

# Time Inconsistency and Social Security

Önsel Emre\*

The University of Chicago

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## Abstract

This paper identifies a new role for social security and, based on this role, proposes a mandatory savings plan. The goal of the mandatory savings plan is to deal with the moral hazard problem created by unobservable heterogeneity in skill across households, combined with the time inconsistency problem of government policy. The time inconsistency problem prevents the government from credibly committing to let consumption of the elderly fall below a minimum threshold. The paper shows that a government without commitment can attain the allocation the committed government implements if and only if it uses a mandatory savings system.

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## 1. INTRODUCTION

This paper identifies a new role for social security, one that deals with the moral hazard problem arising from unobservable heterogeneity in skill, in combination with a time inconsistency problem of government policy. The paper shows that a mandatory savings plan solves this problem with the least resource reallocation.

This paper analyzes social security in a simple two-period model where individuals differ in terms of skill that is unobservable by the government. The role of the government is to insure a minimum standard of living for everyone, but the time inconsistency problem prevents the government from credibly committing to the second-period policy. Besides, due to costly administration and distortionary taxation, the government wants to keep its budget small. In this environment, rational agents, taking the government's lack of commitment into consideration, might choose to undersave for retirement with the expectation that they will be bailed out if they end up with too little resources<sup>1</sup>. The objective of the government in the initial period is to design a policy that guarantees everyone a minimum level of consumption, given that the second-period policy will grant benefits to anyone with consumption below a minimum threshold. This paper shows that the mandatory savings plan attains this objective with the least reallocation.

This paper contributes to the large literature on social security in several respects. First, it provides an alternative rationale for social security. The literature has identified several roles for social security, which are grouped into income redistribution, correcting market inefficiency arising from adverse selection, and paternalism towards myopic agents by Diamond (1977)<sup>2</sup>. This paper identifies a new role, one that survives even when governments have their separate redistribution programs, there are markets and/or government programs that can perfectly insure against income and lifetime uncertainty, and rational agents are perfectly informed about what the future might bring. Second, to the best of my knowledge, this is the first paper studying the problem of undersaving for retirement as a result of governments' commitment problem. Most of the theoretical research on social security studied the problem of undersaving due to time-inconsistent preferences (Feldstein, 1985 and 1987, Imrohroglu et al., 2003, Amador et al., 2006, among others).

Even though it is studied in a new framework, the proposed social security system is not

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<sup>1</sup>This is simply an application of time inconsistency problem introduced by Kydland and Prescott (1977).

<sup>2</sup>More recent and detailed analysis of the rationales for social security is provided by Mitchell and Zeldes (1996). The adverse selection problem in the context of social security is studied by Eckstein et al. (1985).

a new one. In terms of capital formation, the superiority of fully funded social security has been known at least since Feldstein (1974) and Samuelson (1975)<sup>3</sup>. The mandatory savings system of this paper is very much in line with the personal security accounts of Boskin (1986) and Boskin et al. (1988), except that in the present paper, social security savings are raised to the minimum benefit level at retirement. In Boskin's (1986) proposal, each social security payment of a working individual is raised to the minimum contribution level before being saved at social security accounts, which is potentially more distortionary at the lower end of income distribution.

The paper is organized as follows. Section 2 provides the theoretical analysis of social security in a simple two-period model and the third section concludes.

## 2. A TWO-PERIOD MODEL

In this model, preferences and technology are standard and very simple. The novel feature is the objective of the government. The government's goal is to guarantee a minimum level of consumption for everyone with minimal resource reallocation. This section first discusses the planning problem of government with commitment, then turns to the planning problem of government without commitment. If the government cannot commit to the second-period policy, it might revise the initially determined policy in the future and pay anyone with total savings less than a minimum threshold. The government in the first period may not commit to the second-period policy but can design the first-period policy such that, given the second-period policy, the government budget is balanced and the (present value of) total transfers are minimized.

The agents in this economy are individuals, firms, and a government. There are two periods. At the beginning of the first period, a continuum of individuals are born. They work in the first period and are retired in the second period. All individuals die at the end of the second period. Skill level of an individual  $z$ , which is her private information, is drawn at birth from a known distribution  $\Gamma(z)$ , i.i.d. across individuals, and determines her labor productivity. Individuals derive utility from consumption and leisure and have the identical utility function;

$$U(c_1, c_2, l) = \log c_1 + \beta \log c_2 - \theta l \tag{1}$$

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<sup>3</sup>Also, in a simple life-cycle model Kotlikoff (1979) showed that unfunded social security system reduces steady state capital by 20 percent.

where  $c_1, c_2$  are consumption in the first and second periods,  $l$  is labor, and  $0 < \beta < 1, \theta > 0$ .

