Technology Licensing to a Rival

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Abstract

Licensing a new technology implies introducing competition into the market. This has a negative effect on the profit of the incumbent if the demand remains unchanged, i.e., if consumers do not react to competition changes. However, because of the novel content of an innovation, consumers may have different perceptions of the value of a good depending on the market structure (e.g., because of network externalities). Thus, the introduction of a competitor into the market may enhance demand, and consequently have a positive effect on the profit of the incumbent. In a simple setting, we show that the incumbent may decide to license her technology even in the absence of a royalty when the positive effect outweighs the negative one. We then investigate what the optimal royalty rate should be depending on the demand enhancement and the cost of production for the competitor. We show that it may even be profitable for the incumbent to subsidize the competitor when the competitor’s cost of production is too high.

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1 Introduction

Technology licensing is generally viewed as a revenue-generating device. It might be a way of ensuring that a technology will be commercialized by a more efficient firm, for instance, a firm that has a better distribution system in place. In return for the use of her technology, the innovator gets a reward in the form of royalties (Tirole 1988). Even if an innovator sells her own products, it may also be profitable to license a technology that would be incorporated into a product offered in an independent market where the marketing skills of the innovator may not be as strong. In this way, the license has no competitive impact but generates additional revenues for the innovator.

Licensing may also be used for strategic considerations. Rockett (1990) shows that an innovator may use technology licensing to prevent the entry of strong competitors. Before her patent expires, the innovator licenses the technology to weaker competitors. These firms have some time to get a foothold in the market before others can join, and then ultimately, this discourages later entrants. Licensing thus enables the innovator to choose her competitors. Licensing may also be a way to impose a technology on a market where incompatible technologies are candidates to become a standard. The aggressive licensing strategy used by VHS patent holders certainly contributed to the victory of the VHS technology over Betamax on the video market. Many firms have also recognized that cross-licensing is mutually beneficial in order to ensure that the introduction of a technology will not be blocked. This is especially true in markets such as telecommunications, where new products or services incorporate a large number of patented innovations.

Licensing may have a positive impact on demand, so that there may be benefits to licensing in terms of market expansion. Shepard (1987) and Farrell and Gallini (1988) show that licensing may stimulate demand, since consumers would not fear being victims of opportunism on the incumbent’s part because she faces competition. Shepard (1987) demonstrates that licensing acts as a guarantee of the quality of the products, while Farrell and Gallini (1988) show that licensing prevents future price increases. Although these benefits may be substantial, these studies do not consider that an increase in demand may be sufficient to ensure the profitability of a licensing strategy.

Conner (1995) goes further in the analysis of the benefits of market expansion from licensing. She determines the conditions under which it is profitable for an incumbent to license her technology for free to an entrant that will use the technology to introduce his own product and compete on the incumbent’s market. She shows that licensing is a profit-maximizing strategy when network externalities are strong enough, but only if the quality of the entrant’s products is lower than the quality of the incumbent’s products. Under these circumstances, the licensee covers the inferior portion of the market that consists of consumers with low valuations of the product. Thus, the licensee does not steal many customers from the incumbent, and the presence of network externalities results in an increase in willingness-to-pay of the incumbent’s customers.

We follow in Conner’s footsteps in suggesting that the increase in demand resulting from li-
censing may be large enough to induce an innovator to share her technology. Our results establish that it may be profitable for a firm to license her technology to a rival who would compete in the same market in a homogenous-product duopoly setting, even without royalties. We also determine the optimal royalty scheme and show that royalties increase the innovator’s incentive to license a technology. Whereas, in much of the licensing literature, it is assumed that the licensor does not market the technology (e.g., Bousquet et al., 1998), we consider an innovator who markets her technology and can also license it to another firm.

The paper is organized as follows. The model is presented in section 2. We first determine the demand and define the equilibrium quantities and payoffs. In section 3 we study the licensing behavior of the leader in absence of royalties. Section 4 is devoted to the determination of optimal royalties, and section 5 defines the conditions under which the leader licenses her technology.

2 The Model

We consider a two-period model in which an incumbent (leader) has to decide whether to share her technology via a license with a potential licensee (follower).

The timing of the game is as follows:

- First, the leader decides whether to license her technology and determines the optimal level of royalty. The follower then accepts it or not.

