1. If input prices are \( w = 3 \) and \( r = 2 \), and \( q = 10KL \), what is the least cost input combination required to produce 60 units of output? How would input usage change if output is increased to 240 units? Sketch the solutions on a graph.

The solution is

\[
\begin{align*}
L^* &= 2, \\
K^* &= 3.
\end{align*}
\]

If output is increased but input prices remain the same with a Cobb-Douglas function, the input ratio does not change. \( L^* = 4, K^* = 6 \). See Figure 1.

2. If input prices are \( w = 4 \), and \( r = 1 \) and \( q = 4K^{0.5}L^{0.5} \), what is the least cost input combination required to produce 40 units of output? Suppose instead that capital was fixed at 16 units. What would be the implications for labor usage and total cost?

Minimizing cost requires that \( MRTS = \frac{w}{r} \). Since \( MRTS = \frac{MP_L}{MP_K} \), set the ratio of marginal products equal to the ratio of input prices, then substitute into the output constraint.

\[
\begin{align*}
K/L &= 4/1 \\
40 &= 4(4L)^{0.5}L^{0.5} = 8L \\
L^* &= 5 \\
K^* &= 20 \\
C &= 4(5) + 1(20) = 40
\end{align*}
\]
If capital is fixed at 16 units, least cost production is not possible. Instead, labor must be increased to 6.25 units. Total cost increases from $40 to $41.

3. Two firms currently produce the goods $q_1$ and $q_2$ separately. Their cost functions are $C(q_1) = 250 + q_1$ and $C(q_2) = 350 + 2q_2$. By merging, they can produce the two goods jointly with costs described by the function $C(q_1, q_2) = 450 + q_1 + q_2$. Are there scope economies in this case that would justify the merger?

Using the equation for scope economies given in Section 7.5 of the chapter, scope economies exist if $SC > 0$. In this case, scope economies do exist as the following expression is greater than zero for all values of both outputs.

$$SC = \frac{[250 + q_1 + 350 + q_2 - (450 + q_1 + q_2)]}{450 + q_1 + q_2}$$

4. Suppose a perfectly competitive firm has the short-run cost function $C(q) = 125 + q^2$. Use the derivative formula or marginal cost to determine the firm’s output level and profit at prices of $30 and $20. At what price does the firm reach the shut-down point?

The marginal cost equation is $MC = 2q$. When $p = $30, $q^* = 15$, $\pi =$ $100$. When $p = $20, $q^* = 15$, $\pi =$ $-25$. The firm should keep operating since $TR > TVC$. The firm should shut down when $AVC > MC$ (normally, the minimum point of $AVC$). In this case, however, variable cost is linear with slope of 1, and $MC$ is linear with slope of 2, making all positive output levels above the shutdown point.

5. Suppose that the average variable cost of a competitive firm is given by $AVC(q) = 3 + q$, the marginal cost is $MC(q) = 3 + 2q$ and the firm’s fixed costs are known to be $3$. Will the firm be earning a positive, negative or zero profit in the short run when the market price is $9$?

Set $p = MC$ gives $q^* = 3$. Thus $\pi = 6$.

6. Draw a graph showing the average total, average variable, and marginal cost curves for a typical firm. Draw in three prices that result in the firm making positive profits, breaking even, and making negative profits that are less than fixed costs.

See Figure 1. At $p_3$, the firm makes positive profits. At $p_2$, the firm breaks even and at $p_1$, the firm realizes losses that are less than fixed costs.
7. If each competitive firm in an industry has the short-run cost function \( C(q) = 50 + 5q + q^2 \), and the market price is $35, what is the profit-maximizing output level for each firm? What is the total revenue? What are the profits?

Set \( MC = MR \) and solve.

\[
\begin{align*}
MC & = 2q + 5 \\
MR & = 35 \\
2q + 5 & = 35 \\
q^* & = 15, TR = 525, TC = 350, \pi = 175
\end{align*}
\]