The aggregate production is linear in quantity-adjusted labor:

$$Y = \int zl(z) d\Gamma(z)$$

Markets are competitive so the wage rate is equal to one. Individuals can save at a risk-free interest rate  $r$  which is equal to  $1/\beta - 1$ .

The individuals in this society do not like seeing poor people. Specifically, they get a large disutility from seeing people with consumption below a minimum level  $\underline{c}$ . They do not value consumption by others above  $\underline{c}$ . By its tax and benefit policies, the government wants to insure that everyone consumes at least  $\underline{c}$  in each period, i.e.

$$c_n(z) \geq \underline{c} \text{ for all } z \in [\underline{z}, \infty) \text{ and } n = 1, 2 \quad (2)$$

where  $\underline{z}$  is normalized to the ability level of an individual that would earn  $\underline{c}$  if there were no government program. The relationship between  $\underline{z}$  and  $\underline{c}$  is derived from the maximization problem of an individual with  $\underline{z}$  in the absence of government; hence,

$$\underline{z} = \frac{\theta \underline{c}}{1 + \beta} \quad (3)$$

The government's objective is to choose the tax policy  $\tau(zl)$  and mandatory saving rate  $\varphi$  in the first period and transfer policy  $s(zl, a)$  in the second policy, where  $a$  is asset holdings, that minimizes total transfers

$$\int_{\underline{z}}^{\infty} [\max(0, -\tau(zl)) + \beta s(zl, a)] d\Gamma(z) \quad (4)$$

subject to (2), government budget constraint, and the condition that each individual of type  $z$  maximizes its utility given government policy<sup>4</sup>. Tax payments  $\tau(zl)$  can be negative or positive, but transfer payments  $s(zl, a)$  are always nonnegative. The government expects everyone to earn at least  $\underline{c}$ , the labor income of an individual with  $\underline{z}$  in the absence of government. Individuals who do not earn enough to consume  $\underline{c}$  in both periods can be eligible to benefits in the first or second period.

Once the government determines the total amount of transfers (4), it has access to a large set of tax policies to finance. To solve this indeterminacy issue, the government is assumed to

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<sup>4</sup>Given the type of preferences, utility maximization implies incentive compatibility (see Mirrlees, 1971).

collect labor income taxes at the rate of  $\tau_0$  from individuals who do not receive any benefits. In other words, the government tax policy is reduced to

$$\tau(zl) = \begin{cases} \tau(zl), & \text{if } \tau(zl) \leq 0 \\ \tau_0 zl, & \text{if } \tau(zl) > 0 \end{cases} \quad (5)$$

In the first period, the government also chooses the savings rate  $\varphi$  at which it mandates everyone to save their before-tax labor income. Individuals are allowed to consume those savings, with interest, in the second period. In this environment, the government budget constraint is

$$\int_{\underline{z}}^{\infty} \tau(zl) d\Gamma(z) = \beta \int_{\underline{z}}^{\infty} s(zl, a) d\Gamma(z) \quad (6)$$

Individuals in this economy make their decisions at the beginning of first period. Every individual of type  $z$  chooses how much to work  $l$ , consume  $c_1$ , and save  $a$ , given the government income tax policy  $\tau(zl)$ , mandatory saving rate  $\varphi$ , and benefit policy. In the second period, individuals consume,  $c_2$ , their savings, with interest, and the benefits, if any, they receive  $s(zl, a)$ . Individuals are not allowed to have negative worth at the end of first or second period. This implies the following budget constraints for an individual with ability  $z$

$$\begin{aligned} c_1 &= (1 - \varphi) zl - \tau(zl) - a \\ c_2 &= (1 + r) [a + \varphi zl] + s(zl, a) \\ a &> 0 \end{aligned} \quad (7)$$

These conditions also imply the feasibility constraint:

$$\int_{\underline{z}}^{\infty} \left( c_1 + \frac{c_2}{1 + r} \right) d\Gamma(z) = \int_{\underline{z}}^{\infty} zl d\Gamma(z) \quad (8)$$

The government is strategic; it takes into account that its policy choice will affect the choices of private agents. In each period, first the government chooses its policy, then individual allocations are determined in competitive markets. Individual allocation choices will thus be a function of government policy. Letting  $x(z)$  denote the allocations for an individual of type  $z$ ,  $x(z) = (c_1(z), c_2(z), l(z), a(z))$ , an individual allocation rule will be a function mapping government policies  $\tau(zl), \varphi, s(zl, a)$  into individual allocations. The

equilibrium concept in this environment is the following.