- In the second stage, if both firms are in the market, they decide their production level in a Cournot competition.

Using a backward induction argument, we first determine the quantity offered by each firm, depending on the structure of the market and on the demand. Second, we determine whether the follower accepts the technology, and then, the optimal level of royalty.

Demand plays an important role in our setting, as the structure of the market directly influences the demand. We assume that consumers have different perceptions of the value of the good depending on whether only one firm offers the good or two firms offer two similar goods. We first present the detailed demand structure, before turning to the competition stage.

2.1 Consumers’ Behavior and Demand

The perception of the value of a good is based not only on its intrinsic characteristics, but also on marketing cues. This is especially the case for new products whose characteristics and performance are still unknown to consumers.

The demand structure is affected by the decision of the follower to enter or not. Consumers value differently the good offered by a single firm, and the goods offered by two firms. We assume a simple
framework in which each consumer consumes 0 or 1 unit of the good. The good is characterized by
an index $s$ that represents the perception of the value of the good by consumers. A consumer who
is not willing to buy a product offered by a monopoly may be willing to buy it once he can observe
that another firm has decided to market the technology. The fact that another firm is backing the
technology is a signal of the value of the good to the consumer. If the product is just offered by
the monopoly, the perception of the value by consumers is $s_m$, whereas if another firm has entered
the market, the perception of the value by consumers becomes $s_d$. To keep the model as general as
possible, we impose no restrictions on the relationship between $s_m$ and $s_d$. If the value of the good
is positively related to the number of firms on the market then $s_d > s_m$.

The utility of a consumer is

$$U = \begin{cases} 
\theta s_m - p_m & \text{if a monopoly serves the market, and the consumer buys at price } p_m, \\
\theta s_d - p_d & \text{if a duopoly serves the market, and the consumer buys at price } p_d, \\
0 & \text{if the consumer does not buy}, 
\end{cases}$$

where $\theta$ represents the taste parameter of the consumer and is distributed according to a density
function $f(\theta)$ with a cumulative function $F(\theta)$. $F(\theta)$ represents the fraction of consumers with a
taste parameter inferior to $\theta$. We assume that $\theta$ is distributed uniformly on $[0,1]$.

If there is no entry, the leader has a monopoly position in the market. Consumers with utility
smaller than 0 do not buy. Thus, $\theta < p_m/s_m$ represents the fraction of consumers who do not buy
the good and the demand for the monopoly is $D_m(p_m) = N[1 - F(p_m/s_m)]$, where $N$ is the total
number of consumers that we normalize to be 1. As we assume a uniform distribution, the demand
function for the monopoly is $D_m(p_m) = 1 - p_m/s_m$ and the inverse demand function becomes

$$p_m(q) = s_m(1 - q). \quad (1)$$

If the follower accepts the license, the structure of the market is a duopoly. The fraction of
consumers who buy the product increases by $[F(p_m/s_m) - F(p_d/s_d)]$, and thus, the new demand
becomes $D_d(p_d, p_m) = 1 - F(p_m/s_m) + F(p_m/s_m) - F(p_d/s_d)$, which gives the inverse demand
function

$$p_d(q) = s_d(1 - q). \quad (2)$$

2.2 Competition in Quantity

We now derive the quantities provided by the firms when there is no technology licensing (monopoly)
and when there is technology licensing (duopoly). In this section we keep the model as general as
possible and consider that the licensing agreement can include a per unit royalty.

2.2.1 Monopoly

If the follower does not enter, the leader has a monopoly and the demand is given by (1). Thus,
the monopoly maximizes her profit $\Pi^m = p_m(q)q - c_1q$, where $c_1$ is the cost of production per unit.
The monopoly quantity is \( q_m = (s_m - c_1)/2s_m \), where \( c_1 < s_m \leq 3c_1 \), and the monopoly price is \( p_m = (s_m + c_1)/2 \). The monopoly profit is
\[
\Pi^m = \frac{(s_m - c_1)^2}{4s_m}.
\]

