Intermediate Microeconomics 301
Final Exam
Thursday, May 5, 2005

Time: 2 hours.

Instructions. To obtain credit, you must give arguments to support your answer. The numbers in brackets at the start of each question are the numbers of points the questions are worth.

Exercise 1 [15]: Consumer theory
Jacob’s utility function is
\[ U = 10X^2Y \]
The price of \( X \) is \( p_X = $10 \) and the price of \( Y \) is \( p_Y = $5 \), and his income is \( m = $150 \). What is his optimal consumption bundle? Show in a graph.

Exercise 2 [25]: Producer theory
Suppose a production function is given by \( f(K,L) = K^{L^{1/3}} \), and that the price of capital is $10 and the price of labor is $16. The capital is fixed at the level \( K = 4 \).

1. What is the quantity of labor that minimizes the cost of producing any given output?
2. What is the minimum cost of producing \( y \) units of output?
3. What are the marginal cost of production and the average total cost, average variable cost and the average fixed cost?
4. Derive the firm’s short run supply curve.

Exercise 3 [20]: Supply and Demand
The supply curve for T-shirt is given by the equation \( P = 6Q \). The demand curve is given by the equation \( p = 18 - 3Q \).

1. What is the equilibrium price and quantity?
2. At a price of $18, will there be a surplus? Indicate by how much.
3. At a price of $6, will there be a surplus? Indicate by how much.

Exercise 4 [15]: Nash Equilibrium
1. In the following game, players must move simultaneously. What are the Nash equilibria of the game? How many are there?

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<td>M</td>
<td>2.3</td>
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2. Which equilibrium will occur if players play sequentially and player 1 makes her decision first? Why? Give an extensive representation of the game (tree).
3. Which equilibrium will occur if players play sequentially and player 2 makes her decision first? Why? Give an extensive representation of the game (tree).
Exercise 5 [25]: Oligopoly

In a Cournot duopoly, each firm has a marginal cost of 40 (i.e., the total cost for each firm $i$ is $c_i(q_i) = 40q_i$), and market demand is $Q = 100 - 2p$ (recall that $Q = q_1 + q_2$)

1. What are the best response functions of each firm?

2. What is the best output level for each firm?

3. How does the total output level compare to the cartel output level?

4. How much does firm 1 produce if it is the leader in a Stackelberg model? How much does firm 2 produce? Which one gets a higher payoff?