Patents, Search of Prior Art and Revelation of Information

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Abstract

The recent deliverance of “business-method-software” patents has been strongly debated. Examiners of the Patent Office (PTO) are accused of granting patents to non-deserving innovations. This failure is mainly due to the lack of prior art (i.e., existing set of related inventions). However, innovators are liable too, as they do not search for relevant prior art. We, thus, propose a model of bilateral search of information. The innovator can undertake a costly search of prior art, and thus be privately informed of the value of the innovation. He then applies for a patent and reveals all or part of the prior art to the PTO. The latter performs a complementary search of information to assess the patentability of the innovation. We first study the information search game when the PTO does not commit to an ex ante level of screening. We show that an innovator can conceal some information to increase the probability of being granted a patent, and that the PTO makes its screening intensity contingent upon the level of prior art transmitted. Second, we study the ex ante commitment, and we show that it involves equal screening intensity across all applications. This ex ante commitment has two interesting properties. On one hand, it requires a limited commitment power on the part of the PTO and, on the other hand, it induces truthful information transmission on the part of innovators.

Keywords: Patents; Information; Incentives

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1 Introduction

Performances of the Patent and Trademark Office (PTO) have been strongly questioned. Indeed, it has been argued that PTO examiners grant too broad and/or non-deserving patents in new industries (for instance in e-commerce). This is mainly due to their very poor knowledge of the relevant prior art – the existing set of related inventions. In this paper, we focus on the process of gathering information for both innovators and PTO examiners. Our concern is twofold; first, we focus on how to induce innovators to search for relevant prior art information, and second, how to give them incentives to reveal it. We show that an innovator who finds a little piece of evidence against the patenting of his innovation will not always provide all the information he has acquired. Indeed, as the efforts of the innovator and the PTO are complementary, the more information the PTO gets, the more likely she will find out the value of the innovation. In absence of any commitment from the PTO, we find that the screening intensity of the examiners is contingent upon the level of prior art transmitted; the more information transmitted, the higher the screening intensity. We, then, investigate whether the ex ante commitment of the PTO to a certain level of complementary search of prior art information can induce innovators to search for more prior art information, and then to reveal their findings. From a policy implication perspective, we find that the PTO should not commit to different levels of search of prior art information, but, on the contrary, should commit to equal screening intensity across all applications. Indeed, this strategy has two advantages: first, it requires a limited commitment power from the PTO and, second, it induces truthful information transmission from innovators.

The recent surge of “business-method software” patents raises fundamental issues of intellectual property rights (IPR) due to the nature and the novelty of e-commerce. Most e-commerce patents have little prior art: slightly less than half of all patents, over 20 years, cite no non-patent prior art, and the average patent cites about two non-patent prior art items. It is, thus, difficult for the PTO examiners to assess the novelty of the innovation. On the other hand, even though it seems that there is no clear relationship between cited, uncited prior art and patent validity, uncited prior art is a more effective tool for invalidating patents in court than cited prior art (Allison and Lemley (1998)). In other words, it is easier to invalidate a patent on the basis of prior art that is not included in the patent, but has been provided later in court by a third party. Thus, patents with too little prior art information lead to two kinds of inefficiency:

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1 For instance, if the innovator finds evidence that may invalidate the patent if challenged in court.
too many patents are granted, and \textit{ex post} too many patents may be invalidated in court.

In the current US patent system, to be patentable, an innovation must be useful, novel and non obvious.\footnote{In the European patent system, an innovation must be novel, mark an inventive step and be commercially applicable. The process of deliverance of patents in the two systems is different. See Graham, Hall, Harhoff and Mowery (2002) for a detailed description of the two systems.} A patent application must contain references to previous literature and patents upon which the innovation improves or from which it diverges. Thus, innovators must provide information concerning prior art in order to prove that the innovation has not been patented or published prior to the time the patent is filed. Therefore, innovators have a duty of candor in disclosing prior art information,\footnote{In the European patent system, innovators do not have to provide a full list of prior art. The European patent office (EPO) provides a search report to the innovator, who decides whether or not to request the patent examination (Graham, Hall, Harhoff and Mowery (2002)).} but they have no duty to search for prior art before filing an application: they must only disclose the information that is in their possession. Furthermore, according to the doctrine of inequitable conduct, innovators should disclose all the relevant prior art information they have, and should not disclose false information with the intent to deceive the PTO. But the latter has made it clear that applications will not be investigated and rejected based on violation of the duty to disclose prior art, as the PTO is not well-equipped to enforce such a rule (Kesan (2002)). Innovators have, thus, too little incentive to reveal relevant information.

The examination procedure is quite standard, even though it strongly depends on the examiner's judgment. Cockburn, Kortum and Stern (2002) show that there exists heterogeneity between examiners, especially in their search of prior art. In the US patent system, the PTO examiners make a complementary search of prior art information. The burden of searching prior art is on innovators, as well as PTO examiners.

As already mentioned, a key problem in the e-commerce world is the lack of published papers on software programming and the lack of citations in patents. Therefore, it is difficult (but not impossible) to find information concerning existing prior innovations. Because of this weak prior information some patents tend to be granted to non-deserving innovations. Whose fault is it? The quality of the examination has been extensively criticized: some lawyers and economists claim that the PTO is responsible.\footnote{See Beal (1998), Coppel (2000), Kesan (2002), Kesan and Banik (2000), Lemley (2001), and Merges (1999) for a discussion on the relevance of the patent protection system in the e-commerce world.} Indeed, it seems that the PTO lacks the money and information to perform the necessary search of prior art. Then we can blame the PTO for not
doing its job and for granting patents to innovations without enough evidence of their novelty. One can think of a new way of rewarding PTO examiners, or implementing an opposition system similar to what exists in Europe\(^6\) as has been suggested by Merges (1999). But it is not just the PTO examiners’ responsibility; innovators are liable too. Because it is well established that the prior art information is not easily available and that the PTO does not (and cannot) enforce the “doctrine of inequitable conduct,”\(^7\) innovators have no incentive to make the effort to search for prior art and very little incentive to reveal it. Concerning the search of information, any relevant piece of evidence can constitute prior art: it can be a thesis in a French university, a Chinese institution or an Italian institute for instance.\(^8\) This is evidence that there is a costly way to find relevant information, for those who want to find it.\(^9\) This is a classical moral hazard problem. With regard to the revelation of information, we have a classical adverse selection problem.

Thus, a mechanism must be designed to induce innovators to perform the necessary search of prior art, and to reveal it. This new incentive is twofold. On one hand, innovators will have stronger patents in case of lawsuit, and should be able to defend their patents against an accusation of invalidity. On the other hand, the PTO will examine fewer patents.

We propose the following model. We assume that the value of an innovation that has just been discovered can be “good” or “bad,” but is unknown to the innovator and the PTO. By “good,” we mean that the innovation is really novel and, for instance, does not infringe on existing patents, or does not reproduce existing innovations. Furthermore, the innovation can belong to a rich or poor field of prior art. The innovator performs a search for prior art information. He finds a certain amount of prior art (we restrict the amount to only three levels: nothing, intermediate, or full amount) that may be accompanied by a signal that can be either positive or negative. This signal informs the innovator of the value of the innovation. The idea here is that with serious research, the innovator may become better informed about the real nature of his innovation. Aware of this piece of evidence, the innovator reveals some prior art information while applying for a patent. He can reveal all the information he found or just part of it. The PTO receives the revealed information from the innovator, updates her beliefs consequently, and then decides to search for extra information to assess the patentability of

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\(^6\) For details on the opposition system, see Graham, Hall, Harhoff and Mowery (2002).

\(^7\) See Kesan (2002) for details about the law.

\(^8\) On the web site Bountyquest.com, one can post an announcement to find prior art concerning a patent, for a reward from $10,000 to $30,000.

\(^9\) Moreover, intuition suggests that innovators, being familiar with their innovation, know where to search.
the innovation. The complementary search of information provides a signal to the PTO that reveals imperfectly the nature of the innovation. The screening accuracy of the PTO depends on her effort, as well as on prior beliefs about the application submitted. When the PTO increases her effort, the probability of perfect screening increases and the weight of prior beliefs in the decision-making process diminishes; there are fewer mistakes. When the effort is low, the decision-making is more loose, and essentially driven by the PTO’s prior beliefs about the application.

In this setting, we find that an innovator who receives a negative signal about his innovation has no incentive to always report truthfully when he finds the full amount of prior art. He will, thus, prefer not always to report truthfully (mixed strategies equilibrium). This equilibrium can be explained as follows. When the innovator searches with a high level of effort, he finds the true nature of his innovation. On one hand, an innovator who finds that the innovation is “bad” has an incentive not to report all the prior art he found, in order to make the PTO’s screening more difficult, and hoping that potential infringement will go unnoticed by the PTO. On the other hand, an innovator who finds that the innovation is “good” would like to “alleviate” the PTO’s burden, and would transmit all the relevant prior art. However, such a behavior cannot constitute an equilibrium, because patent applications with the full amount of prior art tend to be only good applications, and the PTO should deliver patents without screening them. Anticipating this, bad innovators would then start filing patent applications with a high prior art content. Therefore, the equilibrium cannot be one in pure strategy, but rather one in which bad innovators will randomize the amount of prior art they transmit.

We characterize the optimal effort of the PTO in this setting and we study the properties of this equilibrium. Those equilibrium effort levels, contingent upon the level of prior art delivered, are shown to be dependent upon each other.

We then study a policy in which the PTO can \textit{ex ante} commit to effort levels that are contingent upon the amount of prior art delivered. When, based on the amount of prior art, the PTO commits to different levels of effort, the applications made by bad innovators will always receive the lowest amount of screening by the PTO. We discuss the commitment to different levels of effort by the PTO and show that in this policy, the PTO has high incentives to renege on its commitment.

We then analyze a policy in which the PTO commits to identical levels of screening irrespective of the amount of prior art delivered. We first show that, compared to the non-commitment case, this policy of equal treatment may involve lower effort on the part of the PTO and more revelation on the part of innovators. Therefore, commitment allows the PTO to shift the burden
of research to innovators. Second, since it involves the same screening intensity across all applications, there is no strategic reporting and, therefore, no information loss. Finally, although this policy implies some commitment power on the part of the PTO, the incentives to renge on this commitment are much lower than in the case of different effort levels.

