1. Consider a monopolist that serves 2 groups of consumers. The marginal cost of production is 1.

a. Group 1 has an inverse demand function $p_1 = 10 - q_1$ thus

\[
\begin{align*}
MR_1 &= MC \\
10 - 2q_1 &= 1 \\
q_1^m &= 4.5 \\
p_1^m &= 5.5
\end{align*}
\]

Group 2 has a demand $p_2 = 5 - \frac{1}{2}q_2$

\[
\begin{align*}
MR_2 &= MC \\
5 - q_2 &= 1 \\
q_2^m &= 4 \\
p_2^m &= 3
\end{align*}
\]

b. Consumers’ surplus, producer’ surplus, total welfare.

\[
\begin{align*}
CS_1 &= \frac{10 - 5.5}{2} \times 4.5 = 10.125 \\
PS_1 &= (5.5 - 1) \times 4.5 = 20.25 \\
W_1 &= 10.125 + 20.25 = 30.375 \\
CS_1 &= \frac{5 - 3}{2} \times 4 = 4 \\
PS_2 &= (3 - 1) \times 4 = 8 \\
W_2 &= 4 + 8 = 12
\end{align*}
\]

c. No price discrimination, aggregate demand is

\[
Q = \begin{cases}
   20 - 3p & \text{if } 0 < p < 5 \\
   10 - p & \text{if } 5 \leq p < 10
\end{cases}
\]

and

\[
\begin{align*}
MR &= \frac{20}{3} - \frac{2}{3}Q = 1 \\
Q^m &= \frac{17}{2} = 8.5 \\
p^m &= \frac{20}{3} - \frac{1.17}{3} = 3.8333
\end{align*}
\]

And

\[
\begin{align*}
CS &= \frac{10 - 5}{2} \times 5 + (5 - 3.8) \times 5 + \frac{5 - 3.8}{2} \times (8.5 - 5) = 20.6 \\
PS &= (3.8 - 1) \times 8.5 = 23.8 \\
W &= 20.6 + 23.8 = 44.4 > W_1 + W_2 = 30.375 + 12 = 42.375
\end{align*}
\]
2. The standard monopoly solution is \( Q^* = 40, \ p^* = $200 \), which generates profits of $8,000.

With perfect price discrimination, the monopolist sets \( MC = p \) to determine the best output level. Costs are $4,808. Revenue is the entire area under the demand curve from \( Q = 0 \) to 48, or $10,368 (area \( abcd \) in Figure 1). Profits are $5,560, and consumer surplus is zero.

\[
\begin{align*}
4Q &= 240 - Q \\
Q^* &= 48 \\
R &= (192 \times 48) + (24 \times 48) = 10,368
\end{align*}
\]

![Figure 1](image)

3. At the original price of $5 per rental, Tuan rents 6 videos per month. His total expenditure per month is $30. Firm cost for these rentals is $12, leaving profits of $18 per month. Under the new pricing strategy, Tuan will continue to purchase as long as the flat fee is less than or equal to his consumer surplus. Figure 2 shows that, at a price of $2, consumer surplus is equal to \( \frac{6 \times 12}{2} = $36 \). Thus the firm can charge a monthly fee or $36, plus a $2 fee per movie, and earn profits of $36, or twice the profits of the $5 price with no fee.

Set $MC = MR$ and solve.

$$MR = 5Q^{1/2}$$

$MC = 5$.

$Q^* = 1$

$p^* = 10$

$\pi = 10 - 5 = \$5$.

5. Perloff, fourth edition: problem 26 page 382

The total profit is total revenue minus total cost and tax.

$$\pi = TR - TC = P(Q)Q - \tau Q - C(Q)$$

$$\frac{d\pi}{dQ} = P + \frac{dp}{dQ}Q - \tau - \frac{dc}{dQ}Q = 0$$

$$P + \frac{dp}{dQ}Q = \tau + \frac{dc}{dQ}$$
Set marginal revenue in each market equal to marginal cost to determine the quantities. Plug the quantities into the demand functions to determine prices.

\[ MR_1 = 100 - 2Q_1 = 30 = MC \]
\[ MR_2 = 120 - 4Q_2 = 30 = MC \]
\[ Q_1 = 35; \ p_1 = 65 \]
\[ Q_2 = 22.5; \ p_2 = 75 \]