**Definition 1** *The minimal-transfer equilibrium is an allocation  $X = \{x(z), z \in [\underline{z}, \infty)\}$ , where  $x(z) = (c_1(z), c_2(z), l(z), a(z))$ , and a government policy  $\tau(zl), \varphi, s(zl, a)$  such that the following conditions are satisfied:*

1. *Given  $\tau(zl), \varphi, s(zl, a)$ , the vector  $x(z)$  maximizes (1) subject to the constraints (7), for each  $z \in [\underline{z}, \infty)$ .*
2. *The government policy  $\tau(zl), \varphi, s(zl, a)$  minimizes total transfers (4) subject to the government budget constraint (6), the condition that everyone consumes at least  $\underline{c}$  (2), and the maximization problem of each individual with  $z \in [\underline{z}, \infty)$ .*

The planning problem of the government is described in detail in Appendix A. The allocations solving this planning problem is shown in the following figure<sup>5</sup>.

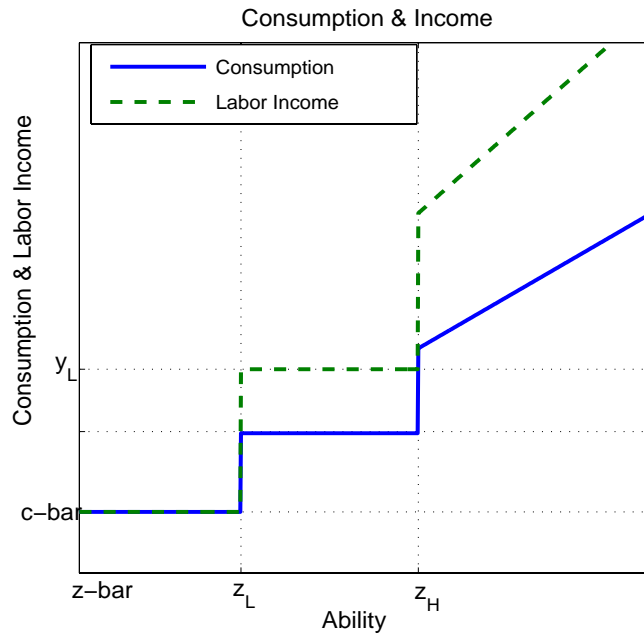


Figure 2.1

In this environment, individuals at the lower end of the distribution who possess different abilities act like the least able individual and, hence, consume the same level of consumption

<sup>5</sup>In this figure, one period is 30 years,  $\beta = 0.365$ ,  $\theta = 4$ . The distribution of ability is a truncated lognormal distribution  $\log(z) \sim N(\mu_z, \sigma_z^2)$  on  $[\underline{z}, 1]$  where  $\mu_z = -0.9363$ ,  $\sigma_z = 0.4581$ , and  $\underline{z} = 0.16$ . In equilibrium,  $\underline{c} = \underline{z}(1 + \beta)/\theta = 0.055$ ,  $\tau_0 = 10.2\%$ .

$\underline{c}$  in both periods. They all receive total benefit payments of  $\underline{c}$ . The government is not able to discriminate those individuals perfectly.

$z_L$  is the ability level of individual who is indifferent between consuming  $\underline{c}$  by working less and consuming more than  $\underline{c}$  by working more. This individual is not subject to income taxes if it chooses to work more. To avoid more people acting like the least able individual, the government does not collect income taxes from individuals with income less than  $y_L$ , the labor income of individual with ability  $z_L$ . Some of the individuals with ability greater than but close to  $z_L$  then chooses to act like the individual with ability  $z_L$  to avoid income taxes.  $z_H$  is the ability level of the most able individual who chooses to pretend like the individual with  $z_L$ .

If the government cannot make a binding commitment to the second-period policy, it might revise the initially determined policy in the second period. The government might choose to grant benefits to individuals that it was not initially planning to provide if the gross return on their savings is less than  $\underline{c}$ ; i.e.,

$$s(zl, a) = \begin{cases} \underline{c} - (1+r)(a + \varphi zl), & \text{if } (1+r)(a + \varphi zl) \leq \underline{c} \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

In the first period, rational individuals anticipate that in the second period, the government will grant benefits to anyone with savings, including interest, less than  $\underline{c}$ . When making their savings and labor supply decisions, individuals take this into consideration.

The initial government may not be committed but is aware of the sequence of events that will follow its policy announcement. Therefore, it designs its tax and saving policies that are consistent with the second-period policy and the maximization problem of individuals. The definition of equilibrium when the government cannot credibly commit is the same as in Definition 1 except that the second condition is replaced by "*Given the second-period policy  $s(zl, a)$ , the first-period policy  $\tau(zl)$ ,  $\varphi$  minimizes (4) subject to the government budget constraint (6), the condition that everyone consumes at least  $\underline{c}$  (2), and the maximization problem of each individual with  $z \in [\underline{z}, \infty)$ .*"

The planning problem of the initial government is described in Appendix A. The following proposition states the key conclusion of this section.

