banking system. Therefore, CREDIT should be more informative about borrowing constraints faced by households in developing countries than in OECD countries.

Another explanatory variable in the regression is per-capita GDP in 1970 (GDP70). This variable is included to capture income effects in human capital accumulation, which are absent in the model due to the specific functional forms assumed.16

Human capital accumulation is also related to the relative size of expenditure in education. Education expenditure may affect positively human capital accumulation because it may reflect low private costs or high quality of education.17 Also, education expenditure may be a proxy for the quality of education. To control for these factors, current expenditure in education as a share of GDP (EDU) is used as another independent variable in the regressions.18

The main results are presented in Table 1. The hypothesis of heteroscedasticity could not be rejected for all the specifications using Lagrange multiplier tests, and all standard errors were computed using White’s robust procedure.

The results for OECD countries show that when GDP70 and EDU are omitted, an increase in LTV and CONSC increase secondary and tertiary school enrollment ratios. However, once GDP70 and EDU are included, some of the results become weaker. In particular, LTV and CONSC become insignificant in the regressions for TER. The results for SEC appear to be more robust, since both LTV and CONSC are positive and statistically significant in all regressions. On the other hand, and as expected, EDU is positively correlated with enrollment ratios, although not significant for TER. In turn, GDP70 has a positive and significant effect only on TER.

Overall the results for the OECD countries lend some support to the hypothesis that increasing borrowing constraints reduce the rate of human capital accumulation. The results for CONSC are more conclusive than those for LTV, and

16In terms of the model of this paper, a simple way to include income effects would be to assume that θ is a function of the stock of human capital, as in Azariadis and Drazen (1991), and thus, indirectly it would be a function of Y. Alternatively, one could assume that individuals enjoy utility from education, in which case the income effect in (21) would be related to the income elasticity of the demand for education.

17There are other fiscal incentives to human capital accumulation not included in data on education expenditure, such as the case of loan guarantees or tax incentives. Unfortunately, there are no data available for these variables.

18In a previous version of this paper I reported the results with other four proxies for education expenditure in the sample of OECD countries: The first two measures were public expenditure in (i) secondary and (ii) tertiary level per student enrolled. The other two measures were public expenditure in (i) secondary and (ii) tertiary level as a share of GDP. Those variables were found not to have a significant effect on h. Their inclusion did not affect qualitatively the effects of Ψ on h.
Table 1
School enrollment ratios and borrowing constraints

<table>
<thead>
<tr>
<th>Regression no.</th>
<th>Constant</th>
<th>Borrowing constraints</th>
<th>Initial GDP (GDP70)</th>
<th>Education exp. (EDU)</th>
<th>( R^2 )</th>
<th>No. obs.</th>
</tr>
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*\( t \)-statistics in parentheses. See Appendix B for data description and list of countries.

\( ^a \) Department variable.

\( ^b \) Proxy for borrowing constraints.
secondary school enrollment ratios appear to be the most affected by borrowing constraints. For secondary school enrollment ratios in developing countries the coefficients on \( CREDIT \) are, as expected, positive and statistically significant. For tertiary school enrollment, similarly to the results obtained for the sample of OECD countries, the correlation between human capital accumulation and \( CREDIT \) is strong, but it becomes insignificant after income is included.

4.2. Growth and borrowing constraints

To analyze the effects of borrowing constraints on growth I follow Jappelli and Pagano (1994) by re-estimating Barro's (1991) regressions including the different proxies for borrowing constraints. Borrowing constraints affect growth by affecting the savings rate and human capital accumulation. In addition, if the proxies for borrowing constraints are correlated with financial market development (such as \( CREDIT \) in developing countries), including \( \Psi \) in growth regressions will capture also the effect stemming from financial deepening. When the measures of schooling are excluded from the regressions, the coefficient on \( \Psi \) represents the total effect of borrowing constraints on growth. When enrollment rates are included, the coefficient of \( \Psi \) captures the effect coming from savings and financial development, and it should decline compared to the regressions that exclude enrollment rates.

Table 2 presents the results for the period 1970–85, where the proxies for borrowing constraints are predetermined. The results using initial GDP and indices of schooling are reported regardless their significance. However, other ‘Barro regressors’ were included only if they were statistically significant. Several results are interesting to note. First, only for \( LTV \) and when the OECD sample is expanded to include developing countries there is a positive and significant relationship between borrowing constraints and growth (see regressions 2.3 and 2.4). These results contrast with those of Jappelli and Pagano (1994), who find a positive effect of borrowing constraints (measured by \( LTV \)) on growth. The main explanation for the conflicting results is that they include some non-OECD

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19These findings are consistent with micro evidence that has found that important determinants of education (attainment and quality) are the financial resources of families. See Behrman, Kletzer, McPherson, and Schapiro (1994).

