Problem Set # 4 (OLG with money)

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ECON 602
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1. Consider a standard pure-exchange 2-pd lived OG economy. There are $N_t$ identical young agents at time $t$ and population grows at the rate $n > -1$, $N_1 > 0$. The endowments are denoted by $w_1$ and $w_2$, both strictly positive. All agents have the utility function

$$U(c_1, c_2) = \beta \ln c_1 + (1 - \beta) \ln c_2$$

with $\beta \in (0, 1)$. Only the initial old are additionally collectively endowed with $M > 0$ units of money.

(a) Write down the problem faced by young agents.
(b) Compute the savings function of an young agent.
(c) Write down the conditions under which this is a Samuelson economy.
(d) Define a competitive equilibrium for this economy.
(e) Write down the law of motion for $z(t)$ where $z(t) \equiv \frac{M}{N_t \rho_t}; z_1 > 0$.
(f) Compute the stationary equilibria for $z(t)$. Is currency “valued” at all the stationary equilibria? Explain.
(g) Explicitly solve for the entire time path of $z(t)$. Consider all possible ranges of $z_1$.
(h) Investigate the stability properties of the stationary equilibria under the assumption that this is a Classical case economy.
(i) Comment on the optimality properties of the stationary equilibria in each case.
(j) Write down the equation describing the entire time path for the price level $p(t)$ for different ranges of $p(1)$.
(k) Comment on the asymptotic behavior of $p(t)$ and of $p(t)/p(t + 1)$.
(l) Suppose $M$ is doubled to $2M$. What is the effect of this change on the savings behavior of young agents? What is the effect on the time path for $p(t)$? Explain your finding.

2. (Computations) Consider an OLG economy with two-period lived agents who all have the utility function $U = \ln c_1 + \beta \ln c_2$. They also have a young endowment of 2 unit and an old endowment of 1 unit. The only asset is money. The government prints money to finance a fixed deficit of $g$ per young person every period.

(a) Set $g = 0.01$ and $\beta = 2$. Compute the equilibrium demand for real money balances. Also compute the optimal choice of $c_1$ and $c_2$.

(b) Set $g = 0.01$ and $\beta = 2$. Draw the steady state seigniorage Laffer curve.

1For this question you will need to use a software like MATLAB, Maple, Mathematica, Excel etc., or run a program in GAUSS and then use the data generated to plot in Excel.
(c) Set $g = 0.01$ and $\beta = 2$. Are there two money growth rates that finance this level of $g$? What are they? Explain. What about welfare at these two money growth rates?

(d) With $\beta = 2$, what is the maximum $g$ that may be financed through money creation in this economy?

(e) Set $g = 0.1$ and $\beta = 2$. Compute the equilibrium demand for real money balances. Also compute the optimal choice of $c_1$ and $c_2$. Compare your answers to (a). Explain.

3. Exercise 7.3 pg. 293 in the Sargent book.