**Proposition 1** *The government without commitment can attain the allocation the government with commitment would implement if and only if it uses a mandatory savings program.*

**Proof.** *In Appendix A. ■*

The goal of the government without commitment is to deal with the people undersaving for retirement. By making use of mandatory savings, the government can control individuals' savings and prevent them from taking advantage of government's commitment problem. The government can set its mandatory saving rate such that individuals who otherwise would not save for old age save at least  $\underline{c}$  through the mandatory savings system. The government with commitment does not pay benefits to anyone earning more than what an individual with  $z_L$  earns, so the mandatory saving rate should be at least  $\beta\underline{c}/y_L$ , where  $y_L$  is the labor income of individual with  $z_L$ . On the other hand, the savings rate can be at most  $\beta/(1+\beta)$  as individuals are borrowing-constrained. In fact, any savings rate between  $\beta\underline{c}/y_L$  and  $\beta/(1+\beta)$ , combined with the accompanying tax policy, can implement the allocation the government with commitment would attain. If the government does not use a mandatory savings program, it cannot prevent people from undersaving for retirement and, hence, cannot attain the allocation the government with commitment would do.

Figure 2.2 compares the allocations and individual utilities the government without commitment can attain. If the government utilizes a mandatory savings system, the allocation is identical to the minimum-transfer-equilibrium allocation in Figure 2.1. If, however, the government does not make use of mandatory savings, more individuals choose to pretend as if of type  $\underline{z}$  by working less, raising the total amount of transfers. The rise in total amount of transfers, in turn, increases the income tax rate  $\tau_0$ . This is because the government not utilizing mandatory savings relies more heavily on income taxation and, hence, makes everyone not eligible to transfers in the second period subject to income taxes in the first period. As a result, in the economy without mandatory savings, everyone except those on the lower end of ability distribution becomes worse off.

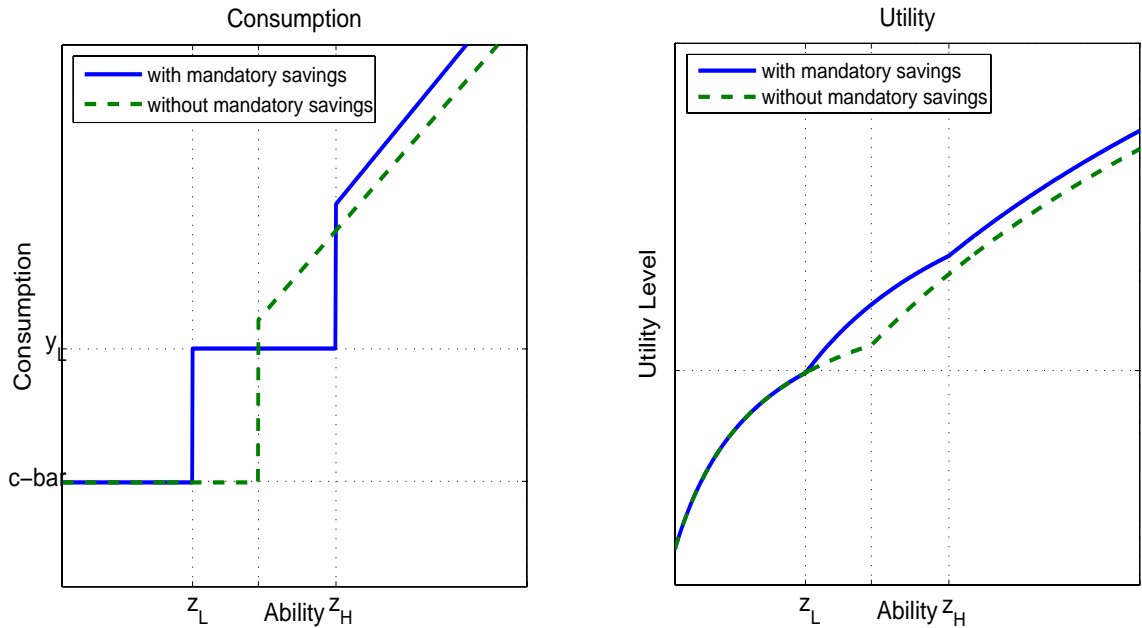


Figure 2.2

### 3. CONCLUSION

Being the largest transfer program of the industrial world, social security can have an enormous impact on economic activity. In an attempt to better understand its impact, I started the paper by seeking the role of social security. In particular, I looked for a role that can survive in an environment where the government has its separate redistribution program and that any earnings or lifetime uncertainty can be perfectly insured. In this environment, the problem of undersaving for retirement, arising from time inconsistent government policy, appeared to be the motive behind social security. Then, I turned to searching for a simple but efficient social security program that can deal with this problem. In a two-period deterministic model where individuals have time-consistent preferences and are heterogenous in skill, I showed that a mandatory savings plan can solve the problem of undersaving for retirement with the smallest budget.