### 2.2.2 Duopoly

If the follower accepts the license offered by the leader, both firms compete in quantity (Cournot competition) and the demand is given by (2). The profit of each firm is
\[
\Pi^d_1 = p_d(q_1, q_2)q_1 - c_1q_1 + rq_2,
\]
\[
\Pi^d_2 = p_d(q_1, q_2)q_2 - c_2q_2 - rq_2,
\]
where \( c_2 \) is the per unit cost of production of the follower, and \( rq_2 \) the amount of the royalty paid by the follower to the leader, where \( r \) is the per unit royalty.\(^2\) The profit maximization gives the following quantities
\[
q_1(r) = \frac{s_d - 2c_1 + c_2 + r}{3s_d},
\]
\[
q_2(r) = \frac{s_d - 2c_2 + c_1 - 2r}{3s_d},
\]
where \( s_d > 2c_1 - c_2 - r \) and \( s_d > 2c_2 - c_1 + 2r \). The duopoly price is \( p_d(r) = (s_d + c_2 + c_1)/3 + r \). For a given \( r \), the optimal profit of each firm is
\[
\Pi^d_1(r) = \frac{(s_d - 2c_1 + c_2 + r)^2}{9s_d} + r\frac{(s_d - 2c_2 + c_1 - 2r)}{3s_d},
\]
\[
\Pi^d_2(r) = \frac{(s_d - 2c_2 + c_1 - 2r)^2}{9s_d}.
\]

### 3 Licensing without Royalties

We first consider the case in which the leader licenses her technology for free (i.e., \( r = 0 \)). This enables us to isolate two opposite effects of technology licensing. From the leader’s point of view, technology licensing first has a negative impact on her profit, since her market share decreases. Second, as demand may increase with the entry of another firm, there is also a potential positive effect on her profit resulting from the expansion of the market. The positive impact may then compensate for the erosion in market share and make licensing attractive, even when it does not bring in additional revenues in the form of dividends.

In this setting, each firm gets a duopoly payoff \( \Pi^d_i(0) = (s_d - 2c_i + c_j)^2/9s_d \), where \( i, j = 1, 2 \) and \( i \neq j \), for producing at least the minimum quantity that can be detected by consumers on the market. Indeed, the demand is defined by the consumers’ perceived value of the good, which is,

\(^1\)We assume that the leader’s cost of production is relatively high.
\(^2\)Empirical studies find that royalties are more common than lump sum fees. See, for instance, Macho-Stadler et al. (1996) and Jensen and Thursby (2001).
in turn, influenced by the number of firms. To convince consumers that they are present in the market, firms must produce at least a minimum quantity, that we denote \( \kappa \), and we further assume that \( \kappa < \frac{1}{3} \). The follower accepts a license as long as he gets a positive profit, i.e., \( \Pi^f_2(0) > 0 \). The leader decides to license her technology if it is worthwhile, as the introduction of a new firm in the market has a positive impact on profits if demand increases sufficiently. Formally, the leader licenses as long as
\[
\Pi^l_1(0) > \Pi^m.
\]

Furthermore, both firms need to produce at least \( \kappa \) to inform consumers that two firms are producing, or equivalently, \( s_d \geq (2c_1 - c_2)/(1 - 3\kappa) \equiv f_1(c_2) \) and \( s_d \geq (2c_2 - c_1)/(1 - 3\kappa) \equiv f_2(c_2) \) must be satisfied.\(^3\)

Inequality (8) holds for certain constellations of parameters \((s_d, c_2)\). Indeed, as long as the introduction of a competitor does not increase demand, i.e., \( s_d < s_m \), the monopoly licenses her technology if \( s_d > (3s_m + c_1)/2 - c_2 \equiv \varphi(c_2) \). On the other hand, if \( s_d \geq s_m \), for high values of the follower’s cost (i.e., \( c_2 > 2c_1 \)), the monopoly always licenses, whereas for lower values (i.e., \( c_2 \leq 2c_1 \)), she licenses only if \( s_d > \phi(c_2) \) where \( \phi(c_2) \) is defined in the appendix.

Overall, licensing occurs if \((s_d, c_2) \in \{(s_d, c_2)/s_d \geq \phi(c_2) \) and \( s_d \geq f_2(c_2)\}\). We represent in a graph \((s_d, c_2)\) the areas where technology licensing occurs.