20Barro (1991) measures schooling at the beginning of the period, which is highly correlated with schooling in subsequent years, and therefore whether initial schooling or average schooling during the period is included does not affect the results. As in other work I use schooling at the beginning of the period.

21For comparison purposes and to provide additional evidence on the robustness of the results, I have run regressions for the period 1960–85. In those regressions schooling indices, initial GDP, and government expenditure are always significant. The results for the proxies of borrowing constraints are broadly in line with those of Table 2. They are available upon request.
countries. In particular, excluding Taiwan and Korea from their sample makes the coefficient on $LTV$ insignificant in all specifications (see, e.g., regression 2.5).

Second, restricting attention only to OECD countries, $LTV$ has no effect on growth regardless the inclusion of schooling variables. $CONSC$, in turn, is positively correlated with growth. For developing countries the evidence also shows a positive effect of the relaxation of borrowing constraints on growth.  

Third, in all regressions, the inclusion of schooling variables reduces the coefficient (in absolute value) of borrowing constraints. This is consistent with the model since it indicates that part of the effects of borrowing constraints on growth are coming through their impact on human capital.

Furthermore, the coefficients on $CONSC$ and $CREDIT$ (regressions 2.5 to 2.8) still remain positive when schooling variables are included. Considering that borrowing constraints increase savings, the coefficients on $CONSC$ and $CREDIT$ could have been expected to be negative. The fact that they are still positive may indicate that $CONSC$ and $CREDIT$ are also capturing other forms of financial development. In addition, it may also indicate that the indirect effect of borrowing constraints on growth through national savings is small.

A summary of the results is presented in Table 3. The table presents the effects of changing each of the three proxies for borrowing constraints by one standard deviation on secondary school enrollment ratios and growth. The coefficient used to calculate the effects on $SEC$ are the ones in the regressions that include $EDU$ and $GDP70$ as regressors. The ‘net effect on growth’ is the effect of borrowing constraints on $SEC$ multiplied by the coefficient of $SEC$ on the growth regressions. On the other hand, the ‘total effect on growth’ is the effect of each proxy for borrowing constraints implied by the growth regressions that exclude schooling indices. The effects on schooling and the net effect on growth are similar for both variables in the OECD sample ($LTV$ and $CONSC$) and higher in developing countries ($CREDIT$). An increase in one standard deviation in $LTV$ and $CONSC$ is associated with an increase in growth of less than 0.1 percentage point, while the effect of $CREDIT$ on growth is about 0.3 percentage point. Since the point estimate of the coefficient of $LTV$ in the growth regression that excludes schooling indices is negative (although statistically insignificant) the total effect of $LTV$ is negative, but rather small. Finally, it is interesting to note that the net effect of $CONSC$ and $CREDIT$ on growth is about one fourth their total effect. As discussed above, this result suggests that $CONSC$ and $CREDIT$ affect positively growth through other channels in addition to secondary school enrollment ratios.

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22This is consistent with work by Gertler and Rose (1991), King and Levine (1993), and De Gregorio and Guidotti (1995), who use $CREDIT$ as proxy for financial development and find that it affects positively growth.

23The effects of borrowing constraints on savings have been examined by Jappelli and Pagano (1994). They find that borrowing constraints affect positively savings.
Table 2
Growth and borrowing constraints, 1970–85

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<tr>
<th></th>
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Dependent variable: per capita GDP growth 1970–85. t-statistics in parentheses. See Appendix B for data description and list of countries.

$^a$ Includes seven non-OECD countries.
$^b$ Excludes Korea and Taiwan from regression 2.4.
$^c$ Includes government expenditure, indices of assassinations, and revolutions and coups as additional regressors.

5. Conclusions

This paper has explored the implications of borrowing constraints in an endogenous growth model where both human and physical capital are the engines of growth, and has shown that ignoring the effects that borrowing constraints may
Table 3
Effects of borrowing constraints on human capital accumulation and growth

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<th>CREDIT</th>
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<td>Total effect on growth</td>
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<td>0.28</td>
<td>1.35</td>
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</table>

All figures are expressed in percent. Effects on SEC and growth are percentage points changes resulting from a one standard deviation change in the variable.

have on human capital accumulation may change the implications for savings and growth from traditional models of the life-cycle hypothesis.

