

A Pecuniary Reason for Income Mixing

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Abstract

Empirical studies have found a high degree of income mixing in American neighborhoods. We give a new explanation of this phenomenon that is based on consumer search. A low price for a given good benefits high valuation buyers more than low valuation buyers. But with search, the probability of obtaining a low price is increasing in the proportion of low valuation buyers. This gives high valuation buyers an incentive to live near low valuation buyers. With many goods, a buyer has an incentive to live near neighbors whose valuations are uncorrelated with hers.

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1 Introduction

It is commonly thought that most cities divide into a fairly small number of areas, some rich and some poor. Those familiar with Boston, for example, might assume that the rich reside in Belmont, Newton, and Brookline, the middle class live in Allston, Brighton, and Somerville, and the poor concentrate in Roxbury and Mattapan. Although there is some truth to this assumption, empirical studies by sociologists show that there is a high degree of income mixing in the typical American neighborhood. After reviewing this evidence, we present a new theory in which income mixing results from consumer search. A buyer who is willing to pay a high price for a given good benefits from living among neighbors who are not willing to pay as much, since this leads local retailers to charge lower prices.

Some evidence for the existence of segregated neighborhoods comes from Michael White's [30, p. 34] survey of 21 U.S. cities in 1980. Of these 21 cities, 17 contained census tracts in which no families were poor. Four of the 21 cities also contained census tracts with 100% poverty rates.

In spite of this evidence, segregation is less prevalent than one might think. Support for this is found in Reynolds Farley's 1977 study, which examines 1970 data for the largest 29 urban areas in the U.S. Farley [17, p. 504] finds that 81% of the average city's variance in family income is variance within the city's census tracts.^{1,2} Even

¹Farley's unit of analysis is urbanized areas. These include both the central city and suburbs, but exclude outlying rural areas [17, pp. 499-500].

²Total variance equals the sum of within-tract and between-tract variance. Variance within census tracts should be a somewhat conservative measure of integration since census tracts were originally intended to be homogeneous. According to the U.S. Census Bureau,

Census tracts usually have between 2,500 and 8,000 persons and, when first delineated, are designed to be homogeneous with respect to population characteristics, economic status, and living conditions. [27]

The first census tracts were delineated for an experimental study of 8 cities in the 1910 census. In 1940, the U.S. Census Bureau adopted census tracts as an official unit of analysis [28]. Migration since then has probably caused an increase in within-tract income variance, although census tracts are still likely to give a smaller measure of within-tract variance than would be obtained from a random grid. Use of a subdivision of census tracts such as city blocks would of course lead to higher measured segregation, although at least for racial segregation the difference appears to be minor [17,

in Dallas, the most segregated of the 29 cities by this measure, 74% of the citywide income variance is within-tract variance. This mixing is due not to income shocks or life cycle effects, but rather to mixing of households with disparate levels of human capital: in Farley's sample, an average of 81% of the variance in years of education of adult males³ is within-tract variance [17, p. 504]. There is also much less segregation by education than by race. Farley finds that the index of dissimilarity⁴ between post-college educated black men and post-college educated white men across census tracts was 91% in Detroit in 1970. In contrast, the index of dissimilarity between post-college educated white men and white men with *no education whatsoever* was only 62% [17, p. 514].

We present a new explanation for income mixing that is based on consumer search. The theory has two motivations. First, search appears to be a plausible model of consumer behavior. Kumar and Leone [21] find that substitution between nearby supermarkets in response to promotions is two to three times weaker than promotion-induced within-store substitution; Walters [29] reports similar findings. This suggests that supermarket customers tend to be imperfectly informed about prices at competing stores. Pratt, Wise, and Zeckhauser [25] find that the prices of given goods vary substantially among nearby stores. Such price dispersion is predicted by search theory but is not a feature of models with complete price information.

The second motivation is evidence that most urban consumers do not stray far from their homes to buy goods and services. In a survey of 1,119 Boston households in 1991, the median distance of shopping trips that began at home was only 0.98 miles.⁵ Median distances for trips whose goals were eating out and banking/personal

p. 502].

³Farley's educational calculations include all and only those men who were aged 25 and over in 1970 [17, p. 500].

⁴This is the minimal proportion of either group that would have to be reallocated across census tracts in order to give the two groups an identical distribution across census tracts.