Consider a given high enough \( s_d \) (i.e., \( s_d > s_m \)), as represented by point \( X \) in figure 1. At this point, the leader does not share her technology and enjoys a monopoly profit. If the cost of production of the follower increases from point \( X \) to point \( Y \), the leader now shares her technology, and thus gets a duopoly profit. As the cost of production for her competitor increases, the leader is more willing to share her technology, as she will produce more than her licensee. On the other hand, if we consider a given level of cost \( c_2 \) (point \( W \) in figure 1), and \( s_d \) increases, we go from a regime where the leader does not share her technology to a regime where she does. This is because the entry of a competitor enhanced the demand enough. This can happen even if the follower’s cost of production is low. Thus, there are two effects: a demand effect and a cost effect.

**Result 1** The leader licenses her technology for free when the entry of a competitor sufficiently enhances demand, and/or his cost of production is high enough, i.e., for \( s_d \geq \max\{\phi(c_2), \varphi(c_2)\}\).

The follower only accepts the technology licensing if \( s_d \geq f_2(c_2) \).

Even in the absence of royalties, the leader finds it profitable to license her technology when the introduction of a competitor increases the demand. As Figure 1 illustrates, the leader’s incentive

\(^3\)Those conditions are more restrictive than the conditions on positive quantities.
to license her technology depends on: (i) the change in the consumers’ perception of the value; (ii) the relative cost of production.

When there is a low cost of production for the follower, the leader prefers to keep her monopoly position unless the demand stimulation is large. If the follower has a cost advantage, the leader expects a large decrease in her market share following the entry of a rival. The demand stimulation thus must be high in order to compensate for the erosion of the market share. As the follower’s cost advantage decreases, the demand stimulation necessary to make licensing profitable for the innovator also decreases. In the case of higher cost of production for the follower, and if the follower produces, the leader prefers to license her technology. The same logic may be used to explain the intuition of the result. If the leader has a cost advantage, she expects only a small reduction of her market share following the entry of a rival. A small demand stimulation is thus sufficient to compensate for the reduction in the market share and to make licensing attractive. As the follower’s cost of production increases, technology sharing is less likely to occur, since the profitability of the follower’s entry decreases.

We also need to consider the behavior of the firms when $s_d < f_1(c_2)$ (the duopoly quantity of the leader is smaller than $\kappa$) and/or $s_d < f_2(c_2)$ (the duopoly quantity of the follower is smaller than $\kappa$). In the former case (respectively, the latter case), the leader (respectively, the follower) can decide to produce $\kappa$ or to produce less, and thus, the demand is no longer represented by (2) but by (1). Indeed, if one of the quantities is too small, consumers cannot correctly infer that two firms are in the market, and they have the demand (1). However, the leader is better off by keeping her monopoly position, i.e., by not licensing her technology in the first place. When the minimum quantity $\kappa$ increases, the area where the leader shares her technology shrinks as the function $f_2(c_2)$ rotates to the left around $c_1/2$.

Thus, the leader shares her technology if (i) the demand enhancement is large enough and/or (ii) the cost for the follower is not too large, and finally (iii) if neither firm has to produce too much to signal his or her presence in the market.

### 4 Determination of Optimal Royalties

We have seen that even in the absence of royalties the leader may be willing to license her technology. We now complete our study and consider the case where the leader chooses an optimal level of licensing. Not surprisingly, we show that the leader’s willingness to license her technology increases. However, the optimal level of licensing does not monotonically increase with the demand enhancement of the market. Furthermore, the leader can even decide to subsidize the follower in order to introduce competition into the market when his cost of production is too high. The former result is essentially due to the fact that the maximization program of the leader is constrained. Indeed, the quantity offered by each firm must be at least as large as $\kappa$. 

7
The leader chooses the level of per unit royalty that maximizes $\Pi_1^{d}(r)$ defined by (6) subject to $q_2(r) \geq \kappa$ and $q_1(r) \geq \kappa$. If none of these constraints are binding (i.e., $q_2(r) > \kappa$ and $q_1(r) > \kappa$), the optimal level of per unit royalty is

$$r^u = \frac{5s_d - 4c_2 - c_1}{10},$$

and the quantities provided are $q_1(r^u) = (5s_d - 7c_1 + 2c_2)/10s_d$, and $q_2(r^u) = (2(c_1 - c_2))/5s_d$. As long as $q_1(r^u) > \kappa$ and $q_2(r^u) > \kappa$, the unique solution is $r^u$. This will arise for values of $s_d > (7c_1 - 2c_2)/5(1 - 2\kappa) \equiv g_1(c_2)$ and $s_d < 2(c_1 - c_2)/5\kappa \equiv g_2(c_2)$. A necessary condition is that $c_1 > c_2$. As long as $c_1 > c_2$, the royalty rate is strictly positive. For values of $c_2 > c_1$, the follower will not produce, as $s_d$ cannot be smaller than $2(c_1 - c_2)/5\kappa$.

