1. Consider an overlapping generations model where all agents except the initial old live for two time periods. They all have an endowment of \( y > 0 \) units of perishable corn when young and nothing when old. At each date \( t \), \( N_t \) young agents are born where \( N_t = (1 + n)N_{t-1} \). Assume corn is perishable and cannot be saved. Furthermore, they all have the following utility function:

\[
U(c_1, c_2) = \beta \log c_1 + (1 - \beta) \log c_2
\]

where \( 1 > \beta > 0 \).

(a) Write down the per-period budget constraints facing a typical young agent if there are no savings instruments. Draw the intertemporal budget constraint. Label the axes.

(b) Write down the problem faced by a young agent at date \( t \). Write down the problem faced by an old agent at date \( t \). Compute the solution to the agent’s problem.

(c) Write down the constraint faced by a social planner in a steady state. Draw the intertemporal constraint facing the planner. What is the problem faced by a social planner?

(d) Compute the choices of \((c_1, c_2)\) that the social planner would pick for this economy. Call this \((c_{1sp}, c_{2sp})\).

Consider the same economy as above except now the initial old have printed $ \( M \). These are intrinsically worthless green pieces of paper (called $) with pictures of presidents on them. Let \( p_t \) denote the price level at \( t \). Let \( z_t \) denote the demand for money for a single young agent at \( t \).

(e) Write down the per period budget constraints facing a typical agent in a steady state. Draw the intertemporal budget constraint.

(f) Compute the steady state real return to holding $1.

(g) Compute the optimal choices of \((c_1, c_2)\) that the agent would choose. Call them \((c_{1m}, c_{2m})\). Do they differ from the ones in 1(g)? Explain.
(h) Would anything change if agents had the endowment vector \((y, 2y)\) and 
\((1 - \beta) - \frac{2\beta}{1+n} < 0\)?

2. Consider a standard pure-exchange 2-pd lived OG economy. There are \(N_t\) identical young agents at time \(t\) and population grows at the rate \(n > -1\), \(N_0 > 0\). The endowments are denoted by \(w_1\) and \(w_2\), both strictly positive. All agents have the utility function

\[ U(c_1, c_2) = \beta \ln c_1 + (1 - \beta) \ln c_2 \]

with \(\beta \in (0, 1)\). Only the initial old are additionally endowed with \(M > 0\) units of money.

(a) Write down the problem faced by young agents.
(b) Write down the problem faced by the initial old agents.
(c) Compute the savings function of an young agent.
(d) Write down the conditions under which this is a Samuelson case economy.
(e) Define a competitive equilibrium for this economy.
(f) Write down the law of motion for \(z(t)\) where \(z(t) = \frac{M}{N_t p_t}; z_0 > 0\).
(g) Define a stationary competitive equilibrium for this economy. Compute the stationary equilibria for \(z(t)\). Is currency “valued” at all the stationary equilibria? Explain.
(h) Explicitly solve for the entire time path of \(z(t)\). Consider all possible ranges of \(z_0\).
(i) Investigate the stability properties of the stationary equilibria under the assumption that this is a Samuelson case economy.
(j) Investigate the stability properties of the stationary equilibria under the assumption that this is a Classical case economy.
(k) Comment on the optimality properties of the stationary equilibria in each case.
(l) Write down the equation describing the entire time path for the price level \(p(t)\) for different ranges of \(p_0\).
(m) Comment on the asymptotic behavior of \(p(t)\).
(n) Suppose \(M\) is doubled to \(2M\). What is the effect of this change on the savings behavior of young agents? What is the effect on the time path for \(p(t)\)? Explain your finding.