Exercise 2.2 page 104

- $\theta$ is uniformly distributed on $[0, 1]$,
- Cost function
  \[ C(q, s) = \frac{cs^2}{2}q \]
- $N = 1$
- Inverse demand function
  \[ p(q, s) = s(1 - q) \]
- Average marginal valuation (social planner)
  \[ \frac{1}{q} \int_0^q \frac{\partial p(x, s)}{\partial s} dx = 1 - \frac{1}{2}q \]
- Marginal valuation (monopoly)
  \[ \frac{\partial p(q, s)}{\partial s} = 1 - q \]
- Comparison
  \[ \frac{1}{q} \int_0^q \frac{\partial p(x, s)}{\partial s} dx > \frac{\partial p(q, s)}{\partial s} \]
- Then under-supply of quality at a given $q$
  \[ s_m < s^* \]
• Optimal quantities?

• Monopoly

\[ Max_{q,s} \{ s(1 - q)q - \frac{cs^2}{2}q \} \]

\[ q^m = \frac{1}{3} \]

\[ s^m = \frac{2}{3c} \]

• Social planner

\[ Max_{q,s} \{ \int_0^q p(q, s)dx - \frac{cs^2}{2}q \} \]

\[ q^* = \frac{2}{3} \]

\[ s^* = \frac{2}{3c} \]

• Thus

\[ q^* = \frac{2}{3} > q^m = \frac{1}{3} \]

\[ s^* = \frac{2}{3c} = s^m \]

• Same quality, lower production