If one of the constraints on the quantities is binding, we find corner solutions in which either the leader produces the minimum required ($q_1(r) = \kappa$) or the follower produces ($q_2(r) = \kappa$). We consider each of these cases. If only the constraint on the quantity produced by the leader is binding, the optimal level of royalty is determined by the equality $q_1(r) = \kappa$, where $q_1$ is defined by (4). The solution is thus

$$r^1_\kappa = -s_d(1 - 3\kappa) + 2c_1 - c_2,$$

and the follower produces $q_2(r^1_\kappa) = (s_d(1 - 2\kappa) - c_1)/s_d$.

If, on the contrary, only the constraint on the quantity produced by the follower is binding, the per unit royalty is defined according to $q_2(r) = \kappa$ with $q_2$ defined by (5), and thus the optimal level of per unit royalty is

$$r^2_\kappa = \frac{s_d(1 - 3\kappa) - 2c_2 + c_1}{2},$$

and the leader produces $q_1(r^2_\kappa) = (s_d(1 - \kappa) - c_1)/2s_d$. One condition in order to get interior or corner solutions is that $r^1_\kappa \leq r^2_\kappa$, or $s_d \geq c_1/(1 - 3\kappa) \equiv g_3$. For values of $s_d < g_3$, there is no production in this duopoly framework. Note that if both constraints are binding, the leader chooses $r^1_\kappa$, such that $q_1(r) = \kappa$ and then $q_2(r^1_\kappa) = (s_d(1 - 2\kappa) - c_1)/s_d \leq \kappa$ (or equivalently $s_d \geq g_3$) must be satisfied.

Thus, depending on the binding constraints, if any, the optimal per unit royalty is

$$r^* = \begin{cases} 
    r^1_\kappa & \text{if } (s_d, c_2) \in \Delta_1 \\
    r^u & \text{if } (s_d, c_2) \in \Delta_2 \\
    r^2_\kappa & \text{if } (s_d, c_2) \in \Delta_3
\end{cases},$$

where $\Delta_1 = \{(s_d, c_2)/s_d \geq g_3, \text{ and } s_d \leq g_1(c_2)\}$, $\Delta_2 = \{(s_d, c_2)/s_d \leq g_1(c_2), \text{ and } s_d \geq g_2(c_2)\}$ and $\Delta_3 = \{(s_d, c_2)/s_d \geq g_3, s_d > g_1(c_2), \text{ and } s_d \geq g_2(c_2)\}$.

The optimal per unit royalty depends crucially on the magnitude of the demand stimulation on one hand and on the cost of production for the follower on the other hand.

Insert figures 2 and 3 over here
In figure 2, we first consider a given low enough cost of production \( c_2 \) (i.e., \( c_2 < c_1 \)), and we investigate the effect of an increase of the demand enhancement \( s_d \). For very low values of \( s_d \), there is a reduction of the demand after the introduction of the competitor, and thus, no technology sharing will occur. For higher values of \( s_d \), technology sharing occurs and at first the royalty rate \( r_{\kappa}^1 \) decreases as \( s_d \) increases. The leader produces just enough to signal her presence in the market \((q_1 = \kappa)\), and thus as \( s_d \) increases, the profit of the leader decreases (through the price and the royalty rate that both decrease). The leader, thus, reduces the level of royalty rate. Then, for higher level of \( s_d \), the royalty rate increases. It is first \( r^u \) and then becomes flatter as \( r^* = r_{\kappa}^2 \).

