1 Duopoly game with linear demand

- Duopoly, \( n = 2 \)
- Market demand: \( D(p) = a - p \)
- Marginal cost: \( c \)
- Firm \( i \)’s profit is

\[
\Pi^i(p_i, p_j) = \begin{cases} 
(p_i - c)(a - p_i) & \text{if } p_i < p_j \\
\frac{1}{2}(p_i - c)(a - p_i) & \text{if } p_i = p_j \\
0 & \text{if } p_i > p_j 
\end{cases}
\]

- What are the best response functions?
- Profit of firm \( i \) for different prices \( p_j \)
- Denote

\[
p^m = \arg\max (p - c)(a - p)
\]

- Graph
• If $p_j < c$, the best response function is

$$B_i(p_j) = \{p_i : p_i > p_j\}$$

• If $p_j = c$, the best response function is

$$B_i(p_j) = \{p_i : p_i \geq p_j\}$$

• If $c < p_j \leq p^m$, the best response function is

$$B_i(p_j) = \emptyset$$

• If $p_j > p^m$, the best response function is

$$B_i(p_j) = \{p^m\}$$
• Thus the best response function is

\[ B_i(p_j) = \begin{cases} 
\{p_i : p_i > p_j\} & \text{if } p_j < c \\
\{p_i : p_i \geq p_j\} & \text{if } p_j = c \\
\emptyset & \text{if } c < p_j \leq p^m \\
p^m & \text{if } p_j > p^m
\end{cases} \]

• The fact that the firm \( i \) has no best response when \( c < p_j \leq p^m \) is an artifact of modeling price as a continuous variable.

• If discrete prices: if \( c < p_j \leq p^m \),

\[ B_i(p_j) = p_j - \epsilon \]

• Graph

• The game has a single Nash equilibrium \((c, c)\)
1. \( p_j < c \)

2. \( c < p_j \leq p^m \)
3.

\[ p_j > p^m \]

4.

\[ B_1(p_2) \]

\[ B_2(p_1) \]