To the best of our knowledge, in the economic patent literature, very little attention (if not none) has been devoted to this problem of search and revelation of prior art information. Patent literature has focused on the importance of patent litigation (Lanjouw and Schankerman (2001)), or settlement in case of patent infringements (Crampes and Langinier (2002)). A great deal of attention has focused on patent rules that affect the value of the patent grant in the context of sequential innovation (Chang (1995), Scotchmer (1996), O’Donoghue (1998), Schankerman and Scotchmer (2001)). On the other hand, scholars in law have been concerned with this problem of the search of prior art information (see Kesan (2002), Lemley (2001), Mergers (1999)).

In the context of the principal-agent model, Levitt and Snyder (1997) put the emphasis on the role of the transmission of information from agent to principal. They show that contracts must sometimes reward agents for announcing bad news. Thus, as we find in our model, the principal commits \textit{ex ante} to an inefficient \textit{ex post} outcome. Also related to our analysis, Khalil (1997)’s model studies the optimal contract when there is no commitment to an audit policy from the principal.

The paper is organized as follows. In section 2 we present the model, the search of prior art strategy for the innovator, and the search of complementary information and granting strategy for the Patent and Trademark Office. Section 3 is devoted to the detailed presentation of the technologies of examination of a patent and research of prior art. In section 4 we show the non existence of equilibrium in pure strategies. We show that equilibrium exists only in mixed strategies in which the innovator does not always reveal the information obtained. In section 5, we derive the optimal level of innovator effort to search for prior art information, according to the revelation strategies. We analyze a policy in which the PTO can \textit{ex ante} commit to a certain level of scrutiny in section 6. In section 7, we compare those two commitment devices (non commitment and \textit{ex ante} commitment) and we derive policy implications.

\section{The Model}

We consider a model with two players: an innovator who has just made an innovation, and the Patent and Trademark Office (PTO) that will judge of the deliverance of a patent. At the
outset, neither the innovator nor the PTO know the (social and private) value of the innovation. We assume that the innovation can be good or bad, and thus will lead to a “good” or “bad” patent if patented. By “good” patent, we mean a patent on an innovation that is very novel (whether it belongs to a novel field or not). A “bad” patent is more likely to be invalidated in court, as it infringes upon existing patents or is not novel enough. Both the innovator and the PTO have prior distribution on the value of the innovation: they believe with probability $p$ that the innovation is good.

Having done the original work, the innovator is, nonetheless, aware of the inventive degree of his innovation. The innovation can be new or, on the contrary, can be an improvement of another innovation, but can still be patented. This, in turn, determines the field of prior art to which the innovation belongs. It can belong to a field where there is very few prior art, that we call a “poor prior art field” or, conversely, to a “rich prior art field”. The innovator knows the prior art field his innovation belongs to. However, in order to get more information concerning the value of the innovation, the innovator must undertake an effort to search for relevant prior art. Thus, the prior art that the innovator finds depends, on one hand, on his effort of research and, on the other hand, on the field to which the innovation belongs. This information will be helpful for the innovator to update his beliefs concerning the value of the innovation. In fact, the observation of the prior art will bring bad or good news. For instance, if the innovator makes enough effort to search for prior art if he believes that he has a good innovation that belongs to a rich field, he will find all the relevant prior art that represents only positive information (good news). On the contrary, if he does very little effort to search for prior art, and believes that it is a bad innovation and there is only few prior art, he will not find any prior art, and thus will not be able to infer any information (positive or negative). All the contingencies will be detailed in the presentation of the different steps of the model.

The innovator then files a patent application that contains his announced amount of prior art. The PTO cannot observe the effort of the innovator nor what he actually found, she can just observe the announced amount of prior art and update in consequence her beliefs. Based on her new beliefs, and on the announced amount of prior art, she can decide to search for more information.

We now detail each of the components of the model.
2.1 Search of Prior Art by the Innovator

At the beginning of the game an innovation has been made. We assume that the private value for the innovator can take three values, $V$, $V'$, and $v$ with $V > V'$ and $V > v$. When the innovation is good and patented, the innovator gets the private value $V$. When the innovation is good but is not granted a patent, its value is $v$. The lower value $V'$ can stem from the fact that the patent delivered by the PTO infringes on an existing patent or can be invalidated in court because of the existence of previous patented or non patented prior art such as scientific publications or notes, recorded experiences, etc. We assume that innovators prefer to have a bad innovation patented rather than none. The idea is that the innovator hopes that the trial costs may discourage entrant to challenge the validity of their patent. Alternatively, a trial may invalidate only part of the claims originally made by the innovator, and the patent would still keep some positive value. Moreover, we assume that $\Delta V = V - V' - v > 0$; that is, the difference in value between a good innovation and a bad innovation is greater when the innovation is patented.

The innovator has a certain ability to search for more information. Each innovator has different ability to search for information, depending on the sophistication on his R&D team. Depending on this ability to search, the innovator will decide whether or not to undertake the prior art search. If he decides to exert a level of effort, he will find a certain amount of prior art that we assume can only take three values $x \in \{0, x_i, \overline{x}\}$. The ability of the innovator generates a probability of finding an amount of prior art $a \in [0, 1]$ and has a desutility $c(a) = \frac{1}{2}a^2k$.

Let $\gamma$ be the probability that the innovation belongs to a rich prior art field, and $(1 - \gamma)$ the probability it belongs to a poor prior art field. Neither the innovator’s effort nor the found prior art can be observed by the PTO. Only the innovator is aware of the prior art information he found that can be the maximum amount ($\overline{x}$) that we normalize to be 1, the minimum amount (0) or an intermediate level of prior art ($x_i < 1$). Let us now explain under what circumstances the innovator will find one of these levels of prior art. First, the innovator will find the maximum amount of prior art $\overline{x} = 1$ with probability $a$ if the prior art actually exists (i.e. in a rich prior art field). However, he can get positive information if the innovation is good, or negative information in the case of a bad innovation. To simplify, we assume that the amount of prior art is the same whatever the value of the innovation but, in the case of a bad innovation, the innovator gets an extra ($\varepsilon$) amount of information that is negative. This negative extra amount of information is enough to state that the value of the patent is bad.\(^{10}\) The innovator will be aware of the value of his innovation after having found the prior art.\(^{11}\) Second, the innovator

\(^{10}\) For instance the innovator can find a piece of evidence that could invalidate his patent if he is sued in court.
\(^{11}\) Our model makes the strong assumption of perfect learning by innovators. This assumption only simplifies the
will find an amount \( x_i < 1 \) of prior art with probability \( a \) in a poor prior art field.

After having searched for more prior art, if the innovator learns the value of his innovation, he can decide to apply for a patent. If he does not find a piece of evidence against patenting, but on the contrary only positive information, he will for sure apply for a patent. On the other hand, if he finds a piece of evidence against patenting (that could invalidate it in court), he may decide not to apply for a patent, or at least not all the time. We could assume indeed, that an innovator that receives a negative signal decides to apply for a patent with a certain probability. This assumption does not alter nor improve our qualitative results, so to simplify we just assume that whatever his type, the innovator applies for a patent.

### 2.2 Search of Complementary Information and Granting a Patent

Once he has discovered prior art information, the innovator files a patent application with his announced level of prior art \( \bar{x} \). The PTO observes the level of prior art and updates her beliefs. She then makes the following decision. If the announced level of prior art is 0, she does not grant a patent claiming that there is not enough information to judge of the novelty of the innovation.\(^\text{12}\) Otherwise, if she observes \( x_i \) or \( \pi = 1 \) she decides to search for complementary information. After having observed \( \bar{x} \), she makes an effort \( E \in [0, 1] \) to search for complementary information at the cost \( C_2(E) \). We define the cost structure in the next section. Therefore, she receives a signal that can be either an informative signal (with probability \( E \)) that the innovation is good (or bad) or a random signal (with probability \( 1 - E \)). If she receives a random signal she will grant a patent with the updated probability of having a good innovation and will not grant a patent with the complementary probability.

To summarize, the timing is the following:

1. The innovator makes two decisions: - the effort to search for prior art - and, once he has learned the value of the innovation, he files a patent application with an announced prior art \( (\bar{x}) \).

2. Then the PTO observes the innovator’s announcement, and decides to search for additional information or not. Finally, the PTO decides whether or not to grant a patent.

\(^\text{12}\)This assumption could be relaxed although it seems natural that if the file is empty, the PTO refuses to consider it.
3 The Technologies of Examination and Research of Prior Art

We now examine in detail the structure of the PTO’s technology of examination of patent, and then the innovator’s technology of research of prior art.

3.1 The Examination Technology of the PTO

The PTO will decide whether or not to search for complementary information and eventually grant a patent based on the received announcement.

If the innovator files a patent application with no prior art at all ($x = 0$), the PTO does not deliver a patent arguing that there is not enough information, whatever her beliefs. If the innovator files a patent application with the intermediate or maximum amount of prior art ($x = x_i, 1$), the PTO will first update her beliefs and then will decide how much effort to put in checking the application of the innovator. The updated beliefs of the PTO depend crucially on the anticipated behavior of the innovator. There are several possible combinations: - either the innovator always reveals the truth - or only the innovator that receives a positive signal reveals the truth, - or only the innovator that receives a negative signal reveals the truth, - or none of them reveal the truth, - or eventually the innovator randomizes his revelation of the truth.

For the time being we just define non-specified updated beliefs that will take different values depending on the anticipations of the PTO. After having received a signal $\bar{x}$, the PTO is able to update her beliefs, and thus now believes that the innovation is good with probability $\mu_{\bar{x}} \equiv \Pr(\text{Good} | \bar{x})$ and no longer $p$.

There is always uncertainty on patent applications made by innovators, some applications are more difficult to judge than others. An important question is whether the PTO should exert a high effort of checking on every innovations or only on part of them. Should the PTO devote more time on an innovation that appears difficult to judge?

Obviously, an examiner who works at the PTO has just a limited amount of time to devote to the examination of an application. We assume that, for a given innovation, she can perfectly discover the nature of an innovation with probability $E$, and with probability $1 - E$ the delivery of the patent is made (randomly) according to the belief of the PTO. The cost of doing the checking effort is given by

$$C_\bar{x}(E) = \frac{K}{\bar{x}(1 - E)}$$ (1)
where $K > 0$ and must be smaller than a cutoff value $K$ that we will define precisely later, to induce to PTO to undertake the necessary search effort.