⁵Trip distances were taken from the 1991 Regional Household-based Travel Survey of the Central Transportation Planning Staff, Massachusetts Department of Transportation. The survey data were kindly provided by Karl Quackenbush.

business were 0.85 miles and 1.01 miles, respectively. These are the median distances that a pedestrian would walk; straight-line distances would be even shorter in most cases.

The evidence that most shopping is fairly close to home suggests that local retail market structures may have a significant effect on the demographic composition of a neighborhood or area. To study this, we build a formal model with two types of agents who differ in their valuations of a single good. The agents choose where to live and then search for the good from time to time among local stores. The search process is modelled as in Diamond [9].⁶ In any model of this type, high valuation buyers will care about the good's price over a larger range than will low valuation buyers. This is because a rising price hurts a buyer only until the price reaches her valuation; above this level, she does not buy the good, so further price increases do not affect her. But with search, the average price of a good is an *increasing* function of the proportion of high valuation buyers in an area. For this reason, high valuation buyers benefit more than low valuation buyers from an increase in the proportion of low valuation buyers in their neighborhood. This force gives rise to integration between buyers of different valuations.

A similar intuition holds in the case of many goods. In this case, a buyer's welfare is high when the correlation is low between her valuations and the prices she faces for different goods and services. Since search creates a correlation between prices and the valuations of *other* buyers in the same search market, it behooves a buyer to reduce the correlation between her valuations and those of her neighbors.

The model helps to explain income mixing because there is evidence that a buyer's valuations tend to be associated with her income. Hoch *et al* [20] find that demand at supermarkets is less price-sensitive in neighborhoods that contain more expensive houses, better educated residents, and fewer minorities. This suggests that a buyer's

⁶Diamond takes the distribution of buyer types as given. In our model, it is endogenously determined by agents' residential decisions.

valuations tend to be an increasing function of her income. Consistent with this, Marvel [22] and Maurizi and Kelley [23] find that search intensity is decreasing in income. Carlson and Gieseke [8] and Goldman and Johansson [19] also find a significant (but nonlinear) relationship between income and search intensity.

The evidence from White [30] and Farley [17], cited above, indicates that the degree of integration varies, with some neighborhoods being quite homogeneous. This may be because forces for integration coexist with incentives to segregate. Our model assumes that, in addition to the force for integration that comes from search, agents also have some incentive to live with others of the same type. We find that the combination can lead to multiple equilibria, some mixed and others not.

2 Literature Review

Prior theoretical work has provided a number of explanations for both segregation and integration. Bénabou [3] shows that segregation can occur if the cost of attaining high skills is decreasing in the proportion of school district residents who do so. Segregation can also result if either the poor are credit constrained, education is a normal good, or the children of well educated parents benefit more from high community human capital than the children of less educated parents (Bénabou [4, 5]). Durlauf [10, 11] shows that segregation ensues if communities can control who enters and school quality depends on both the income tax base and on the average level of human capital in the district. The poor would like to move to rich communities in order to benefit from the better schools, but are not permitted.

Epple, Filimon, and Romer [16] and Fernandez and Rogerson [18] show that segregation occurs if a household's willingness to accept higher taxes in return for a given improvement in public goods is a monotonic function of its income. Epple and Romer [15] show that a local system of lump sum grants to households, financed by linear property taxes, will also lead to segregation since wealthier families prefer less

redistribution.

There are also several prior explanations for income mixing. In the models of Bond and Coulson [7] and Miyao [24], mixing occurs because housing is heterogeneous and the rich and poor have different tastes. In Bond and Coulson’s model [7], housing differs by age and the rich prefer newer homes. In Miyao [24, pp. 458-460], housing differs by distance from the city center. The rich prefer to live further away, where lower land prices permit them to buy larger plots.⁷ In both models, mixing is stable if agents’ preference for segregation is sufficiently weak.

These two models differ from ours in that one group does not *benefit* from living with the other; the rich would prefer to see the poor replaced by other rich families. In this sense, our model is closer to that of de Bartolome [2]. Integration occurs in his model because the children of parents with low education benefit more from having intelligent schoolmates than do children from highly educated families. Intergroup spillovers also lead to integration in the models of Berglas [6] and Stiglitz [26], where high and low skill labor are complementary inputs in the community’s production function.