This paper abstracted from lifetime uncertainty against which the current social security program insures. Since people face mortality risk in real life, a mandatory savings plan should actually be formed in combination with mandatory annuitization at retirement. Voluntary annuitization does not solve the problem; if the government leaves annuitization as voluntary, (some) households will tend to use up their mandatory savings early at retirement and be

eligible to minimum retirement income later in life.

## Appendix: Proofs and Solutions

Solution of the Planning Problem of Government with Commitment

Going backwards, we first solve the maximization problem of individuals, given the government policy;

$$V(z) = \max_l \left\{ (1 + \beta) \log \frac{y(zl)}{1 + \beta} - \theta l \right\} \quad (10)$$

where, using the Euler equation,  $c_1$  and  $c_2$  are replaced by  $c_1 = c_2 = y(zl) / (1 + \beta)$ .  $y(zl)$  is the net income of an individual with labor income  $zl$ :

$$y(zl) = zl - \tau(zl) + \beta s(zl) \quad (11)$$

where  $s(zl, a)$  is replaced by  $s(zl)$ . Using a simple argument, we can show that any benefit policy that is a function of asset holdings cannot be the minimal-transfer allocation. If benefit payments were a function of asset holdings, consumption in the first and second periods would not be equal to each other. Suppose that benefit payments were decreasing with asset holdings. Then, individuals would consume more in the first period and save less for the second period. In particular, there would be an individual consuming more than  $\underline{c}$  in the first period but eligible for benefits in the second period. The benefit payments to that individual could be reduced or totally avoided if benefit payments were only a function of labor income, which would decrease the first-period consumption and increase savings.

Multiplying the first order condition (FOC) of (10) by  $l$  and rearranging, we can find that

$$V'(z) = \frac{\theta l}{z} \quad (12)$$

where the left hand side (LHS) is equal to  $V'(z)$  by the Envelope Theorem. The second-order condition (SOC) of (10) is satisfied if

$$\frac{d(zl(z))}{dz} = l(z) + zl'(z) > 0 \quad (13)$$

In what follows, the maximization problem (10) will be represented by the conditions

(12)-(13)<sup>6</sup>.

Now we can turn to the planning problem of government in the first period. Since the government is minimizing total transfers, there will be an individual who is able enough so that it is not eligible for any benefits and, hence, does pay income taxes at the rate of  $\tau_0$ . For this individual,  $y(zl) = (1 - \tau_0)zl$  and the individual maximization problem indicates that (13) does not bind. Let  $z_H$  be the lowest ability level for such an individual. The individual with  $z_H$  will be indifferent between being eligible for a tax or transfer scheme by working less and paying income taxes at the rate of  $\tau_0$  by working more; i.e.,

$$(1 + \beta) \log \frac{y(z_H l(z_H))}{1 + \beta} - \theta l(z_H) = (1 + \beta) \log \frac{(1 - \tau_0)z_H}{\theta} - (1 + \beta) \quad (14)$$

where the LHS is the utility that individual can get by working less, while the right hand side (RHS) is the utility the individual obtains by supplying labor  $l = (1 + \beta) / \theta$  and paying income taxes at the rate of  $\tau_0$ . Individuals with  $[z_H, \infty)$  will not be receiving any benefits so they will not enter into the objective function of the government, except through the indifference condition (14). The government will set the only policy variable affecting their decision problem,  $\tau_0$ , so as to balance its budget, given the total benefit payments to be transferred.

The planning problem of government can be solved by utilizing the Optimal Control Theory. We can choose  $V(z)$  as the state variable and  $m(z)$  as the control variable, where  $m(z) = zl(z)$ . Using these definitions, we can also write  $y(zl(z))$  as a function of  $V(z)$ ,  $m(z)$ , and  $z$ . Rather than incorporating the condition (13) into the planning problem, I will solve the planning problem of the government without (13) and, then, check if (13) is satisfied. We can write the planning problem of government as

$$\begin{aligned} & \max_{\underline{z}} \int_{\underline{z}}^{z_H} \left[ m(z) - (1 + \beta) \exp \left( \frac{1}{1 + \beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right) \right] d\Gamma(z) & (15) \\ & s.t. \\ & V'(z) = \frac{\theta m(z)}{z^2}, \\ & V(\underline{z}) = (1 + \beta) \log \underline{c} - (1 + \beta), \\ & \exp \left[ \frac{1}{1 + \beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right] \geq \underline{c}, \\ & \text{and (6), (13), (14).} \end{aligned}$$