The rationale for this cost function goes as follows. If the PTO wants any potential innovation to be scrutinized with perfect accuracy, the cost will be infinite. We also assume that the level of cost is influenced by the quantity of prior art, $\bar{x}$, transmitted by the innovator. A lower amount of information tends to raise the overall cost of examination.

The PTO maximizes her objective function

$$\max_{E}[B(E; \mu_{\bar{x}}) - C_{\bar{x}}(E)]$$

where her gross benefit is

$$B(E; \mu_{\bar{x}}) = \mu_{\bar{x}}[E\bar{W}_G + (1 - E)(\mu_{\bar{x}}\bar{W}_G + (1 - \mu_{\bar{x}})\bar{W}_R)] + (1 - \mu_{\bar{x}})[E\bar{W}_R + (1 - E)(\mu_{\bar{x}}\bar{W}_G + (1 - \mu_{\bar{x}})\bar{W}_R)]$$

This gross benefit depends on the level of effort of the PTO and will be different depending on her updated beliefs. In the first term of $B(E; \mu_{\bar{x}})$, the PTO finds the exact nature of the innovation that generates a value $\bar{W}_G$ to society when the innovation is good (and a patent is granted), whereas it generates a value $\bar{W}_R$ when she rejects a bad innovation. When $E < 1$, the PTO introduces the possibility of type I and type II errors in the granting process. More precisely with probability $\mu_{\bar{x}}(1 - \mu_{\bar{x}})$ the examiner refuses a patent to a deserving innovation and generates a social value $\bar{W}_R$ (type I error). Alternatively, he may award a patent to a non-deserving innovation and create a social value $\bar{W}_G$ (type II error).

We assume that granting a patent on a good innovation is always better than refusing a patent on a bad innovation that in turn is better than refusing a patent on a good innovation that is better than granting a patent on a bad innovation. In other words, the social value of granting a patent on a good innovation is very high. Formally, $\bar{W}_G > \bar{W}_R > \bar{W}_R > \bar{W}_G$. This assumption implies that $\bar{W}_G - \bar{W}_G > \bar{W}_R - \bar{W}_R$, and thus we define $\Delta W = \bar{W}_G - \bar{W}_R - \bar{W}_G + \bar{W}_R$ that is always positive.

The cost of effort of the PTO is parameterized by the amount of information delivered by the innovator and we make the explicit assumption that the PTO’s cost is lowered when the innovator provides more complete information, i.e. the greater $\bar{x}$ is. We believe that this “complementarity” between the quality of the information provided by the innovator and the PTO efficiency is an important aspect of this relationship.13

13Alternatively, we could assume that the innovator may provide the PTO with too much “non relevant” information in order to discourage serious checking. In each case, the strategy of the innovator amounts to increase the search costs of the PTO in order to obtain a less accurate decision.
From (2), the generic first order condition can be written as

$$\mu_x (1 - \mu_x) \bar{x}\Delta W = \frac{K}{(1 - E)^2}. \quad (4)$$

Solving (4) with respect to $E$ shows that the generic level of effort is given by

$$E_x^* = \frac{\mu_x (1 - \mu_x) \bar{x}\Delta W - \sqrt{K\mu_x (1 - \mu_x) \bar{x}\Delta W}}{\mu_x (1 - \mu_x) \bar{x}\Delta W} < 1$$

where $K < \bar{K} = \bar{x}\Delta W \min\{\mu_x (1 - \mu_x), \frac{1}{4}\}$.

The PTO will exert a high level of scrutiny on a fraction $E_x^* > 0$ of the applications that contain a level $\bar{x}$ of prior art, whereas a fraction $1 - E_x^*$ will not be scrutinized seriously.$^{14}$

3.2 The Technology of Research of Prior Art of the Innovator

The innovator makes two decisions: first, he determines the level of effort he wants to make to search for prior art and, second, he communicates all or part of the prior art information he eventually got after searching for it.

We first analyze the announcement of the innovator once he has found some prior art and has received a signal about the value of his innovation. Then, we determine the optimal level of effort that the innovator makes to find the appropriate level of prior art.

3.2.1 Announcement of the Innovator

We first determine what must be the (strategic) announcement of the innovator. Once the innovator has made a certain level of effort, he will find prior art information, that can be 0, $x_i$ or 1, and a signal about the value of the innovation. In the case of a good innovation, in a rich prior art field (respectively in a poor prior art field), with probability $a$ the innovator will discover $x = 1$ (respectively $x = x_i$) and a positive signal, and with probability $(1 - a)$ he will discover $x = 0$ and thus he is unable to infer the value of the innovation. Conversely, if the innovation is bad, the innovator will get the same structure of information depending on the prior art field but he will get a negative signal, and will thus know that there exists a piece of information that is not in favor of patenting.

As the innovator cannot report more than he has actually found, if he gets no information ($x = 0$) he will report nothing $\bar{x} = 0$.

$^{14}$It is important to notice that our model introduces some restriction on the parameters. In particular, the term $\mu (1 - \mu) x\Delta W$ should be strictly greater than $K$. We assume that the value of an innovation for society is high so that the condition underlined is true for almost all possible beliefs of the PTO about the innovation.
If the innovator receives an intermediate level of information \((x = x_i)\), he can announce \(\bar{x} = 0\) or \(\bar{x} = x_i\). There is no gain to announce nothing since the PTO does not deliver a patent if the patent application has no prior art information. Thus, whatever the value of the innovation, he should always report truthfully, \(\bar{x} = x_i\).

If the innovator gets all the information \((x = 1)\) he can announce either \(\bar{x} = 0\), \(\bar{x} = x_i\) or \(\bar{x} = 1\). We first compare the gain from reporting something and the gain from not reporting anything. As we have already mentioned it, the innovator is always better off by revealing some prior art rather than nothing. So, announcing \(\bar{x} = 0\) is a strictly dominated strategy that will never been chosen by the innovator. Second, we have to determine under what circumstances the innovator will prefer to report truthfully. An innovator that gets only positive signal on top of the prior art gets an expected gain from reporting truthfully of

\[
E_1\bar{V} + (1 - E_1)[\mu_1\bar{V} + (1 - \mu_1)v],
\]

where \(\bar{V}\) represents the profit of the innovator if he patents a good innovation, \(v\) the profit if the innovation is not granted a patent and \(E_1\) the level of effort undertaken by the PTO when she observes \(\bar{x} = 1\). If he reports \(\bar{x} = x_i\) instead, he gets

\[
E_i\bar{V} + (1 - E_i)[\mu_i\bar{V} + (1 - \mu_i)v],
\]

where \(E_i\) is the level of effort of the PTO when she observes \(\bar{x} = x_i\). The innovator compares the two expected payoffs to decide whether or not to report truthfully. He then reports truthfully as long as

\[
E_1\bar{V} + (1 - E_1)[\mu_1\bar{V} + (1 - \mu_1)v] > E_i\bar{V} + (1 - E_i)[\mu_i\bar{V} + (1 - \mu_i)v].
\]  

(5)

If the innovator receives a negative signal on top of the prior art he found, he gets an expected gain from reporting truthfully

\[
(1 - E_1)\mu_1\bar{V},
\]

where \(\bar{V}\) represents the profit of the innovator if he patents a bad innovation. If he reports \(\bar{x} = x_i\) his payoff is

\[
(1 - E_i)\mu_i\bar{V}.
\]

Thus, the innovator who gets bad information reports truthfully if

\[
(1 - E_1)\mu_1 > (1 - E_i)\mu_i.
\]  

(6)

Equations (5) and (6) represent the conditions under which both types report truthfully.
3.2.2 Search of Prior Art

The innovator determines first whether to make a level of effort in order to find the relevant prior art, according to his ability to searching for more prior art. As we solve the game by the end, we characterize the expected profit of the innovator after having determined the optimal level of effort of the PTO and the optimal amount of prior art that the innovator will decide to reveal. This is detailed is section 5.

4 Optimal Report of Prior Art

In this section we characterize the optimal report of the innovator once he has discovered prior art information accompanied of a signal. We derive the Subgame Perfect equilibrium.

4.1 Is it optimal to Report Truthfully?

Imagine first that both types report truthfully. We can thus update the beliefs of the PTO,

\[
\begin{align*}
\Pr(Good \mid \bar{x} = 1) &= \mu_1 = \frac{p\gamma}{p\gamma + (1-p)\gamma} = p, \\
\Pr(Good \mid \bar{x} = x_i) &= \mu_i = \frac{p(1-\gamma)}{p(1-\gamma) + (1-p)(1-\gamma)} = p, \\
\Pr(Good \mid \bar{x} = 0) &= p.
\end{align*}
\]

where the posterior beliefs are in fact equal to the prior beliefs.

The PTO chooses the optimal levels of effort \(E_1^t\) and \(E_i^t\) when she observes the intermediate level of prior art \((\bar{x} = x_i)\) or the maximum level of prior art \((\bar{x} = 1)\). If she observes \(\bar{x} = 1\), the first order condition (4) becomes

\[
p(1 - p) \Delta W = \frac{K}{(1 - E)^2},
\]

that gives the optimal level of effort

\[
E_{1t}^* = \frac{p (1 - p) \Delta W - \sqrt{p (1 - p) \Delta W K}}{p (1 - p) \Delta W},
\]

where \(t\) stands for truthful. This effort is increasing (respectively decreasing) with the prior belief \(p\), if \(p\) is smaller (respectively bigger) than 1/2.

If she observes the intermediate level of prior art \(\bar{x} = x_i\), the cost of acquiring information borne by the PTO increases and the incentive constraint becomes

\[
p(1 - p) \Delta W = \frac{K}{x_i (1 - E)^2},
\]
and the optimal level of effort of the PTO is

\[ E_{it}^* = \frac{x_i p (1 - p) \Delta W - \sqrt{px_i (1 - p) \Delta W K}}{x_i p (1 - p) \Delta W}, \] (8)

where, here again, the optimal level of effort is increasing with \( p \) for values of \( p < 1/2 \) and decreasing with \( p \) as \( p > 1/2 \).

