Economics 301  
Spring 2008  
Problem set # 7: answers

1. For the production function $Q = 5L^5K - L$, when $K = 2$, $Q = 10L^5 - L$. $MP_L = 5L^{-5} - 1$. Thus, $MP_L = 0$ when $L = 25$.

2. The production function is $Q = 2K + L$. $AP_L = 2K/L + 1$. $MP_L = 1$.

3. In the Cobb-Douglas production function $Q = AK^\alpha L^\beta$, as long as the output elasticities are positive, the marginal products cannot be negative. For example, increases in labor will always have the marginal effect $\beta Q/L$.

   a) No diminishing marginal returns to labor. With $K$ fixed at any level, marginal product of labor is constant at 10.

b) Diminishing marginal returns to labor. With capital fixed at 2 units, the production function becomes $Q = 1.414L^{1/2}$. Marginal product of labor, calculated using the derivative formula, is $MPL = 0.707L^{-1/2}$, which decreases as $L$ is increased.

5. Perloff, fourth edition: problem 26 page 176  
   a. $Q = L + K$. For $t > 1$

   
   $f(tK,tL) = tL + tK = tf(K,L)$

   Constant returns-to-scale.
b. \( Q = L^\alpha K^\beta \). For \( t > 1 \)

\[
f(tK, tL) = (tL)^\alpha (tK)^\beta = t^{\alpha + \beta} f(K, L)
\]

If \( \alpha + \beta > 1 \) increasing returns-to-scale. If \( \alpha + \beta = 1 \) constant returns-to-scale. If \( \alpha + \beta < 1 \) decreasing returns-to-scale.

c. \( q = L + L^\alpha K^\beta + K \). For \( t > 1 \)

\[
f(tK, tL) = tL + (tL)^\alpha (tK)^\beta + tK
\]

to be compared with \( tf(K, L) = tL + tL^\alpha K^\beta + tK \). Thus, \( tL + (tL)^\alpha (tK)^\beta + tK > tL + tL^\alpha K^\beta + tK \) is equivalent to \( t^{\alpha + \beta} f(K, L) > tL^\alpha K^\beta \) and the answer is the same that the answer of b.

a) \( AFC = 10/q. \ MC = 10. \ AVC = 10. \ AC = 10/q + 10 \). See Figure 2

![Figure 2](image)

b) \( AFC = 10/q. \ MC = 2q. \ AVC = q. \ AC = 10/q + q \). See Figure 3
c) $AFC = 10/q$. $MC = 10 - 8q + 3q^2$. $AVC = 10 - 4q + q^2$. $AC = 10/q + 10 - 4q + q^2$. See Figure 4