The model of this paper has shown that borrowing constraints will reduce time devoted to formal education and growth. Finally, absent growth effects, savings increase with borrowing constraints, as emphasized in the traditional life-cycle models. Once growth effects are incorporated, borrowing constraints may reduce savings.

The empirical evidence presented here suggests that tight borrowing constraints are negatively correlated with enrollment ratios and growth, lending support to the main theoretical implications from the model developed here. In the case of human capital, when other variables are included – initial income and education spending – the effects of borrowing constraints on human capital accumulation only remain robust for secondary school enrollment ratios, regardless the proxy for borrowing constraints and the sample of countries used. For growth, the evidence shows that borrowing constraints reduce growth, except when $LTV$ is used as a proxy for the degree of borrowing constraints, in which case the effect is insignificantly different from zero.

An important motivation for this paper is whether a high savings rates, presumably caused by borrowing constraints, may translate in faster growth. There are other areas that have received great attention in the literature on savings, and their implications for growth are also not clear. An important case, as reflected in Modigliani’s remarks quoted in the introduction of this paper, is the role of precautionary savings. In this case, the capital market imperfection is the inability of individuals to fully insure against random income fluctuations. Although higher uncertainty may increase the savings rate, one cannot easily conclude that will also increase output growth. Higher uncertainty will lead to higher savings in the form of highly liquid and low productive assets with consequent negative effects on growth (Bencivenga and Smith, 1991). The analogy with this paper is that there is more than one asset that can be used for wealth accumulation, and the resulting assets composition may be harmful for growth. In this paper, borrowing constraints result in a combination between physical and human capital that leads to lower growth. In the case of precautionary savings, the lack of
insurance results in a composition between liquid (low productivity) and illiquid (high productivity) assets that leads to lower growth.

It could be argued, analogously to Barro (1974), that allowing parents to make transfer to their children could overcome the credit market imperfections. Indeed, following Altig and Davis (1989), one can conclude that transfers from parent to children will ameliorate the effects of borrowing constraints on education (by allowing to have higher consumption and more education when young), but will not eliminate them. The reason is that borrowing constraints, contrary to lump sum taxes as in Barro (1974), are distortionary, thereby affecting the marginal decision between consumption and education.

The model could be used to analyze other relevant issues. For example, following Loury (1981), and recently Caballé (1991) and Glomm and Ravikumar (1992), one could assume that parents decide the amount and/or quality of the education received by their offsprings. This issue is relevant in poor countries, where the trade-off between working and education exists since childhood. Parents facing borrowing constraint may provide low levels and quality of education. This could in turn have important effects on the dynamics of income distribution. Finally, another relevant extension would be to exploit the relationship found for savings and growth. The paper shows that even at the individual level, savings may increase with growth. If in addition savings is a cause for growth, there is a potential for multiple equilibria and low-savings traps.

Appendix A: Proof of lemma and propositions

Proof of Lemma 1

Substituting \( A \) from (14) into (13), the equilibrium value of \( h \) is given by

\[
h = \frac{\psi + \beta(1 + \beta)(\psi + \Omega) - 1 - \delta/(1 + r)}{\psi + \beta(1 + \beta)(\psi + \Omega) + \Omega}.
\]  

(A.1)

Using Eq. (8) and \( \sigma \) from (15), we can define \( \Omega = \Omega(h) \) as follows:

\[
\Omega = \frac{(2 - h)^\phi}{(1 + r)^{(1+\sigma-\phi)/(1-\sigma)}} - 1.
\]  

(A.2)

\footnote{On this topic see Hall (1986), Altig and Davis (1989), Guiso and Jappelli (1992), and Laitner (1993).}

\footnote{In a previous version of this paper I show, in a simplified version of the model, that the main results of the paper still hold when preferences are specified as in Barro (1974). Details are available upon request.}

\footnote{See on this point Becker (1965), Loury (1981), Banerjee and Newman (1991), and Galor and Zeira (1993).}
Denoting the RHS of (A.1) as $F(\Omega)$, (A.1) can be rewritten as $h = F(\Omega(h)) \equiv G(h)$.

The equilibrium is a fixed point of $G(h)$. To show the fixed point exists and is unique define $P(h) \equiv h - G(h)$. Since $G(h) \in (0,1)$, then $P(0) < 0$ and $P(1) > 0$. By continuity of $P$ there must be at least one $h$ such that $P(h) = 0$. This proves existence of at least one equilibrium.