These two levels of effort can be compared and thus we can posit the first set of results:

**Result 1** When the innovator reports truthfully, the effort of the PTO depends on the amount of prior art transmitted by the innovator, \( \bar{x} \).

She always exerts a higher effort when she receives more prior art, \( E_{1it}^* > E_{it}^* \) when the uncertainty is the biggest, i.e. for \( p \in [\underline{p}, \bar{p}] \).

Moreover, whatever the amount of prior art provided her effort is maximized when \( p = \frac{1}{2} \).

**Proof.** see appendix

The intuition of this first set of results is the following. The bigger the uncertainty \( (p = \frac{1}{2}) \) the more likely the PTO will try to get more information whenever she observes the intermediate or the full amount of prior art. When the uncertainty is very small, for values of \( p \) very close to 0 or to 1, the PTO believes that the innovation is either bad or good, and it is thus too costly for her to search for more information.

We now describe the best revealing strategy of the innovator. The innovator reports truthfully or not depending on the PTO’s decision of prior art search.

**Result 2** There exists no equilibrium in pure strategies in which an innovator who obtains the full amount of prior art and receives a negative signal reports truthfully when the uncertainty is the biggest (i.e. for \( p \in [\underline{p}, \bar{p}] \)).

The proof is given in the text. An innovator who receives the full amount of prior art with a positive signal about the patentability of his innovation has no interest in transmitting a lower amount of prior art. Indeed, knowing that his innovation is good, he is better off fostering the chances that the PTO will discover the exact type of his innovation. Formally, the inequality (5) is always satisfied as \( \nabla > v \). By the same reasoning, an innovator who finds the
full amount of prior art on his innovation but obtains a negative signal would never transmit his prior art completely if he believes that the PTO believes he is honest. This can be seen by comparing the benefit of truthfully reporting \((1 - E^*_tt) pV\) and the benefit of non-truthfully reporting \((1 - E^*_tt) pV\). It is smaller since \(E^*_tt > E^*_tt\) for \(p \in [p_i, 1]\), and for \(p \in ]p_1, p_i]\cup ]p_i, 1[\) as \(E^*_tt = 0\) in this latter case. Thus equation (6) is never satisfied for \(p \in ]p_1, p_i[\). This completes the proof of result 2.

However for values of \(p\) very small or on the contrary close to 1, i.e. for \(p \in [0, p_i]\cup [p_i, 1]\), as the PTO does not make any effort whatever the information received (\(E^*_tt = E^*_tt = 0\)), the innovator is indifferent between revealing or not. If we assume that the innovator always reveals the truth when he is indifferent, we have an equilibrium in which the innovator always reveals the truth, and the PTO makes no complementary effort. This is only true when there is almost no uncertainty.

**4.2 Is It Optimal to Always Lie?**

We have shown that when the uncertainty is the biggest, there exits no equilibrium in pure strategies in which the innovator who receives a negative signal reports truthfully when he gets the full amount of prior art. Another pure strategy for this innovator would be to always report the intermediate amount of prior art. To see whether this can be an equilibrium, we need to compute the updated beliefs of the PTO when she receives the patent application.

Consider first that the PTO believes that the innovator who gets a negative signal always retains some information. In this setting the updated beliefs of the PTO become

\[
\Pr(Good \mid \bar{x} = 1) = \frac{p \gamma}{p \gamma} = \mu_1 = 1,
\]

\[
\Pr(Good \mid \bar{x} = x_i) = p \frac{1 - \gamma}{1 - \gamma p} = \mu_i < p,
\]

\[
\Pr(Good \mid \bar{x} = 0) = p.
\]

If the PTO observes the maximum amount of prior art, she prefers not to make any effort to search for more prior art as there is no uncertainty anymore concerning who reveals the full amount of prior art. If she observes the intermediate level \(x_i\), the first order condition (4) becomes

\[
\mu_i (1 - \mu_i) \Delta W = \frac{K}{x_i (1 - E)^2}.
\]
that gives
\[
E_i^* = \frac{\mu_i (1 - \mu_i) x_i \Delta W - \sqrt{\mu_i (1 - \mu_i) x_i \Delta W}}{\mu_i (1 - \mu_i) x_i \Delta W} < 1.
\]
The PTO makes a positive effort as long as \((p, \gamma) \in \Psi_l = \{(p, \gamma)/p \in \overline{a(\gamma), \overline{p}}\}\). Thus, if \((p, \gamma) \in \Psi_l\), then \(E_i^* > E_1^* = 0\). Notice that in this case, not only the uncertainty must be high enough, but the probability of the innovation being in a rich prior field must not be too high. A maximum is reached for \(p = 1/(2 - \gamma) > 1/2\) that solves \(dE_i^*/dp = 0\). Otherwise, if \((p, \gamma) \in \{(p, \gamma)/p \in [0, \overline{p(\gamma)}] \cup \overline{p(\gamma), 1}\}\), \(E_i^* = E_1^* = 0\).

What is now the best revealing strategy of the innovator? An innovator who receives a positive signal about his innovation has no incentive to deviate from revealing the truth as the PTO infers that only good types reveal the truth. Formally, the inequality (5) is always satisfied as \(\nabla > v\). Nevertheless, an innovator who receives a negative signal is expected to always lie and does not report his full prior art information. Thus we should have \((1 - E_i)^\mu_i > (1 - E_1)^\mu_1\). This inequality is equivalent to \((1 - E_i^*)^\mu_i > 1\) that is never satisfied. Thus, the innovator may decide to report truthfully in order to fool the PTO that believes that only good types report the high level of prior art. The innovator who gets a negative signal can get a patent per chance, when the PTO is not sure about the signal she receives. Thus the innovator will only get an expected profit. If now he decides to report all the prior art he found, he will get a patent for sure, as the PTO believes that he is a good type that reports truthfully. This is not an equilibrium.

We consider now that the PTO believes that the innovator who receives a good signal always lies, and that the innovator who receives a bad signal reports truthfully. Using the same argument, we can show that the PTO will not make any effort when she observes the high level of prior art, but she will not give a patent either. When she observes the intermediate level of effort, she will make a positive effort. A good type will not deviate whereas a bad type will deviate as he can get a patent from fooling the PTO. This is not an equilibrium.

The PTO can believe that none of the types will report truthfully. By the same token we show that this cannot be an equilibrium.

**Result 3** There exists no equilibrium in pure strategies when the ex ante uncertainty is the biggest and when the probability of the innovation being in a rich prior art field is not too high.

\[15\] The formula are given in appendix.
4.3 Is It Optimal to Not Always Report Truthfully?

When the uncertainty about the value of the innovation is at its biggest level, innovators have no incentive to report truthfully, as they have no incentive to always lie. Thus it can be an equilibrium to report sometimes truthfully.

We consider a Perfect Bayesian Equilibrium in which only the innovator who receives a negative signal with the maximum amount of prior art decides to randomize his report decision. With probability $\theta$ he reports truthfully and with the complementary probability he reports the intermediate amount of prior art. An innovator who gets the full amount of prior art with a positive signal reports truthfully.

When the PTO observes the maximum amount of prior art, it can come from an innovator who has received a positive or a negative signal. When she receives an intermediate level of prior art, it cannot come from an innovator who has received a positive signal with the full prior art. But it can come from other types of innovators: an innovator who has received an intermediate level of prior art and a positive or negative signal, or an innovator who has received the maximum amount of prior art and a negative signal, and does not report truthfully.

In this case the updated beliefs of the PTO become

$$
\mu_1(\theta) = \Pr(\text{Good}/\bar{x} = 1) = \frac{p \gamma}{p \gamma + \theta (1-p) \gamma} = \frac{p}{p + \theta (1-p)},
$$

$$
\mu_i(\theta) = \Pr(\text{Good}/\bar{x} = x_i) = \frac{p(1-\gamma)}{1-\gamma + (1-\theta) (1-p) \gamma},
$$

$$
\Pr(\text{Good}/\bar{x} = 0) = p.
$$

An innovator who receives a negative signal and the full amount of prior art must be indifferent between revealing the full amount and getting $\mu_1(1-E_1)\bar{V}$ and revealing the intermediate level of prior art and getting $\mu_i(1-E_i)\bar{V}$. Thus, there exists a value of $\theta$ that must satisfy

$$
\mu_1(\theta)(1-E_1) = \mu_i(\theta)(1-E_i).
$$

We can posit the following result.

**Result 4** Whenever the equilibrium levels of effort of the PTO are such that $E_1 > E_i$, there exists a unique probability $\theta^*$ that satisfies (9):

$$
\theta^* = \frac{(1-E_1)(1-p \gamma) - (1-\gamma)p(1-E_i)}{(1-p)[(1-\gamma)(1-E_i) + \gamma(1-E_1)]}
$$
Moreover, $\theta^*$ has the following properties,

- $\theta^*$ decreases with $p$, and increases with $\gamma$.
- $\theta^*$ is decreasing with $E_1$ and increasing with $E_i$.

**Proof.** It consists in the computation of the derivatives. It is straightforward to compute that 
\[
\frac{\partial \theta^*}{\partial p} < 0, \quad \frac{\partial \theta^*}{\partial \gamma} > 0, \quad \frac{\partial \theta^*}{\partial E_1} < 0, \quad \text{and} \quad \frac{\partial \theta^*}{\partial E_i} > 0. 
\]

When the prior beliefs about the value of the innovation are not too small nor too big, the level of effort of the PTO as she observes the full amount of prior art is decreasing with the optimal probability $\theta^*$. The intuition is the following. As the PTO increases her effort to search for more information when she is checking a patent application with the full amount of prior art, the screening process will be more accurate. Indeed, the PTO will grant less patents to bad innovations. Thus, an innovator who has received a negative signal and the full amount of prior art will prefer not to report truthfully, and thus the probability of reporting truthfully decreases.

In the same vein, the effort of the PTO when she observes the intermediate level of prior art is increasing with $\theta^*$, or in other words decreasing with $(1 - \theta^*)$. As the PTO monitors more carefully applications that contain the intermediate level of prior art, the innovator who finds all the prior art and a negative signal will prefer to report truthfully.

We now have to define the optimal levels of effort of the PTO, and verify that we have an equilibrium.