3 The Model

The game is played in continuous time. There is a single neighborhood that contains many identical apartments, each large enough to house a single agent. Agents are of two types: 1 and 2. We assume that apartments are rented on a competitive spot market. In addition, there are more agents of either type than there are apartments in the neighborhood. These two assumptions imply a critical property: at any point in time, the rent on an apartment makes its inhabitant indifferent between living there and living outside the neighborhood. Moreover, the neighborhood is always inhabited

⁷Miyao’s model explains mixed cities but not mixed neighborhoods; the city is divided into a poor, central area and a rich, suburban area. This result is still nontrivial since Miyao assumes that the two groups prefer not to live in the same city.

solely by the type (or types) that most prefers living there to living outside.

The model's consumption component is based on Diamond's search model [9]. There is a single consumption good, which is produced at zero marginal cost. While living in the neighborhood, an agent receives random opportunities to consume a single unit of this good. These opportunities occur according to a Poisson process with arrival rate c . On receiving such an opportunity, an agent starts to visit neighborhood stores. The timing of these visits is also stochastic: visits occur according to a Poisson process with arrival rate a . On receiving a visitor, a firm chooses a price to offer. The agent either accepts the offer or rejects it and continues to search. If she accepts, the agent consumes the good and waits for her next consumption opportunity. If a type i agent pays the price p , her utility is $u_i - p$. The valuation of type 2 agents is the higher one: $u_2 > u_1 > 0$.

Because of the model's construction, the number of neighborhood stores can be ignored; one can imagine either a fixed number or free entry with fixed costs. We assume that only neighborhood residents visit neighborhood stores, and that they visit only neighborhood stores. This assumption can be relaxed, but income mixing requires that there be some effect of an agent's residence on the stores she visits.

Agents may also care about the neighborhood's demographic composition, which is captured at time t by the proportion s_t of type 2 neighborhood residents. A type i agent receives a flow of utility at the rate $d_i(s_t)$. This function can be fairly general; we assume only that a higher proportion of type 2 agents leads to an increase in their utility *relative* to that received by type 1 agents. That is, $d_2(s_t) - d_1(s_t)$ is increasing in s_t . This property is consistent with a number of different stories. One is that the two groups simply dislike each other, as in Miyao [24]; this implies the stronger condition that d_2 is increasing while d_1 is decreasing. Or both types may prefer to live with type 2's, with type 2's having the stronger preference: $d'_2 > d'_1 > 0$.⁸ Alternatively,

⁸Both types may instead prefer to live with type 1, such that type 1's have the stronger preference: $0 > d'_2 > d'_1$.

there may be educational externalities as in Bénabou [3, 5]. The two groups may also differ in their tastes for public goods, as in Ellickson [13], Epple and Romer [15], Epple, Filimon, and Romer [16], and Fernandez and Rogerson [18]. The functions d_i would then capture, in reduced form, the effects of neighborhood composition on the bundle of public goods that is selected.

An agent's utility depends on consumption, demographics, and rent. Consider an agent of type i who lives in the neighborhood during the time period $[t, t + dt]$. Let R_t be the neighborhood rent level. The agent's payoff during this period is simply $(d_i(s_t) - R_t)dt$, plus $u_i - p$ if she buys the consumption good at price p during the period. These payoffs are discounted at the constant rate $r > 0$. (I.e., if a payoff of 1 is received at time t , its discounted value is e^{-rt} at time zero.)

An agent's utility from living outside the neighborhood is normalized to zero. This means that the functions d_i actually capture the *net* benefit from living in the neighborhood for an agent who pays no rent and does not consume. This net benefit can be either positive or negative, depending on unmodelled features of the neighborhood and surrounding areas.

4 Solving the Model

Step 1: Exogenous Demographics

We solve for steady states,⁹ so time subscripts can be dropped. The first step is to determine what happens in the search market for any *given* proportion s of high valuation agents. Theorem 1 does this, largely following Diamond's analysis [9]. Let $V_i(s)$ be the expected discounted sum of consumption payoffs (less prices payed) of a type i neighborhood resident who is waiting for a consumption opportunity. We will refer to $V_i(s)$ as the player's *consumption utility*.¹⁰

⁹These are subgame perfect equilibria in which the neighborhood type distribution and price distribution do not change over time.

