Intermediate Microeconomics 301
Second Mid-Term
Elements of answer

Exercise 1 [20]: Capital and labor to be perfect complements:

\[ q = f(L, K) = \min\{L, K\} \]

Constant return-to-scale? For \( k > 1 \), \( f(kL, kK) = k \min\{L, K\} \).

Exercise 2 [20]: The firm chooses \( L \) and \( K \) such that

\[ \begin{aligned} \min_{L,K} 2L + K \\
\text{s.t. } q = 2KL \end{aligned} \Rightarrow \min_{L} 2L + \frac{q}{2L} \]

FOC gives

\[ 2 - \frac{q}{2L^2} = 0 \text{ and thus } L^* = \sqrt[2]{q} \]

Furthermore

\[ K^* = \frac{q}{2L^*} = \sqrt[2]{q} \]

And the cost function is

\[ C(q) = 2L^* + K^* = 2\sqrt[2]{q} \]

If now \( K = 2 \), the cost function is

\[ C = 2L + 2 \]

where \( L = \frac{q}{2K} \) thus

\[ C(q) = \frac{1}{2}q + 2 \]

Exercise 3 [30]: \( C(q) = 125 + q^2, \ MC = 2q, \ AVC = q \)

1. The individual supply of each firm is \( q \) such that \( p = MC = 2q \) thus \( q = \frac{p}{2} \) for \( p > \min AVC = 0 \).

2. if \( p = $40 \), the profit-maximizing output level for each firm is \( q = 20 \), the total revenue is:

   \[ TR = pq = 40 \times 20 = 800 \]

   and the profit is \( \Pi = TR - TC = 800 - 125 - 20^2 = 275 \).

Exercise 4 [30]: The inverse demand function is \( p = 60 - Q \) and the supply function is \( Q = p \)

1. The initial equilibrium is such that \( S = D \) and thus \( 60 - Q = Q \), i.e., \( Q^* = 30 \) and \( p^* = 30 \).

2. At the equilibrium price, the consumer’s surplus is \( CS = \frac{60-30}{2} \times 30 = 450 \). The producer’s surplus is \( PS = \frac{30 \times 30}{2} = 450 \) thus the welfare is \( W = 900 \).

3. A specific tax of $2 is imposed, thus price becomes \( p' = 32 \), the quantity is \( Q = 28 \) and thus the new CS is \( CS' = \frac{60-32}{2} \times 28 = 392 \), the producer surplus is \( PS' = (32 - 28) \times 28 + \frac{28+28}{2} = 504 \). Thus the welfare is now \( W' = 392 + 504 = 896 < 900 \). Or \( \Delta = W' - W = 4 \).