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<sup>6</sup>The derivations of (12) and (13) are in detail described in Mirrlees (1971) and Salanié (2003).

where the term in square brackets in the objective function is the negative of the total benefit payments an individual with  $z$  receives. The first constraint is the condition that individuals maximize their utility given government policy. The second condition is that the least able individual should earn at least  $\underline{z}(1 + \beta)/\theta$  in order to receive benefit payments. The next constraint is the condition that consumption must be equal to or greater than  $\underline{c}$ . The last three constraints are the government budget constraint, sufficiency for individual maximization problem, and the indifference condition. The Hamiltonian of the problem (15) is

$$H(z) = \left\{ m(z) - (1 + \beta) \exp \left[ \frac{1}{1 + \beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right] \right\} \gamma(z) \\ + \lambda(z) \left( \frac{\theta m(z)}{z^2} \right) + \mu(z) \left\{ \exp \left[ \frac{1}{1 + \beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right] \right\}$$

where  $\lambda(z)$  is the costate variable and  $\mu(z)$  is the multiplier of the constraint (2). For this problem, the maximum principle conditions are

$$\lambda'(z) = \left[ \gamma(z) - \frac{\mu(z)}{1 + \beta} \right] \exp \left[ \frac{1}{1 + \beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right], \quad (16) \\ V'(z) = \frac{\theta m(z)}{z^2}, \\ \gamma(z) + \lambda(z) \frac{\theta}{z^2} - \left[ \gamma(z) - \frac{\mu(z)}{1 + \beta} \right] \frac{\theta}{z} \exp \left[ \frac{1}{1 + \beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right] = 0, \\ \exp \left[ \frac{1}{1 + \beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right] \geq \underline{c}, \\ \mu(z) \left\{ \exp \left[ \frac{1}{1 + \beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right] - \underline{c} \right\} = 0, \quad \mu(z) \geq 0$$

In this economy, there are individuals whose ability is so low that they cannot earn enough to consume  $\underline{c}$  in both periods, so the constraint (2) will bind for some individuals. For those individuals, the fourth condition in (16) will be satisfied with equality. That is, they will all consume equal to  $\underline{c}$  in both periods. Taking the derivative of this poverty constraint, which is satisfied with equality, with respect to  $z$  and using the first and second conditions in (15), we can show that the labor income  $m(z)$  will be constant and equal to the least able individual's income  $m(\underline{z}) = \underline{c} = (1 + \beta)\underline{z}/\theta$ .

The next question is if there is anyone whose poverty constraint does not bind but is eligible to benefits. If so, for those households, the third condition in (16) will solve for  $\lambda(z)$ . The derivative of  $\lambda(z)$  with respect to  $z$  should be equal to the one in the first line of (16). This is satisfied only if  $\lambda(z) = 0$ . However,  $\lambda(z)$  can be equal to zero only at the end point  $z_L$ , as otherwise it would violate the first condition in (16). The end condition implies that  $H(z_L) - V'(z_L)\lambda(z_L) = 0$  and  $\lambda(z_L) = 0$ . Thus,

$$\frac{m(z_L)}{1+\beta} = \exp\left[\frac{1}{1+\beta}\left(V(z_L) + \theta\frac{m(z_L)}{z_L}\right)\right] = \frac{z_L}{\theta} \quad (17)$$

Using the continuity of  $V(z)$  at  $z_L$ , and (17), we can find that  $z_L$  satisfies

$$\log\left(\frac{z_L}{\theta}\right) = \log \underline{c} + \left(1 - \frac{z_L}{\theta}\right) \quad (18)$$

Given that  $H(z) = 0$  for  $z \geq z_L$ , we look for the smallest  $z_H$  satisfying  $m(z_H) = y(z_H)$  and the indifference condition (14); i.e.,  $z_H$  solves

$$\log\left(\frac{(1-\tau_0)z_H}{\theta}\right) = \log\left(\frac{z_L}{\theta}\right) + \left(1 - \frac{z_L}{z_H}\right) \quad (19)$$

The tax rate  $\tau_0$  in this condition (19) will be the one satisfying the government budget constraint (6). Therefore, the minimal-transfer allocation, denoted by  $*$ , will be

$$(c^*(z), l^*(z)) = \begin{cases} \left(\underline{c}, \frac{1+\beta}{\theta} \frac{z}{z_L}\right), & \text{for } z \leq z_L \\ \left(\frac{z_L^*}{\theta}, \frac{1+\beta}{\theta} \frac{z_L^*}{z}\right), & \text{for } z_L^* < z \leq z_H^* \\ \left(\frac{(1-\tau_0^*)z}{\theta}, \frac{1+\beta}{\theta}\right), & \text{for } z > z_H^* \end{cases} \quad (20)$$

where  $c^*(z) = c_1^*(z) = c_2^*(z)$ .