Before turning to the description of the objective functions of the PTO, it is worth describing her equilibrium belief about the quality of the innovation. We use the probability that the innovator announces the full amount of prior art $\theta^*$, to define the updated beliefs of the PTO. By replacing this value $\theta^*$ in the updated probabilities $\mu$ gives the equilibrium beliefs. We obtain
\[
\mu^*_1 = \frac{p[(1 - \gamma)(1 - E_i) + \gamma(1 - E_1)]}{1 - E_1} > p, \quad (10)
\]
when the full amount of prior art has been revealed and
\[
\mu^*_i = \frac{p[(1 - \gamma)(1 - E_i) + \gamma(1 - E_1)]}{1 - E_i} < p, \quad (11)
\]
when only the intermediate amount of prior art has been reported by the innovator.
We have not yet computed the equilibrium efforts of the PTO. These efforts take into account the fact that the innovator does not behave truthfully. From the last section, we know that the first order condition of the PTO (see appendix for the second order condition), when she receives $\tilde{x} = 1$, can be written as:

$$\frac{\partial B(E_1; \mu_1^*)}{\partial E_1} = \frac{K}{(1 - E_1)^2}$$

(12)

Whenever she receives $\tilde{x} = x_i$, it is

$$\frac{\partial B(E_i; \mu_i^*)}{\partial E_i} = \frac{K}{x_i(1 - E_i)^2}$$

(13)

In this setting, we first notice that the aggregate effort levels of the PTO when she receives the full or the intermediate levels of prior art are not independent anymore in the case of randomized reports. Indeed, if the innovator anticipates that the PTO will increase her level of effort if he reports the full amount of prior art, then he prefers to “retain” more information and make incomplete application. In equilibrium this should be anticipated by the PTO who would then raise her level of effort $E_i$.

In order to find the effort $E_1$ of the PTO, the incentive condition (12) must be solved with respect to the effort level, using the equilibrium prior $\mu_1^*$ and the randomization parameter $\theta^*$. We obtain a level of effort $E_1$ that depends on $E_i$

$$E_1^* (E_i) = 1 - \sqrt{\frac{K - p (1 - E_i) (1 - \gamma) (p (1 - E_i) \Delta W (1 - \gamma) + W_G - W_R)}{\sqrt{\gamma} p (1 - p \gamma) \Delta W}}.$$  

(14)

It is interesting to first notice that $E_1$ is decreasing with $E_i$. It means that the PTO should be consistent and that an increase of the effort when there is little prior art should trigger an increase in $E_1$. This increase in $E_1$ is necessary to make up for the increased number of strategic (and bad) applications made in the category with abundant prior art.

In the same vain, we can compute the aggregate effort $E_i$ as a function of $E_1$, using (13), $\mu_1^*$, and the randomization parameter $\theta^*$. We obtain

$$E_i^* (E_1) = 1 - \sqrt{\frac{K - (1 - E_1) px_i \gamma (p \gamma (1 - E_1) \Delta W + W_G - W_R)}{\sqrt{px_i (1 - \gamma) (p \gamma + 1 - p) \Delta W}}}.$$  

(15)

and the derivative of $E_i^* (.)$ with respect to $E_1$ is also negative.

Using the reaction functions (14) and (15) it is not possible to obtain a general explicit expression for the equilibrium levels $E_i^*$ and $E_1^*$ of the PTO. However, we can state the following result:
**Result 5** *(No commitment)* There exists a semi-separating equilibrium in which the innovator who receives a negative signal with the maximum amount of prior art randomizes his revelation.

Furthermore, the effort of the PTO when receiving the full amount of prior art is bigger than when she just receives the intermediate level, $E_1^* > E_i^*$ when the uncertainty is the biggest and for intermediate values of $\gamma$ (i.e., for a constellation of parameters such that $(p, \gamma) \in \Upsilon \cap \Psi$).

**Proof.** see appendix

Thus, there exists an equilibrium in which an innovator who receives the full amount of prior art and a negative signal randomizes his decision to report the full amount of prior art. In this case, the PTO intensifies her complementary search of information when she receives more prior art, when the uncertainty is the biggest, and for a relatively high probability that the innovation belongs to a rich prior art field.

It is interesting to study how the various parameters of the model will impact the aggregate effort levels $E_1^*$ and $E_i^*$ performed by the PTO. The next result summarize this study.

**Result 6** *(No commitment)* The levels of effort of the PTO exhibit the following comparative statics:

- $E_1^*$ is increasing with $\Delta W$, whereas an increase of $\Delta W$ has an ambiguous effect on $E_i^*$.
- $E_i^*$ is increasing with $p$, whereas an increase in $p$ has an ambiguous effect on $E_i^*$.
- $E_i^*$ is decreasing with $K$, whereas an increase in $K$ has an ambiguous effect on $E_1^*$.
- $E_1^*$ (respectively $E_i^*$) is decreasing (respectively increasing) with $x_i$.
- An increase in $\gamma$ has an ambiguous effect on the levels of effort chosen by the PTO.

**Proof.** see appendix

We now define the optimal *ex post* and *ex ante* benefits of the PTO. The *ex post* benefit depends on the prior art information received by the PTO. When she receives $\tilde{x} = 1$ (respectively...
\( \bar{x} = x_i \), her gross benefit is \( B(E^*_1; \mu^*_1) \) (respectively \( B(E^*_i; \mu^*_i) \)) as defined by equation (3) with the appropriate \( E^*_1, E^*_i, \mu_1(\theta^*) \) and \( \mu_i(\theta^*) \). The net benefits are obtained by subtracting the cost \( C_1(E^*_1) \) (respectively \( C_i(E^*_i) \)) to the gross benefits.

The ex ante benefit of the PTO corresponds to the benefit she expects to get at the very beginning of the game, after anticipating correctly what will be the issue of the game. Thus,

\[
B_{\text{ex ante}} = a^* \left[ \gamma(p + (1 - p)\theta^*) (B(E^*_1; \mu^*_1) - \frac{K}{1 - E^*_1}) + (\gamma(1 - p) - (1 - \gamma)) (B(E^*_i; \mu^*_i) - \frac{K}{x_i(1 - E^*_i)}) \right],
\]

where \( a^* \) is the probability that the innovator discovers some prior art information. After some algebraic manipulation, the ex ante benefit of the PTO becomes

\[
B_{\text{ex ante}} = a^* \left[ (1 - E^*_1) \gamma p (1 - \gamma) - (1 - E^*_i)(1 - \gamma)(1 - p(1 - \gamma)) + \gamma(1 - E^*_1) \Delta W p + p \gamma(1 - E^*_1)(1 - E^*_i)(1 - \gamma)(1 - \gamma) \right] x_i \gamma K + (1 - \gamma) K x_i (1 - \gamma)(1 - E^*_i) + (1 - a)^2 k (17)
\]

5 Search of Prior Art for the Innovator

The very first decision of the innovator is to choose whether or not to search for more information in order to find the relevant prior art and also to be aware of the value of his innovation. The benefit of the innovator is

\[
\Pi(a) = a [\gamma p(E_1^* V + (1 - E_1)(\mu_1 V + (1 - \mu_1) v)) + \gamma(1 - p) \theta(1 - E_1) \mu_1 V + \gamma(1 - p)(1 - \theta)(1 - E_1) \mu_1 V + (1 - \gamma)(1 - p)(1 - E_1) \mu_i V + (1 - \gamma)p(E_i V + (1 - E_i)(\mu_i V + (1 - \mu_i) v)) + (1 - a) p v],
\]

The maximization program of the innovator is thus

\[
\text{Max}_a^i \Pi(a),
\]

that gives

\[
a^* = \frac{1}{k} [p \Delta V [(1 - p)(\gamma E^*_1 + (1 - \gamma) E^*_1) + p] + V] \]

where \( \Delta V = V - v - \underline{V} \).

It is interesting to notice that the level of effort of the innovator is always decreasing when the value of a non-patented innovation, \( v \), increases. Indeed we have
\[
\frac{\partial a^*}{\partial v} = -\frac{1}{2k}[p(1 - p)(\gamma E_1 + (1 - \gamma)E_i) + p] < 0.
\]

This is not surprising since an increase in \(v\) raises the cost of doing research and patent the innovation relative to directly sell the innovation.

It is an increasing function of the value of a good patented innovations

\[
\frac{\partial a^*}{\partial V} = \frac{1}{2k}[p(1 - p)(\gamma E_1 + (1 - \gamma)E_i) + p] > 0,
\]

but we cannot conclude for a bad patented innovation

\[
\frac{\partial a^*}{\partial V} = \frac{1}{k}[p(1 - p)(\gamma E_1 + (1 - \gamma)E_i) + p^2 - 1].
\]

The innovator is “sensible” to the efforts made by the PTO as

\[\frac{\partial a^*}{\partial E^*_1} > 0 \text{ and } \frac{\partial a^*}{\partial E^*_i} > 0.\]

The innovator’s effort varies with the amount of intermediate prior art that he can get, \(x_i\), as well as with the probability of having a good innovation, \(p\), and the probability of having an innovation that belongs to a rich prior art field, \(\gamma\). Using result 6 and for \(y = x_i, p, \gamma\), we study how the optimal level of effort of the innovator varies with these parameters,

\[
\frac{da^*}{dy} = \frac{\partial a^*}{\partial y} + \frac{\partial a^*}{\partial E^*_1} \frac{\partial E^*_1}{\partial y} + \frac{\partial a^*}{\partial E^*_i} \frac{\partial E^*_i}{\partial y}.
\]

According to result 6, an increase in \(x_i\) induces the PTO to intensify her research effort \(E^*_1\), whereas she devotes less effort to search for complementary prior art when she has received the full amount of prior art, \(\partial E^*_i/\partial x_i < 0\). Thus, there are two effects that work in opposite direction. On one hand if the intermediate level of prior art that the innovator can find increases, due to the increase in the PTO’s effort if the intermediate amount is reported, the innovator intensifies his effort. But, on the other hand, due to the decrease in the PTO’s effort if the report is the full amount of prior art, the innovator reduces his effort. It is not clear what effect is bigger than the other, and we cannot conclude.