¹⁰If the agent consumes at times t_1, t_2, \dots and pays the corresponding prices p_1, p_2, \dots , her realized

Theorem 1 shows that the consumption utility of type 2 buyers is constant or decreasing in s while the consumption utility of type 1 buyers is identically zero. This will generate income mixing below, when we treat s as endogenous.

Theorem 1

There is a unique stationary equilibrium for each (exogenously given) $s \in [0, 1]$. The consumption utility of a type 1 resident is identically zero. There are constants $h > 0$, $0 < s_1 < s_2 < 1$, and $\bar{v}_2 > 0$, as well as decreasing functions $\lambda(s) \in [0, 1]$ and $v_2(s) \in [0, \bar{v}_2]$ and an increasing function $p_2^(s) \in (u_1, u_2]$, such that:*

1. *If $s \in [0, s_1]$, all firms charge the low price u_1 . The consumption utility of a type 2 resident is $\bar{v}_2 > 0$.*
2. *If $s \in (s_1, s_2)$, there is a two price equilibrium. A proportion $\lambda(s)$ of firms charge the low price u_1 . This proportion decreases from one to zero as s goes from s_1 to s_2 . The rest of the firms charge the high price $p_2^* = p_2^*(s) > u_1$, which increases continuously to u_2 as s rises from s_1 to s_2 . The consumption utility of a type 2 resident is $v_2(s)$, which decreases continuously from \bar{v}_2 to zero as s increases from s_1 to s_2 .*
3. *If $s \in [s_2, 1]$, all firms charge the valuation u_2 of type 2 agents. The consumption utility of a type 2 agent is zero.*

Proof: *The analysis largely follows Diamond [9] and is therefore omitted (but is available on request).¹¹*

consumption utility is $\sum_{n=1}^{\infty} e^{-rt_n} (u_i - p_n)$. $V_i(s)$ is the expectation of this sum given s .

¹¹The analysis differs from Diamond's in two ways. First, the two types of buyers enter the search market at exogenous rates in Diamond's model. Our inflow rates are partly endogenous: they equal the rate c of consumption opportunities times the measure of non-searching neighborhood residents of a given type. Diamond's model also has an additional Poisson process that terminates search among buyers who have not yet purchased. This is necessary to prevent the number of type 1 searchers from going to infinity when all firms charge the high price. Such a process is not necessary in our model because the measure of potential searchers is limited by the population of the neighborhood.

The intuition for Theorem 1 is as follows. The consumption utility of a low valuation, type 1 resident is always zero because no firm ever charges less than her valuation, u_1 . If there are sufficiently few high valuation residents, no firm is willing to charge above u_1 and lose all of its low valuation customers. All firms thus offer u_1 and any type 2 agents receive a strictly positive consumption utility. When the proportion of high valuation residents reaches s_1 , there are enough of them that firms are indifferent between charging their reservation price p_2^* and the low price u_1 . As the proportion of high valuation residents grows past s_1 , some firms switch to the higher price p_2^* , while others still charge the low price u_1 . Further influxes of high valuation residents lead more and more firms to shift to the high price. This reduces the consumption utility of high valuation residents, which raises the high price since it is their reservation price.¹² When the proportion of high valuation residents reaches $s_2 < 1$, the proportion of firms charging the high price p_2^* reaches one and the consumption utility of high valuation agents hits zero. The high price p_2^* itself reaches the valuation u_2 of high valuation agents at this point as well: since high valuation agents receive zero utility from continuing to search, their reservation price equals their valuation. Further influxes of high valuation agents have no effect on the price distribution or on their consumption utility.

