This is a closed book exam. You may use a calculator. Write down all major steps.

(1) [20 pts]:
   a. Please write your name [2 pts] and ID number [2 pts] clearly on the cover of your answer book, and return this exam page [1 pt].
   b. What is a monotonic preference? [5 pts]
   c. What is the price elasticity of demand? [5 pts]
   d. Prove or disprove the claim that average cost increases whenever marginal cost is below the average cost [5 pts].

(2) [10 pts] Mr. A plans to sell ice-cream cone and Coke during the Homecoming parade at Ohio State on Friday, November 29, 1999. An ice-cream cone weighs .25 lb, a can of Coke weighs 0.5 lb. Selling an ice-cream cone takes 1.5 minutes and makes $2 profits, and selling a can of Coke takes 0.5 minute and makes $1 profits. Suppose the parade lasts one hour, Mr. A can carry only b lbs (b > 0 is a fixed number), and the crowds are large enough to consume any quantity that Mr. A wishes to sell. Formulate Mr. A's optimization problem.

(3) [20 pts] Suppose a consumer has $80 income, which is to be spent on only two goods, corn flakes and orange juice. Corn flakes cost $2 per box while orange juice costs $1 per can. Suppose also that the consumption of corn flakes is rationed (not more than 30 boxes per person) by the government due to a drought. Please answer the following:
   (a) The constraints in his U-Max problem. [4 pts]
   (b) The maximum consumptions for orange juice and Corn flakes. [6 pts]
   (c) The bundle on the budget equation with maximum consumption of Corn flakes. [5 pts]
   (d) Suppose the optimal consumption of corn flakes is less than 30 boxes (rationing has no effects on the consumer). Compute his marginal rate of substitution of corn flakes for orange juice (i.e. with orange juice on the vertical axis) at the optimal bundle. [5 pts]

(4) [20 pts] Suppose a firm's production function is \( Q = \frac{1}{10^{10}}KL \), the wage rate and rental rate are \( w = $10 \) and \( r = $250 \). The marginal productivities are \( MP_L = \frac{1}{20}\sqrt{K} \) and \( MP_K = \frac{1}{20}\sqrt{L} \). Derive the firm's cost function.

(5) [20 pts] Suppose a firm's demand function is \( Q = 24 - P \), its cost function is \( TC(Q) = 5Q^2 \). Its marginal revenue and marginal cost are \( MR(Q) = 24 - 2Q \) and \( MC = 10Q \).
   (a) Calculate the firm's maximal profits and optimal supply [10pts].
   (b) Suppose now the firm is a price taker, and \( SRTC(Q) = 5Q^2 \) is its short-run cost function. Its short-run marginal cost is \( SRMC(Q) = 10Q \). Derive its short-run supply function [10pts].

(6) [10 pts] Suppose utility \( u(L, I) \) depends on leisure \( L \) and income \( I \), the wage rate is \( w = $10 \) per hour (the price of the good called income is \( P_I = \frac{1}{w} = 1/10 \)), so the constraint is: \( L + I/10 = 16 \) (i.e., one allocates 16 hrs between \( L \) hrs of leisure and I/10 hrs of work). Suppose \( w \) increases to $1000 per hour (\( P_I \) decreases from 1/10 to 1/1000). Suppose Mr. B decided to work less. Are \( L \) and \( I \) substitutes or complements [2 pts]? Is income an inferior good or a normal good [8 pts]?
Answers to Midterm Exam

(1) See your notes.

(2) Max \( C + 2I \)
    s.t. \( 0.5C + 0.25I \leq b \), \( 0.5C + 1.5I \leq 60 \), \( C, I \geq 0 \).

(3) Let \( x \) denote corn flakes, \( y \) denote orange juice.
    (a) \( 2x + y \leq 80 \) (budget ),
        \( x \leq 30 \) (ration),
        \( x, y \geq 0 \) (Non-negative).
    (b) \( x_{\text{max}} = 30, \ y_{\text{max}} = 80 \)
    (c) \( x = 30, \ y = 20 \).
    (d) \( \text{MRS (of } x \text{ for } y \text{)} = \frac{P_x}{P_y} = 2 \).

(4) Step 1. \( \text{RTS} = \frac{\text{MP}_L}{\text{MP}_K} = \frac{K}{L} \);
    Step 2. The equal slope condition and constraint equation \( \begin{cases} \text{RTS} = \frac{W}{r} \\ f(K, L) = Q \end{cases} \)
        are:
        \( \begin{cases} \frac{K}{L} = \frac{10/250}{1/10} \quad \ldots \ldots \text{(i)} \\ \sqrt{\frac{L}{K}} = Q \quad \ldots \ldots \text{(ii)} \end{cases} \).
    Step 3. Solving for \( (K, L) \) in (i) and (ii), one has \( K^* = 2Q, L^* = 50Q \).
    Step 4. Plugging into objective, one has \( \text{TC(Q)} = rK^* + wL^* = 1000Q \).

(5) Part (a). Step 1. \( p = 24 - Q \), \( \pi(Q) = R(Q) - TC(Q) = (24 - Q)Q - 5Q^2 \).
    Step 2. The optimal condition \( \text{MR} = \text{MC} \) is: \( 24 - 2Q = 10Q \).
    Step 3. Solve for \( Q \) in the above condition, one has \( Q^* = 2 \), and \( \pi^* = 24 \).

Part (b). Step 1. For a price taker, the \( \text{MR} = p \);
    Step 2. The optimal condition \( p = \text{SRMC} \) is: \( p = 10Q \);
    Step 3. Solve for \( Q \) (as a function of the price \( p \)) in the above equation, one gets:
        \( Q^* = S(p) = p/10 \), which is the short-run supply function.

(6) As the price for other good decreases, the demand for a complement good rises, and that for a substitute good falls. Since Mr. B works less, his consumption of leisure rises. Therefore, \( I \) and \( L \) are complements. Though Mr. B works less, his consumption of income could increase or decrease. Let \( h_0 \) and \( h \) denote his old and new working hours, \( (h_0 - h) = \Delta h = \Delta h > 0 \); \( I_0 \) and \( I \) denote his old and new incomes. Then, \( I = 1000 \cdot (h_0 - \Delta h) > I_0 = 10 \cdot h_0 \) if and only if \( 0.99 \cdot h_0 > \Delta h \). Therefore, more information is needed in determining whether income is normal or inferior.