**Result 7** The effort of the innovator is

- increasing (decreasing) with the value of the patented (non-patented) innovation,
- increasing with the optimal levels of effort of the PTO.
6 Ex ante commitment of the PTO

In this section, our investigation is twofold. First, we investigate whether an ex ante commitment policy to research effort is preferable to a no-commitment policy (the actual policy). Second, we examine whether the ex ante commitment policy induces more truthful revelation from bad innovators, or whether it should allow innovators to retain some information when they discover that their innovation does not deserve a patent. The answer to that question is not obvious and depends on a trade-off. On one hand, a policy that encourages truthful behavior will limit the power of incentives that can be given to foster research effort. On the other hand, a policy that gives incentives to make high level of effort will probably make truthful revelation impossible.

The PTO can choose to ex ante commit to exert levels of effort. We denote by $E_{1c}$ and $E_{ic}$ the levels of effort that the PTO exerts when the prior art provided is, respectively, the full amount and the intermediate amount. Two cases must be considered, whether $E_{ic} \geq E_{1c}$ or $E_{ic} < E_{1c}$.

Let first consider the case where $E_{ic} < E_{1c}$. While finding the full amount of prior art, innovators learn the type of their innovation. Therefore only those who actually learn that their innovation is good will report the full amount of prior art. The other fraction of innovators who learn that their innovation is bad will actually decide to transmit only the intermediate amount of prior art. The next result is the first step to find the optimal commitment policy of the PTO.

Result 8 When the PTO commits ex ante to given levels of scrutiny $(E_{ic}, E_{1c})$, with $E_{ic} < E_{1c}$, it is always possible to find a policy that strictly dominates this policy.

Proof. Let $A = (E_{1c}^A, E_{ic}^A)$ be a policy such that $E_{ic}^A < E_{1c}^A$. In such a policy, the equilibrium beliefs of the PTO are

\[
\begin{align*}
\mu_{1c}^A &= 1 \\
\mu_{ic}^A &= p \frac{1 - \gamma}{1 - \gamma p} = \mu_i < p
\end{align*}
\]

Now, consider a policy $A'$ that announces $(E_{ic}^{A'}, E_{1c}^{A'})$ and is such that $E_{ic}^{A'} = E_{ic}^A$ and $E_{1c}^{A'} = E_{1c}^A - \varepsilon$ with $\varepsilon > 0$ but such that $E_{ic}^{A'} > E_{ic}^{A'}$. In this new policy, upon receiving $\tilde{x} = 1$, the PTO will check with accuracy $E_{1c}^{A'} > E_{ic}^{A'}$. Thus, innovators with bad signal will never choose to report $\tilde{x} = 1$ even if they can. Thus,

\[
\mu_{1c}^{A'} = 1.
\]
The innovators who receive a bad signal will report $\tilde{x} = x_i$ so that

$$\mu_i^{A'} = p \frac{1 - \gamma}{1 - \gamma p} < p.$$ 

The PTO exerts effort $E_{ic}^{A'}$, pays a cost $C(E_{ic}^{A'}) = 1/(1 - E_{1c}^{A'})$ and follows her judgement. It is obvious that the policy $A'$ dominates the policy $A$ since they have the same screening efficiency but $A$ is more costly than $A'$ as $C(E_{ic}^{A'}) < C(E_{1c}^{A})$ even though $C(E_{ic}^{A}) = C(E_{ic}^{A})$. ■

A policy that involves $E_{ic} < E_{1c}$ has two major drawbacks. First, as it has been demonstrated in result 8, it is cost dominated by another policy with a lower gap between $E_{1c}$ and $E_{ic}$. We believe that this drawback is not so acute. A second drawback is that the commitment problem of the PTO is very intense when $E_{ic} < E_{1c}$ because the gains of reneging on her commitment are very high. Indeed, in this context, the PTO would gain $C(E_{1c}^{A})$ monetary units by reneging on her commitment without altering the quality of screening.

The commitment policies involving $E_{ic} \geq E_{1c}$ induce truth telling and do possess such a commitment problem. However, this one has a much lower intensity. Indeed, in a truth telling equilibrium, it is certain that $(E_{ic}, E_{1c})$ are ex post inefficient. However, the gains from reneging on her commitment (i.e., switching to $E_{it}^{*}$ and $E_{it}^{*}$ defined in (7) and (8)) are expected to be much lower. Relative to the case $E_{ic} < E_{1c}$, the PTO faces a low intensity commitment problem when $E_{ic} \geq E_{1c}$. We now proceed to the analysis of this case.

When $E_{1c} = E_{ic}$ the innovator who learns the nature of his innovation is indifferent between reporting a high level of prior art (truthful reporting) and reporting only part of the prior art found (strategic reporting). For definitiveness, we make the assumption that when the innovator is indifferent, he reports truthfully. Given that the innovator is induced to be truthful, and if the innovator reports some prior art information, the gross benefit function of the PTO is

$$B_c(E_{1c}, E_{ic}) = \gamma B(E_{1c}; p) + (1 - \gamma) B(E_{ic}; p)$$

where $c$ stands for commitment, and $B_j(E_{jc}; p)$ for $j = 1, i$ are defined in equation (3).

The ex ante commitment contract of the PTO that induces information search and truthful revelation by the innovator is solution of the following program:

$$\max_{E_{1c}, E_{ic}} a(E_{1c}, E_{ic}) [B_c(E_{1c}, E_{ic}) - \gamma \frac{K}{(1 - E_{1c})} - (1 - \gamma) \frac{K}{x_i (1 - E_{ic})}]$$

subject to

$$E_{1c} \leq E_{ic}$$

(18)
The next result simplifies the analysis of the program of the PTO when she wants to induce truthful reports.

**Result 9 (ex ante commitment)** If the PTO wants to induce truthful reports, the unique optimal commitment policy is such that $E_{1c} = E_{ic}$.

We know that if they exist, the *ex post* efficient levels of effort are such that $E^*_1 > E_i^*$. By committing to levels of effort *ex ante*, the PTO would like to reduce *ex post* inefficiencies as much as possible. More precisely, the PTO would like to raise $E_{1c}$ as much as possible. Therefore, among all the possible levels of effort that verify $E_{1c} \leq E_{ic}$, the ones that minimize *ex post* inefficiency are such that $E_{1c} = E_{ic} = E_c$.

Using this result, the program of the PTO can now be stated as

$$\max_{E_c} a(E_c) [B_c(E_c) - \frac{\gamma K}{1 - E_c} - \frac{(1 - \gamma) K}{x_i (1 - E_c)}]$$

where

$$a(E_c) = \frac{1}{k} [p\Delta V[(1 - p)E_c + p] + V]$$

and $B(E_{1c}; p) = B(E_{ic}; p) = B(E; p)$, thus

$$B_c(E_c) = -p(1 - p)\Delta W(1 - E_c) + p(W_G - W_R) + W_R$$

We cannot obtain an analytical solution due to the complexity of the *ex ante* program. Let $B_{\text{commit}}(E_c)$ denote the benefit of the PTO when she commits *ex ante* to a level of effort $E_c$

$$B_{\text{commit}}(E_c) = a(E_c) [B_c(E_c) - \frac{\gamma K}{1 - E_c} - \frac{(1 - \gamma) K}{x_i (1 - E_c)}]. \quad (19)$$

The levels of effort of the PTO when they are performed *ex post* generate too few incentives on part of the innovators. By committing to effort *ex ante* the PTO can threaten to lower the innovators’ benefit. The commitment thus helps the PTO to shift, to some extent, the burden of the information search to innovators.

Moreover, the commitment under study induces truthful revelation and does not involve a waste of information. It is important to notice that we have interpreted our model in term of “too few” relevant information transmitted. However, by making an intensive search, the innovator finds an important amount of related (but possibly not relevant) information. Alternatively, we could assume that an innovator has the option to transmit “too much” related but not relevant information, when he discovers that his innovation is not good. If after examination, the PTO is able to judge the relevance of the information transmitted (and we believe she could), then it is also possible to commit to a mechanism that will allow truthful information transmission.
7 Non Commitment versus Commitment

Our ultimate goal in this study is to investigate whether, and under what kind of circumstances, a policy in which the PTO commits ex ante to certain levels of effort is more desirable than a policy in which she does not commit ex ante. In order to do so, we need to compare the benefits of the PTO in the commitment case (19) and non commitment case (16). Due to the complexity of these equations, we cannot compare them for all the values of \( \gamma \) and \( p \). We thus consider two extreme cases: (i) when \( E_1^* \) tends toward \( E_i^* \) and (ii) when \( E_i^* \) tends toward 0.

(i) When \( E_1^* \) tends toward \( E_i^* \), the equilibrium values of the no commitment case converges toward a unique value. Recall that we only have an equilibrium for \( E_1^* > E_i^* \), which is only true for a certain constellation of parameters i.e., for \( p > p(\gamma) \) as implicitly defined in the appendix. Thus, when \( E_1^* \) tends toward \( E_i^* \), the benefit of the PTO in the no commitment case converges towards the benefit of the PTO in the commitment case. Furthermore, as \( E_c \) is the unique solution that maximizes the commitment benefit (19), whatever the values of \( E_1^* \) and \( E_i^* \), the commitment benefit is higher than the no commitment benefit.

(ii) Let consider now the second case, when in fact the parameter values \((p, \gamma)\) are such that \( E_i^* \) tends toward 0, i.e. \( p > R'(\gamma) \). Let see one extreme of this function when \( p \) tends toward 1. In this case, we can check that the commitment benefit is higher than the non commitment benefit.

We summarize those results:

**Result 10** When there is few prior art information available (\( \gamma \) relatively small for a given \( p \)), and when there is more prior art information (\( \gamma \) relatively high for a given \( p \)), the PTO should commit ex ante to a certain level of scrutiny.

The ex ante commitment allows the PTO to force the innovator to reveal all the information he finds. However, she cannot induce him to make the right level of effort to search for it. On the other hand, when it is possible to find enough prior art information, a no commitment from the part of the PTO induces the innovator to intensify his search of prior art, but bad innovators are tempted not to reveal all the information they find.
References


Appendix

Proof of Result 1

We first determine for which values of the prior beliefs the PTO decides to make an effort to search for complementary information depending on the prior art information received from the innovator. Second, we compare the different levels of effort.