Solution for the Planning Problem of Government without Commitment

When the government has not commitment, the maximization problem of individuals is

$$V(z) = \begin{cases} \max_l \left\{ (1+\beta) \log \frac{y(zl)}{1+\beta} - \theta l \right\} & \text{if } \varphi zl \geq \beta \underline{c} \\ \max_l \{ \log [y(zl) - \beta \underline{c}] + \beta \log \underline{c} - \theta l \} & \text{if } \varphi zl < \beta \underline{c} \end{cases} \quad (21)$$

where in the first line,  $c_1$  and  $c_2$  are replaced by  $c_1 = c_2 = y(z_l) / (1 + \beta)$ ,  $y(z_l)$  is as defined in (11). It can easily be shown that (12) and (13) can represent the maximization problem (21). Also, the ability level of the lowest skilled individual who is not eligible for any transfer scheme,  $z_H$ , satisfies the indifference condition (14).

The planning problem of the initial government can be written as

$$\begin{aligned}
& \max_{\underline{z}} \int_{\underline{z}}^{z_H} [m(z) - y(m(z))] d\Gamma(z) & (22) \\
& s.t. \\
& V'(z) = \frac{\theta m(z)}{z^2}, \\
& V(\underline{z}) = (1 + \beta) \log \underline{c} - (1 + \beta), \\
& y(m(z)) \geq (1 + \beta) \underline{c}, \\
& y(m(z)) = \begin{cases} \beta \underline{c} + \underline{c}^{-\beta} \exp \left[ V(z) + \theta \frac{m(z)}{z} \right], & \text{if } z < z_L \\ (1 + \beta) \exp \left[ \frac{1}{1 + \beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right], & \text{if } z \geq z_L \end{cases} \\
& \text{and (6), (13), (14).}
\end{aligned}$$

where  $z_L$  is the ability level of individual who is indifferent between consuming  $\underline{c}$  in the second period by working less and consuming more than  $\underline{c}$  by working more. Other variables are as defined before. The Hamiltonian of the problem (22) for  $\underline{z} \leq z < z_L$  is

$$\begin{aligned}
H(z) = & \left\{ m(z) - \underline{c}^{-\beta} \exp \left[ V(z) + \theta \frac{m(z)}{z} \right] - \beta \underline{c} \right\} \gamma(z) \\
& + \lambda(z) \left( \frac{\theta m(z)}{z^2} \right) + \mu(z) \left\{ \underline{c}^{-\beta} \exp \left[ V(z) + \theta \frac{m(z)}{z} \right] - \underline{c} \right\}
\end{aligned}$$

where  $\lambda(z)$  and  $\mu(z)$  are as defined above. The corresponding maximum principle conditions are

$$\begin{aligned}
\lambda'(z) &= [\gamma(z) - \mu(z)] \underline{c}^{-\beta} \exp \left[ V(z) + \theta \frac{m(z)}{z} \right], \\
V'(z) &= \frac{\theta m(z)}{z^2}, \\
\gamma(z) + \lambda(z) \frac{\theta}{z^2} - [\gamma(z) - \mu(z)] \frac{\theta}{z} \underline{c}^{-\beta} \exp \left[ V(z) + \theta \frac{m(z)}{z} \right] &= 0, \\
\underline{c}^{-\beta} \exp \left[ V(z) + \theta \frac{m(z)}{z} \right] &\geq \underline{c}, \\
\mu(z) \left\{ \underline{c}^{-\beta} \exp \left[ V(z) + \theta \frac{m(z)}{z} \right] - \underline{c} \right\} &= 0, \quad \mu(z) \geq 0
\end{aligned} \tag{23}$$

Using the same arguments above, we can show that for individuals whose poverty constraint bind, the labor income  $m(z)$  will be constant and equal to the least able individual's income  $m(\underline{z}) = \underline{c} = (1 + \beta) \underline{z} / \theta$ . Those individuals will consume  $\underline{c}$  in both periods. The next question is if there is anyone whose poverty constraint does not bind in the first period but is eligible for benefits in the second period. If so, for those individuals, the third condition in (23) will solve for  $\lambda(z)$ . Using the arguments above, we can show that  $\lambda(z_L) = 0$ , where the end point  $z_L$  is determined by the equality of  $V(z)$  at  $z_L$  in (21) and solves (18). The Hamiltonian and the corresponding maximum principle conditions for  $z_L \leq z \leq z_H$  can be written similarly. The maximum principle conditions indicate that the constraint (2) does not bind for anyone with  $z \geq z_L$ . Using the same arguments, we can show that  $\lambda(z) = 0$  for  $z \geq z_L$ . This implies that  $H(z_L) = 0$  and  $z_L$  satisfies (17). Given that  $H(z) = 0$  for  $z \geq z_L$ , the smallest  $z_H$  satisfying  $m(z_H) = y(z_H)$  solves (19). Therefore, the allocation the government without commitment implements are the same as (20).