In figure 3, we consider a given level of \( s_d \) (such that \( s_d > g_3 \)) and we investigate the effect of an increase in the cost of production of the follower. The royalty rate is decreasing, and eventually, for a high cost of production, becomes negative. Thus, the leader needs to subsidy entry. We summarize those findings:

**Result 2** For a given level of cost of production \( c_2 < c_1 \) the optimal level of royalty rate \( r^* \) is first decreasing and then increasing with \( s_d \). On the other hand, for a given level of \( s_d > g_3 \), \( r^* \) is always decreasing. Furthermore, for high values of \( c_2 \), the leader needs to subsidy entry.

Overall, for very low values of \( c_2 \), the leader will either produce the minimum level of output for values of \( s_d < 2c_1 - c_2 \) (which is larger than \( g_1(c_2) \)) or will prefer to set the optimal level of royalty \( r^u \) for \( s_d > g_1(c_2) \). The former case arises because the follower has a lower cost of production and, thus, is more efficient, and the leader’s profit is decreasing with \( c_2 \). For higher values of \( c_2 \), but still smaller than \( c_1 \), as \( s_d \) increases, the leader proposes either \( r_{\kappa}^1 \) or \( r^u \), and eventually \( r_{\kappa}^2 \). When both firms have very similar costs and the demand enhanced by the introduction of a duopoly in the market is fairly large, the leader prefers to let the follower produce as little as possible. As the cost of the follower increases, the leader even subsidizes the follower in order to produce the minimum required \((r_{\kappa}^2 < 0)\).

## 5 Licensing with Royalties

When the leader chooses an optimal level of royalty, she will license her technology if \( \Pi_1(r^*) > \Pi^m \), as long as the follower accepts the license. The follower produces if his profit is non-negative. Thus, the follower always accepts as the leader proposes a contract in which each firm produces at least \( \kappa \). The constellation of parameters for which the leader proposes to license her technology is broader compared to the case where the technology is licensed for free. The optimal level of per unit royalty is

\[
r^{**} = \begin{cases} 
  r_{\kappa}^1 & \text{if } (s_d, c_2) \in \Delta_1 \cap S_1, \\
  r^u & \text{if } (s_d, c_2) \in \Delta_2 \cap S_2, \\
  r_{\kappa}^2 & \text{if } (s_d, c_2) \in \Delta_3 \cap S_3,
\end{cases}
\]
where $S_1 = \{(s_d, c_2)/s_d \geq s'_1(c_2)\}$, $S_2 = \{(s_d, c_2)/s_d \geq s'_2(c_2)\}$ and $S_3 = \{(s_d, c_2)/s_d \geq s'_3(c_2)\}$, where $s'_k(c_2)$ for $k = 1, 2, 3$ is defined in the appendix.

**Result 3** The leader licenses her technology at the optimal royalty rate $r^{**}$ and the follower accepts the technology licensing. For a high level of cost of production for the follower, the leader subsidizes entry.

The results formulated here can be explained intuitively as follows.

Insert figures 4 and 5 over here

Let us first consider that the demand is sufficiently enhanced at a given level $s_d > s_m$. For a very low cost of production of the follower (point $X_1$ in figure 4), the leader produces only the minimum level $\kappa$, to let consumers know that two firms are in the market. Even though it would be more profitable for the leader to have only the follower producing and to get the appropriate level of royalty, she must signal that two firms are producing. As the cost of production of the follower increases (from $X_1$ to $Y_1$), the leader increases her production and both firms produce duopoly quantities. As the cost of production of the follower increases even more (point $Z_1$ in figure 4), the leader sets the level of royalty such that the follower produces just enough to signal that two firms are in the market. However, for a fairly high cost of production, the leader must subsidize the follower to enter the market.

Figure 5 provides the same graph as in figure 4, except that now we look at how the change in the demand enhancement affects the technology sharing for a given cost of production for the follower ($c_2 < c_1$). For a relatively low value of $s_d$ (i.e., low demand enhancement), the leader prefers not to share her technology (point $W_2$ in figure 5). As the demand enhancement increases (from point $W_2$ to point $X_2$), the leader offers a level of royalty such that she produces just the required amount to let consumers know that two firms are in the market, and the follower produces more as he has a lower cost of production. However, as the demand enhancement increases even more (point $Y_1$), both firms produce the optimal duopoly quantities. Finally, for a high demand enhancement (point $Z_2$), the leader offers a royalty rate such that the follower produces just enough to signal that two firms are in the market.
References