To prove uniqueness I first will show that $P'(h)$ is positive:

$$P'(h) = 1 - \frac{dG}{dh} = 1 - \frac{dF}{d\Omega} \frac{d\Omega}{dh}.$$  

Solving the derivatives we have

$$\frac{dF}{d\Omega} = -\frac{\Psi - 1 - \delta/(1 + r)}{[1 + \beta(1 + \beta)](\Psi + \Omega)^2} > -1,$$

(A.3)

$$\frac{d\Omega}{dh} = -\frac{\alpha(1 + \Omega)}{2 - h} < 0.$$  

(A.4)

$dF/d\Omega > -1$, since when it is negative the numerator of (A.3) is less than $\Psi$ and the denominator is greater than $\Psi$ (recall that $\Psi > 1$). The inequality in (A.4) is trivial. Note also that the sign of $dF/d\Omega$ is independent of $\Omega$ and $h$, and hence $G$ is either increasing or decreasing in $h$, but not both. Regarding the slope of $F$ we can distinguish two cases:

- $dF/d\Omega > 0$. In this case $dG/dh$ is negative for all $h$, and hence, $P'(h)$ is positive.
- $dF/d\Omega < 0$. In this case $dF/d\Omega > -1$. Using the assumption of the lemma, which is equivalent to $d\Omega/dh > -1$, we have that $dG/dh = (dF/d\Omega)(d\Omega/dh) < 1$. Therefore, $P'(h)$ is positive.

Given that always $P'(h)$ is positive assume there are two fixed points, $h_1$ and $h_2$ ($h_1 < h_2$). Using the mean value theorem we know there exists $h_3$ in $(h_1, h_2)$, such that $P''(h_3) = (P(h_2) - P(h_1))/(h_2 - h_1) = 0$, since $P(h_1) = P(h_2) = 0$. This is a contradiction with the fact that $P'$ is strictly positive. This proves that the equilibrium must be unique. □

Proof of Proposition 1

(i) Totally differentiating (A.1):

$$\frac{d\Omega}{d\Psi} = \frac{\partial h}{\partial \Psi} + \frac{dF}{d\Omega} \frac{d\Omega}{dh} \frac{dh}{d\Psi}.$$  

(A.5)

It is easy to verify that $\partial h/\partial \Psi > 0$ (this is the 'partial equilibrium effect'), and therefore, a necessary and sufficient condition for $dh/d\Psi$ to be positive is that $(dF/d\Omega)(d\Omega/dh) < 1$. After using Eqs. (A.3) and (A.4), it can be verified
that this condition holds if $2 - h > \alpha(1 - \Omega)$. The rest of the proposition, (ii), (iii), and (iv), are immediate from (A.1), (15), and (17), respectively. □

Proof of Proposition 2

First, note that a constant value of $h$ implies constant values for $\sigma$, $\gamma$, and $\Omega$. Therefore, the effects of changes in $\Psi$ on the savings rate are equal to the effects of $\Psi$ in $f$ [see (19)], keeping $h$, $\gamma$, and $\Omega$ constant. Differentiating it can be verified that $ds_t/d\Psi$ is negative. □

Appendix B: Data sources

The source of the data used in Section 4.1 are (data are in an appendix available upon request):

- LTV: Maximum loan-to-value ratio for the purchase of a house in the 1970s, from Jappelli and Pagano (1994).
- CREDIT: Claims from the banking system on the nonfinancial private sector over GDP, lines 32d over 99b of IFS. Average 1969–71.

The regressions of Section 4.2 use the Barro (1991) data set augmented with the data on CREDIT, CONSC, and LTV.

Sample of OECD countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Turkey, United Kingdom, and United States. In Table 2 the sample of OECD countries was augmented with the following countries: Israel, Korea, Malaysia, Philippines, Taiwan, Thailand, and Mexico.

Sample of developing countries: Algeria, Argentina, Barbados, Bolivia, Brazil, Burma, Burundi, Cameroon, Central African Republic, Chile, Colombia, Costa Rica, Cyprus, Dominican Republic, Ecuador, Egypt, El Salvador, Ethiopia, Fiji, Gabon, Ghana, Guatemala, Guyana, Haiti, Honduras, India, Indonesia, Iran, Israel, Jamaica, Jordan, Kenya, Madagascar, Malawi, Malaysia, Malta, Mauritius, Mexico, Morocco, Nepal, Nigeria, Pakistan, Panama, Paraguay, Peru,
Philippines, Rwanda, Senegal, Sierra Leone, Singapore, South Korea, Sri Lanka, Sudan, Tanzania, Thailand, Togo, Trinidad Tobago, Tunisia, Uganda, Uruguay, Venezuela, Zaire, and Zambia.

References

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