First, we study the function \( E^*_\text{it}(p) \), for \( \overline{x} = \{x_i, 1\} \). We can rewrite the optimal efforts

\[
E^*_\text{it}(p) = 1 - \frac{\sqrt{K}}{\sqrt{p(1-p)}\sqrt{\overline{x}}\Delta W}
\]

that is first increasing and then decreasing with \( p \). As the second derivative is negative, we have a concave function that reaches a maximum for \( p = 1/2 \) (the last part of result 1 is thus proved). Thus, there exist values of \( p \) such that \( E^*_\text{it}(p) = 0 \). Furthermore those values belong to the interval \([0, 1]\) as the limits of \( E^*_\text{it}(p) \) when \( p \) tends toward 0 or 1 is \(-\infty\). Those values are thus

\[
\begin{align*}
p_{\overline{x}} &= \frac{\overline{x}\Delta W - [(\overline{x}\Delta W)^2 - 4\overline{x}K\Delta W]^{\frac{1}{2}}}{2\overline{x}\Delta W} \\
p_{\overline{x}} &= \frac{\overline{x}\Delta W + [(\overline{x}\Delta W)^2 - 4\overline{x}K\Delta W]^{\frac{1}{2}}}{2\overline{x}\Delta W}
\end{align*}
\]

if \( K < \Delta W/4 \). It is easy to check that \( p_{\overline{x}} < p_i < \frac{1}{2} < p_i < p_{1} \).

Thus, for values of \( p \in [0, p_i], \overline{x} \in [\overline{x}_1, 1] \), the PTO does not make any effort. For values of \( p \in [p_{\overline{x}}, p_i] \cup [p_1, \overline{x}] \), the PTO does not make any effort after observing the intermediate amount of prior art, but makes the optimal level of effort \( E^*_\text{it}(p) > 0 \) if the full amount has been revealed. And finally, for values of \( p \in [p_{\overline{x}}, p_1] \), the PTO makes the appropriate levels of effort.

Thus, as for values of \( p \in [p_{\overline{x}}, p_1] \), the PTO makes the levels of effort \( E^*_\text{it} \) and \( E^*_\text{it} \), we now compare those levels. Let rewrite the difference between the two equilibrium efforts

\[
E_{1t} - E^*_\text{it} = \frac{p(1-p)\Delta W - \sqrt{p(1-p)\Delta WK}}{p(1-p)\Delta W} - \frac{xp(1-p)\Delta W - \sqrt{px(1-p)\Delta WK}}{xp(1-p)\Delta W}
\]

which can be reduced to

\[
\frac{\sqrt{xp(1-p)WK} - x\sqrt{p(1-p)WK}}{xp(1-p)W}
\]

The numerator of this expression is positive if \( \sqrt{xp(1-p)\Delta W} > x\sqrt{p(1-p)\Delta W} \). Since both
terms in the previous inequality are positive, we can square both side and make the difference, we obtain \((1 - x) p (1 - p) x \Delta W\) that is strictly positive, and thus we have proven that \(E_{it}^* > E_{it}\) when the uncertainty is the biggest. ■

**Is it optimal to always lie?**

It is easy to check that the PTO makes a positive effort

\[
E_{i}^* = \frac{\mu_{i} (1 - \mu_{i}) x_{i} \Delta W - \sqrt{\mu_{i} (1 - \mu_{i}) x_{i} \Delta W}}{\mu_{i} (1 - \mu_{i}) x_{i} \Delta W}
\]

The PTO makes a positive effort as long as \(p \in \left[\gamma, \bar{p}(\gamma)\right]\) where

\[
\gamma = \frac{(1 - \gamma) x_{i} \Delta W + 2 K \gamma - \left[\left((1 - \gamma) x_{i} \Delta W + 2 K \gamma\right)^2 - 4 K ((1 - \gamma) x_{i} \Delta W + \gamma^2 K)\right]^{\frac{1}{2}}}{2 (1 - \gamma) x_{i} \Delta W + \gamma^2 K},
\]

\[
\bar{p}(\gamma) = \frac{(1 - \gamma) x_{i} \Delta W + 2 K \gamma + \left[\left((1 - \gamma) x_{i} \Delta W + 2 K \gamma\right)^2 - 4 K ((1 - \gamma) x_{i} \Delta W + \gamma^2 K)\right]^{\frac{1}{2}}}{2 (1 - \gamma) x_{i} \Delta W + \gamma^2 K}.
\]

We show that \(\gamma\) and \(\bar{p}(\gamma)\) are increasing with \(\gamma\).

**Concavity of the benefit functions of the PTO**

In the case of mixed strategies, we derive that the benefit functions are concave in their own argument (namely \(E_{1}\) and \(E_{i}\)). If the PTO receives the full amount of prior art \(x = 1\), or the intermediate \(x = x_{i}\), her payoff are respectively \(B(E_{1}; \mu_{1}) - \frac{K}{(1 - E_{1})}\) and \(B(E_{i}; \mu_{i}) - \frac{K}{x_{i} (1 - E_{i})}\), where the gross benefit functions \(B(.)\) are defined by (10) and (11). After rewriting these benefit functions, we calculate the first derivative, and the second derivative. When the PTO receives the full amount of prior art (we find a similar equation for the intermediate level), the second order condition is

\[
SOC: \ K - p^2 (1 - E_{1})^2 (1 - \gamma)^2 \Delta W \gamma (1 - E_{i}) (1 - \gamma) (W_{G} - W_{R}) < 0
\]

that is always negative, at least for the equilibrium values. We thus have shown that the benefit functions are concave. This is the first step of the proof of the existence of solutions.

**Subgame Perfect Bayesian Equilibrium (Proof of Result 5)**

There exists a semi-separating equilibrium in which the innovator that finds the full amount of prior art and a negative signal decides to randomize his revelation decision.

Equations (9), (14) and (15) must be satisfied.
From the first equation, \( \mu_1(\theta)(1 - E_1) = \mu_i(\theta)(1 - E_i) \), we derive an unique \( \theta^* \in [0, 1] \),

\[
\theta^* = \frac{(1 - E_1)(1 - p\gamma) - (1 - \gamma)p(1 - E_i)}{(1 - p)[(1 - \gamma)(1 - E_i) + \gamma(1 - E_1)]}
\]

\( \theta^* < 1 \) is always satisfied if \( E_1^* > E_i^* \) and \( \theta^* > 0 \) if \( p < \overline{p}(\gamma) = \frac{1 - E_1}{(1 - \gamma)(1 - E_i) + \gamma(1 - E_1)} \). This condition is in fact equivalent to checking that \( \mu_1(\theta^*) < 1 \). Assume for the moment that \( E_1^* > E_i^* \) and that \( p < \overline{p} \) (we will determine under what circumstances this condition holds in the last part of the proof).

We need to check that

i. there exists a solution \((E_1^*, E_i^*)\), where \( E_1^* \in [0, 1] \) and \( E_i^* \in [0, 1] \).

ii. that this solution is unique on the interval considered.

Before doing so, recall that the two functions \( E_1(E_i) \) and \( E_i(E_1) \) are decreasing. Let rewrite these functions as follows:

\[
E_1(E_i) = 1 - \phi_1(E_i)^{\frac{1}{2}} \\
E_i(E_1) = 1 - \phi_i(E_1)^{\frac{1}{2}}
\]

By analogy to the Cournot model, we call these functions ‘reaction functions’. Some assumptions are crucial to have solutions, in particular to insure real solutions, we need to impose that \( \phi_1(E_i) \geq 0 \) and \( \phi_i(E_1) \geq 0 \). So the crucial assumptions are the following: \( E_j(1) < 1 \) (or equivalently \( K > 0 \)), \( E_j(0) < 1 \) (to insure that \( \phi_j(0) \geq 0 \), for \( j = i, 1 \). Thus, this insure that the solutions are always smaller than 1. Let see now in detail the study of existence and unicity of positive solutions.

i. **Existence**

   We have already shown that each benefit function of the PTO is concave in its effort. We then have to show that each of the solutions must be positive. Ideally, we need to have \( E_1^{-1}(0) > E_i(0) \) and \( E_i^{-1}(0) > E_1(0) \). However, because of the complexity of our reaction functions, we will not find explicitly the constellation of parameters \((\gamma, p)\) such that these two conditions hold.

   Let first consider a weaker condition. According to our reaction functions, if \( E_1(1) > 0 \) and \( E_i(1) > 0 \), it implies that \( E_1(0) > 0 \) and \( E_i(0) > 0 \) and thus, there exist interior solutions between 0 and 1. We can easily derives a constellation of parameters such that these two conditions are satisfied. However, these conditions are very restrictive, and if one of them, or even both are not satisfied, we can still have positive solutions (we will consider that problem later). It is easy
to check that $E_1(1) > 0$ if $p \in [r'_1(\gamma), r''_1(\gamma)]$ where $r'_1(\gamma) = (\Delta W - \sqrt{(\Delta W)^2 - 4K\Delta W})/2\gamma \Delta W$ and $r''_1(\gamma) = (\Delta W + \sqrt{(\Delta W)^2 - 4K\Delta W})/2\gamma \Delta W$. Similarly, $E_i(1) > 0$ is satisfied if $p \in [r'_i(\gamma), r''_i(\gamma)]$, where $r'_i(\gamma) = (x_i\Delta W - \sqrt{(x_i\Delta W)^2 - 4x_iK\Delta W})/2(1 - \gamma)x_i\Delta W$ and $r''_i(\gamma) = (x_i\Delta W + \sqrt{(x_i\Delta W)^2 - 4x_iK\Delta W})/2(1 - \gamma)x_i\Delta W$. We can represent these functions in a graph $(\gamma, p)$ (figure 1). Thus, for a constellation of parameters $(\gamma, p) \in \{(\gamma, p)/p \geq r'_i(\gamma), p \geq r'_1(\gamma)\}$, a positive pair of efforts exists. However, just on the left of $r'_i(\gamma)$ for instance, we can still have a candidate pair. So in fact, the constellation $(\gamma, p)$ such that $p = r'_i(\gamma)$ corresponds to the $E_i(1) = 0$. For a given $p$, the limit $E_i(1)$ decreases with an increase of $\gamma$ as $\partial E_i(1)/\partial \gamma < 0$ for $p < 1/2(1 - \gamma)$. Thus we can state that there exists a function $R_i(\gamma)$ such that for $p \leq R_i(\gamma)$, the optimal solution $E^*_i \leq 0$. Thus, if we start from point $A$ in the figure, and we increase $\gamma$, first the optimal solution can still be positive, but as the limit decreases, there is one value of $\gamma$ for which the optimal solution becomes negative. By the same token, the same reasoning applies to $E_1(1)$, and thus, there exists a function $R_1(\gamma)$ such that for $p < R_1(\gamma)$, the optimal solution $E^*_1 < 0$.