#### Proof of Proposition 1

The solution for the planning problem of government without commitment above showed that the government, utilizing mandatory savings, can implement the allocation the government with commitment would attain. Therefore, the proof only shows that the minimal-transfer allocation cannot be attained if the government does not use mandatory savings.

When the government does not use mandatory savings,  $\varphi = 0$  and the individual maximization problem (21) indicates that if an individual receives benefits in the second period, it receives them at the amount of  $\underline{c}$ . In the first period, everyone is capable of earning  $\underline{c}$  and the government can set its tax system such that nobody receives benefit payments in the

first period. Therefore, the objective of government in the first period reduces to minimizing government transfers in the second period, which simply means minimizing  $z_L$ .  $z_L$  is the ability level of an individual who is indifferent between saving or not saving for the second period and satisfies

$$\log [y(z_L l(z_L)) - \beta \underline{c}] + \beta \log \underline{c} - \theta l(z_L) = (1 + \beta) \log \frac{(1 - \tau_0) z_L}{\theta} - (1 + \beta) \quad (24)$$

The initial government's planning problem, then, is

$$\begin{aligned} & \max_{\underline{z}} - \int_{\underline{z}}^{z_L} \beta \underline{c} d\Gamma(z) & (25) \\ & \text{s.t.} \\ & V'(z) = \frac{\theta m(z)}{z^2}, \\ & V(\underline{z}) = (1 + \beta) \log \underline{c} - (1 + \beta), \\ & \underline{c}^{-\beta} \exp\left(V(z) + \theta \frac{m(z)}{z}\right) \geq \underline{c}, \\ & \text{and (6), (13), (24).} \end{aligned}$$

The constraints and variables in (25) are the same as in (22). The Hamiltonian of this problem is

$$H(z) = -\beta \underline{c} \gamma(z) + \lambda(z) \left( \frac{\theta m(z)}{z^2} \right) + \mu(z) \left\{ \exp \left[ \frac{1}{1 + \beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right] - \underline{c} \right\}$$

The corresponding maximum principle conditions are

$$\begin{aligned}
\lambda'(z) &= -\frac{\mu(z)}{1+\beta} \exp \left[ \frac{1}{1+\beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right], \\
V'(z) &= \frac{\theta m(z)}{z^2}, \\
\lambda(z) \frac{\theta}{z^2} + \frac{\mu(z)}{1+\beta} \frac{\theta}{z} \exp \left[ \frac{1}{1+\beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right] &= 0, \\
\exp \left[ \frac{1}{1+\beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right] &\geq \underline{c}, \\
\mu(z) \left\{ \exp \left[ \frac{1}{1+\beta} \left( V(z) + \theta \frac{m(z)}{z} \right) \right] - \underline{c} \right\} &= 0, \quad \mu(z) \geq 0
\end{aligned} \tag{26}$$

Using the same arguments above, we can show that for individuals whose poverty constraint bind, the labor income  $m(z)$  will be constant and equal to the least able individual's income  $m(\underline{z}) = \underline{c} = (1+\beta)\underline{z}/\theta$  and their consumption in each period will be  $\underline{c}$ . The third condition in (26) indicates that  $\lambda(z)$  will be zero whenever the poverty constraint does not bind. That is, if an individual's poverty constraint does not bind, that individual will not be eligible for benefits in the second period. The terminal condition will be determined by the indifference condition for the most able individual eligible for benefits (24),  $z_L$ :

$$\log \left( \frac{(1-\tau_0)z_L}{\theta} \right) = \log \underline{c} + \left( 1 - \frac{\underline{z}}{z_L} \right) \tag{27}$$

The allocations solving this planning problem, denoted by  $\hat{\cdot}$ , are

$$\left( \hat{c}(z), \hat{l}(z) \right) = \begin{cases} \left( \underline{c}, \frac{1+\beta}{\theta} \frac{\underline{z}}{z} \right), & \text{for } \underline{z} \leq z \leq \hat{z}_L \\ \left( \frac{(1-\hat{\tau}_0)z}{\theta}, \frac{1+\beta}{\theta} \right), & \text{for } z > \hat{z}_L \end{cases} \tag{28}$$

where  $\hat{c}(z) = \hat{c}_1(z) = \hat{c}_2(z)$ .

Comparing (18) and (27), we can easily see that  $\hat{z}_L > z_L^*$ . Therefore, the allocations in (28) are different from the allocations (20). ■

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