Step 2: Endogenous Demographics

We now consider the full model, in which the proportion s of high valuation residents is endogenous. A type i agent who enters the neighborhood at time t and remains forever receives the continuation payoff $V_i(s) + (d_i(s) - R)/r$: her consumption utility V_i plus the present discounted value of her demographic utility $d_i(s)$ less the rental rate R . Let us refer to this as $U_i(s, R)$ and let $\Delta(s)$ be the relative payoff to

¹²Firms remain indifferent between the two prices despite the increase in the high price. This is because they shift to the high price fast enough as s rises that the proportion of active searchers who have low valuations actually rises despite the increase in s . This is possible because the shift to p_2^* forces low valuation buyers to visit more stores before they buy, while high valuation buyers always purchase at the first store they visit.

type 2 agents from living in the neighborhood, $U_2(s, R) - U_1(s, R)$. (The rental term drops out of $\Delta(s)$, since it affects both types equally.) Since the rental market is competitive, the neighborhood will be inhabited by the type or types that has the highest continuation payoff from living there. Hence, the neighborhood contains only type 2's if $\Delta(s)$ is positive; only type 1's if it is negative; and possibly a mixture if it is zero.

$\Delta(s)$ is the sum of two terms. One, $V_2(s) - V_1(s)$, is decreasing or constant in s , since an increase in the proportion of type 2 agents lowers the consumption utility of type 2 agents relative to that of type 1. The other term, $(d_2(s) - d_1(s))/r$, is increasing in s , since type 2 agents benefit more in nonconsumption terms from an increase in their proportion. This means that $\Delta(s)$ can in principle cross zero any number of times as s goes from zero to one. If it crosses zero from above (i.e., if its slope is negative where it crosses zero), the intersection is a stable equilibrium under myopic dynamics. This is because a small increase in s leads $\Delta(s)$ to fall below zero, whereupon type 1 agents displace type 2 agents, causing s to fall back; similarly, type 2 agents displace type 1 agents whenever s is slightly below the point of intersection.

For example, consider the case in which $(d_2(s) - d_1(s))/r$ equals a constant, say γ . If γ is negative but not less than $-\bar{v}_2$, there is a *unique* equilibrium, which features income mixing. An example appears in Figure 1. For s below point A , high valuation agents receive higher utility from living in the neighborhood than low valuation agents, so s increases. Similarly, s decreases if it exceeds A , so A is the unique equilibrium.

If $d_2(s) - d_1(s)$ is increasing in s , there can be multiple equilibria, some integrated and some segregated. A case with one mixed and two segregated equilibria is shown in Figure 2. There is a segregated equilibrium at point A : if s is positive but close to zero, high valuation agents receive lower payoffs than low valuation agents from living in the neighborhood, so s declines to zero. The segregated equilibrium at C is also stable by a similar argument. Finally, the mixed equilibrium at point B is the only other stable point, since this is the only point at which $\Delta(s)$ intersects zero from

above.

The general case is as follows:

Theorem 2 *With endogenous s , there may be segregated equilibria at $s = 0$ or $s = 1$ (or both). There may also be any number of mixed equilibria for s between s_1 and s_2 . There cannot be mixed equilibria outside of the range (s_1, s_2) .*

Proof

The first two claims follow from the preceding discussion. The last holds because $V_2(s) - V_1(s)$ is strictly decreasing only for s between s_1 and s_2 , so $\Delta(s)$ can be decreasing only in this range; and stability of a mixed equilibrium requires that $\Delta(s)$ be locally decreasing.

Figure 3 gives an (admittedly *ad hoc*) example with multiple mixed equilibria. For clarity, the graph is limited to the interval $s \in [0.5, 0.6]$. Each of the points A , B , C , and D is a stable equilibrium, since in each case $\Delta(s)$ crosses zero from above.

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$\Delta(s)$ $\Delta(s)$ $\Delta(s)$
 $\Delta(s)$ $\Delta(s)$ $\Delta(s)$
s s s s s s
A A B C
A B C D

$$\frac{d_2(s) - d_1(s)}{r}$$

Figure 1) Simulation for the parameters $(c, r, a, u_1, u_2) = (1, 1, 2, 1, 3)$ and $(d_2(s) - d_1(s))/r$ equal to the constant -0.5 .

Figure 2. Simulation for the parameters $(c, r, a, u_1, u_2) = (1, 1, 2, 1, 3)$ and $(d_2(s) - d_1(s))/r$ equal to $-1.5 + 2s$.

Figure 3. Simulation for the parameters $(c, r, a, u_1, u_2) = (1, 1, 2, 1, 3)$ and $(d_2(s) - d_1(s))/r$ equal to $\frac{1}{42} \sin(10 \arctan(50(s - 0.55))) + \frac{1}{4.2} \arctan(50(s - 0.55)) - 0.46$.