Thus, we have defined explicit functions, $R_i(\gamma)$ and $R_1(\gamma)$ such that there exists a pair of efforts $(E^*_1, E^*_i) \in [0, 1] \times [0, 1]$, that we represent in figure 1, for a constellation of parameters $(\gamma, p) \in \Upsilon = \{(\gamma, p)/p \geq R_i(\gamma), p \geq R_1(\gamma)\}$ (set that is less restrictive than the previous set).
For some values of $p$, $E_1^* > 0$ is satisfied, whereas $E_i^* > 0$ is not. In other words, the PTO makes an effort $E_1$ after observing the maximum prior art, but no effort at all after observing the intermediate level. Equation (9) becomes $\mu_1(\theta)(1 - E_1) = \mu_i(\theta)$ and we derive

$$\theta_1^* = \frac{1 - p - E_1(1 - \gamma p)}{(1 - p)(1 - E_1 \gamma)}$$

that belongs to $[0, 1]$ if $p < \overline{p}_1 = \frac{1 - E_1}{1 - E_1 \gamma}$. We check that $\overline{p}_1 > r_1''$ and thus $\theta_1^* \in [0, 1]$.

For another constellation of parameters, none of the inequalities $E_1^* > 0$ and $E_i^* > 0$ are satisfied. Thus the PTO will not make any effort and the innovator is indifferent if $\mu_1(\theta) = \mu_i(\theta)$ which is only satisfied for $\theta = 1$. This is actually the equilibrium in pure strategies that we have define earlier.

ii. Uniqueness

To show that the solution is unique, we need to show that $\left| \frac{\partial^2 (B_1(\cdot) - C(\cdot))}{\partial E_1^2} \right| > \left| \frac{\partial^2 (B_1(\cdot) - C(\cdot))}{\partial E_i^2 \partial E_i} \right|$. After computation of these two second derivatives, a simple comparison shows that this is true as long as $E_1^* > E_i^*$.

We thus have shown that there exists a unique pair of solution $(E_1^*, E_i^*) \in [0, 1] \times [0, 1]$ for $(\gamma, p) \in \Upsilon$. 

Figure 2:

For some values of $p$, $E_1^* > 0$ is satisfied, whereas $E_i^* > 0$ is not. In other words, the PTO makes an effort $E_1$ after observing the maximum prior art, but no effort at all after observing the intermediate level. Equation (9) becomes $\mu_1(\theta)(1 - E_1) = \mu_i(\theta)$ and we derive

$$\theta_1^* = \frac{1 - p - E_1(1 - \gamma p)}{(1 - p)(1 - E_1 \gamma)}$$

that belongs to $[0, 1]$ if $p < \overline{p}_1 = \frac{1 - E_1}{1 - E_1 \gamma}$. We check that $\overline{p}_1 > r_1''$ and thus $\theta_1^* \in [0, 1]$.

For another constellation of parameters, none of the inequalities $E_1^* > 0$ and $E_i^* > 0$ are satisfied. Thus the PTO will not make any effort and the innovator is indifferent if $\mu_1(\theta) = \mu_i(\theta)$ which is only satisfied for $\theta = 1$. This is actually the equilibrium in pure strategies that we have define earlier.

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We thus have shown that there exists a unique pair of solution $(E_1^*, E_i^*) \in [0, 1] \times [0, 1]$ for $(\gamma, p) \in \Upsilon$. 

Figure 2:
Thus far, we have assume that \( E_1^* > E_i^* \) We now have to show for which constellation of parameters \((p, \gamma)\) the inequality \( E_1^* > E_i^* \) holds, inside the constellation of parameters \( \Psi \). Let first re-define the functions \( E_1(E_i) \) and \( E_i(E_1) \) as

\[
E_1(E_i) = 1 - \phi_1(E_i, p, \gamma)^{\frac{1}{2}}
\]

\[
E_i(E_1) = 1 - \phi_i(E_1, p, \gamma)^{\frac{1}{2}}
\]

We then determine the value of \( E_1 \) (respectively \( E_i \)) when the reaction function \( E_1(E_i) \) (respectively \( E_i(E_i) \)) cuts the function \( E_i = E_1 \). In other words, we determine \( E \) such that \( E_1(E) = E \), which is \((1 - E)^2 = \phi_1(E, p, \gamma)\), and \( E_i(E) = E \) which is \((1 - E)^2 = \phi_i(E, p, \gamma)\). As \( E_1 = E_i \), we can state that there exists a value of \( E \) such that \( \phi_1(E, p, \gamma) - \phi_i(E, p, \gamma) = 0 \). The solution of this equation is \( E(p, \gamma) \). If we plug this solution back into the previous equation we end up with \( \phi_1(E(p, \gamma), p, \gamma) - \phi_i(E(p, \gamma), p, \gamma) = 0 \) that does only depend on \( p \) and \( \gamma \). Let rename this equality \( F(p, \gamma) = 0 \). If we totally differentiate this last function, we can determine the sign of the function \( p(\gamma) \) that represents all the values of \((p, \gamma) \) \( (p = p(\gamma)) \) such that \( E_1^* = E_i^* \). indeed,

\[
\frac{dp}{d\gamma} = -\frac{\partial F}{\partial \gamma} \frac{\partial F}{\partial p}
\]

Thus \( \frac{dp}{d\gamma} < 0 \) as long as \( \text{sign}(\frac{\partial F}{d\gamma}) = \text{sign}(\frac{\partial F}{dp}). \) [to show]

So we have defined the function \( p(\gamma) \) such that \( E_1^* = E_i^* = E \). Let’s depart from this point.

Assume that \( E_1^* = E + \varepsilon \) where \( \varepsilon \) is small. We thus have \((1 - E)^2 + \varepsilon = \phi_1(E, p, \gamma)\) and \((1 - E)^2 = \phi_i(E, p, \gamma)\), that gives \( \phi_1(E(p, \gamma), p, \gamma) + \varepsilon - \phi_i(E(p, \gamma), p, \gamma) = 0 \). Thus, \( E_1^* > E_i^* \) for \( p > p(\gamma) \).

And lastly we have to check that \( p < \overline{\theta}_0(\gamma) \) in order to get an equilibrium (i.e., \( \theta^* > 0 \)). Thus \( p \) must be bigger than \( p(\gamma) \) as well as smaller than \( \overline{\theta}_0(\gamma) \), so \((\gamma, p) \in \Psi = \{\gamma, p/p > p(\gamma) \) and 
\( p < \overline{\theta}_0(\gamma) \}).\]

We can thus state that there exists a unique pair of solution \((E_1^*, E_i^*) \in [0, 1] \times [0, 1] \) and \( \theta^* \in [0, 1] \) such that \( E_1^* > E_i^* \) for \((\gamma, p) \in \Psi \cap \Psi \).

**Proof of Result 6**

In absence of analytical formulation for the efforts, we need to study how the functions \( E_1^*(E_i) \) and \( E_i^*(E_1) \) are affected by a change in one parameter to determine the impact of the change of this parameter on the equilibrium values. Let us call \( y \) this parameter, that can be \( \Delta W \) (with \( W_G - W_R = \text{constant} \)), \( p \), \( x_i \), \( \gamma \) and \( K \). Let us rewrite \( E_1^*(E_i) = 1 - \sqrt{\phi_1(y)} \) and \( E_i^*(E_1) = 1 - \sqrt{\phi_i(y)} \) where \( \phi_1(y) \) and \( \phi_i(y) \) are defined in equation (14) and (15). Thus, \( \frac{dE_i^*}{dy} = \)
\[-\frac{1}{2}(\phi_j(y)) - \frac{1}{2}\phi_j'(y)\] where \(j = 1, i\). We then show that \(\phi_1'(y) < 0\) for \(y = \Delta W, p\) (for \(p < 1/2\gamma\)), \(\phi_1'(y) = 0\) for \(y = x_i\), and \(\phi_1'(y) > 0\) for \(y = K, \gamma\) (for \(\gamma > 1/2p\)). Thus the function \(E_1^*(E_i)\) is decreasing (respectively increasing) with \(K, \gamma\) (respectively with \(\Delta W, p\)) and is constant with \(x_i\).

By the same token, we find that \(\phi_1'(y) < 0\) (respectively \(> 0\)) for \(y = \Delta W, x_i, p\) (for \(p < 1/2\)), \(\gamma\) (for \(\gamma < (2p - 1)/2p\)) (respectively for \(y = K\)). Thus, the function \(E_1^*(E_1)\) is increasing with \(\Delta W, x_i, p, \gamma\) (respectively decreasing with \(K\)). We thus define how the equilibrium values change after a change in the functions.

**Simple case where \(W_G = W_R\)**

It is possible to compute the equilibrium levels of effort of the PTO contingent upon the amount of prior art transmitted by the innovator in the special case \(W_G = W_R\). After some basic algebra we obtain

\[
E_1^* = 1 - \frac{\sqrt{|x_i(1-p) - p(1-x_i\gamma - \gamma)|}}{\sqrt{p\gamma x_i (1-p) \Delta W}}
\]

and

\[
E_i^* = 1 - \frac{\sqrt{(1-p\gamma)(x_i - px_i + px_i\gamma - p + p\gamma)}}{\sqrt{(1-p) px_i (1-\gamma)(1-p + p\gamma) \Delta W}}
\]

It can be verified that these expressions verify result 5 as long as the efforts belong to \([0, 1]\).
This result underlies the fact that with a no-commitment policy, the PTO will be provided by the innovators with less information than is actually available to them.
The innovator makes an effort e.
He observes x and a signal.
He reveals x to The PTO.
The PTO observes x and decides to make an effort E.
The PTO grants a patent with a certain probability.

Figure 4: Timing of the Game