

# An Econometric Analysis of the Structure of Commodity Futures Prices

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## Introduction

Agricultural commodity derivative markets continue to expand as agricultural producers, end-users, and speculators look for risk management and speculative investments. The pricing of derivatives has been the subject of numerous economic articles. Two of the more well-known option pricing models, the Black and Black-Scholes models, assume that the volatility of spot prices increases proportionally to the square root of time. This assumption is reasonable for stocks and currencies, but is inconsistent with mean reversion in commodity prices. Many agricultural commodity markets exhibit mean reversion to production costs (Bessembinder et al.), which suggests that the price volatility around this production cost reaches a maximum value. If this is true, and if price volatility is incorrectly assumed to increase in proportion to the square root of time beyond this maximum value, the fair value of long-term options will be overestimated.

## General Representation of Commodity Futures Pricing Model

Futures and spot prices are assumed to be determined by  $N$  underlying factors. The risk-neutral evolution process for the  $N$  underlying factors  $Y_t$  is given by:

$$dY_t = (K'_0 - K' Y_t) dt + \sum_{i=1}^N S_i dW_{i,t}$$

where  $K'_0$  is an  $(N \times 1)$  vector,  $K'$  is an  $(N \times N)$  matrix,  $\Sigma$  is an  $(N \times N)$  matrix,  $S_i$  is an  $(N \times N)$  diagonal matrix with diagonal elements defined as  $[S_i]_{ii} = a_i + b_i Y_t$ ,  $a_i$  is an  $(N \times 1)$  vector,  $b_i$  is an  $(N \times N)$  matrix, and  $dW_t$  is an  $(N \times 1)$  vector of independent Brownian motions. This definition of  $b$  follows Duffee, and is the transpose of the  $b$  matrix defined in most of the literature (e.g., Dai and Singleton). The historical evolution process follows a similar form. Seasonality is incorporated in the model via  $K'_0$ .

The spot price for the commodity is defined as

$$P_t = \exp[\theta_0 + \theta' Y_t]$$

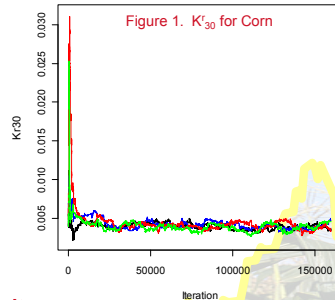
and futures prices for contracts maturing at time  $(t + \psi)$  are given by

$$F_{t,\psi} = E[P_{t+\psi}] = E[\exp[\theta_0 + \theta' Y_{t+\psi}]] = \exp[A^F_{t,\psi} + B^F_{t,\psi} Y_t]$$

where  $\theta_0$  is a constant,  $\theta = [\theta_1, \dots, \theta_N]$  is an  $(N \times 1)$  vector of constants, and the coefficients  $A^F_{t,\psi} \in \mathbb{R}$  and  $B^F_{t,\psi} \in \mathbb{R}^N$  are the solutions to ordinary differential equations with boundary conditions.

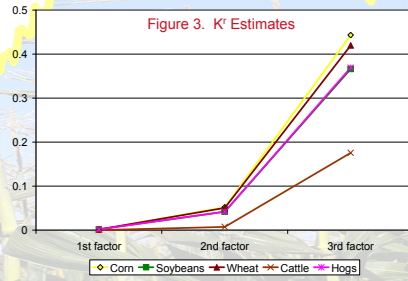
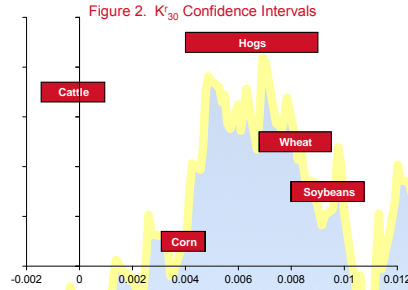
## Method

We utilize a three-factor structure of commodity futures prices. To estimate the models, commodity futures prices for available maturities will be analyzed by means of Markov-Chain Monte Carlo (MCMC) methods. MCMC methods have recently become popular for the estimation of continuous-time derivative pricing models because the main historical impediment to their usage (i.e., computer power) is no longer a significant constraint, and they provide important advantages over more traditional methods (Johannes and Polson). For the present study, MCMC methods are particularly well suited for at least two reasons. First, theoretical pricing models are based on constant-time-to-maturity futures prices, whereas observed commodity futures contracts are based on constant maturity dates. The MCMC approach allows us to draw inferences about constant-time-to-maturity futures prices from constant-maturity-date futures price data. Second, MCMC methods allow us to easily handle the facts that spot commodity prices exhibit seasonality and that maturity dates for most commodity futures contracts are unevenly distributed throughout the year.



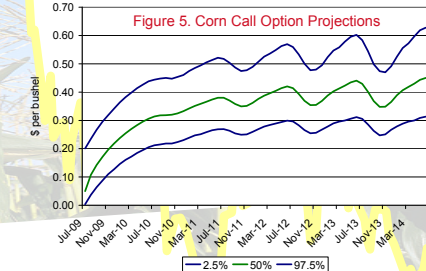
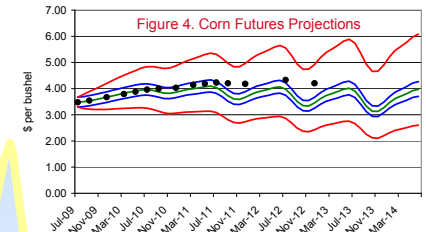
## Results

For corn, soybeans, wheat, cattle, and hogs, MCMC estimation of the model was considered to have converged when the highest Gelman and Rubin's R statistic for the model parameters was smaller than 1.1. Convergence was achieved with between 2 million and 4 million iterations for each commodity. Figure 1 shows the set of iterations for  $K'_{30}$ , part of the  $K'_0$  vector, for corn. The key parameters for examining the hypothesis of mean-reverting processes are  $K'_0$  and  $K'$ . Figure 2 shows the confidence intervals for  $K'_0$  across all commodities, but cattle, indicate mean-reverting processes ( $K'_{30} > 0$ ). Figure 3 shows the speed of that reversion is slowest for the 1st factor, but increases for each additional factor.



## Projections

The model estimates can be used in combination with current futures price data to project the values of longer-term futures and options. Figures 4 and 5 show the projections from the corn model for futures and "at-the-money" call options, based on July 1, 2009 corn prices.



## Selected References

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