Appendix

License without royalties:

The leader licenses as long as $\Pi_I'(0) > \Pi^m$, where $\Pi^m$ and $\Pi_I'(0)$ are defined by equations (3) and (6). If $s_d < s_m$, $\Pi_I'(0) > \Pi^m$ as long as $s_d > (3s_m + c_1)/2 - c_2$. On the other hand, if $s_d \geq s_m$, $\Pi_I'(0) > \Pi^m$ is equivalent to having $q_1^2 s_d > q_m^2 s_m$ and, thus, we only need to verify that $q_1 > q_m$. This last inequality is equivalent to $2s_m(-2c_1 + c_2) > s_d(s_m - 3c_1)$. Depending on the signs of $(-2c_1 + c_2)$ and $(s_m - 3c_1)$, this inequality does hold. As we assume that $s_m \leq 3c_1$, if $c_2 > 2c_1$, $\Pi_I'(0) > \Pi^m$, whereas if $c_2 \leq 2c_1$, $\Pi_I'(0) > \Pi^m$ only if $s_d \geq \phi(c_2)$, where $\phi(c_2) = \frac{1}{8s_m}(8(-s_m c_2 + s_m^2 + c_1^2) + (s_m - c_1)^2 - 3(s_m - c_1) \sqrt{(3s_m + 3c_1)^2 - 4s_m(c_1 + 4c_2)})$. Indeed, $(s_d - 2c_1 + c_2)^2/9s_d - (s_m - c_1)^2/4s_m > 0$ if $s_d < \phi'(c_2)$ and $s_d > \phi(c_2)$, where $\phi'(c_2) = \frac{1}{8s_m}(8(-s_m c_2 + s_m^2 + c_1^2) + (s_m - c_1)^2 + 3(s_m - c_1) \sqrt{(3s_m + 3c_1)^2 - 4s_m(c_1 + 4c_2)})$. We can check that $\phi(c_2)$ is a decreasing and convex function.

License with royalties:


At the optimal level of royalty, the profit of the leader is

$$\Pi_1^d(r^*) = \begin{cases} 
-s_0^2(1-5\kappa(1-\kappa)) + s_d(c_1(3-7\kappa)-c_2(1-2\kappa)-c_1(2c_1-c_2)) & \text{if } (s_d, c_2) \in \Delta_1 \\
\frac{5s_0^2 - 10s_0c_1 + 9c_1^2 - 8c_1c_2 + 4c_2^2}{s_d} & \text{if } (s_d, c_2) \in \Delta_2 \\
\frac{1}{4}s_0^2(1-5\kappa^2) - 2s_d(c_1 - 2c_1(c_1 - c_2)) + c_1^2}{s_d} & \text{if } (s_d, c_2) \in \Delta_3.
\end{cases}$$

We compare $\Pi_1^d(r^*)$ and $\Pi^m$ for each area of the parameters, i.e., if $(s_d, c_2) \in \Delta_1, \Delta_2$ or $\Delta_3$.

- For $(s_d, c_2) \in \Delta_1$ the optimal level of royalty is $r_1^1$, and thus, we compare $\Pi_1^d(r_1^1)$ and $\Pi^m$. $\Pi_1^d(r_1^1) > \Pi^m$ if $s_d < s_1''(c_2)$ and $s_d > s'_1(c_2)$ where $s_1''(c_2) = \frac{1}{27\kappa}(E - \sqrt{E^2 + 4DF})$ and $s'_1(c_2) = \frac{1}{27\kappa}(-E + \sqrt{E^2 + 4DF})$ with $D = 4s_m\kappa(1 - \kappa)$, $E = (2s_m(1 - 2\kappa)(3c_1 - 2c_2) - (s_m^2 + c_1^2))$, and $F = 4s_m c_1(c_1 - c_2)$. It is easy to show that $E^2 + 4DF > 0$ as $4DF > 0$ as long as $c_1 - c_2 > 0$. Furthermore, $s'_1(c_2)$ is a convex function with $s'_1(0) < g_1(0)$ and $s'_1(0) > g_3$ for $\kappa \in [0.25, 0.6]$. Thus, the set of parameters for which the optimal royalty rate is $r_1^1$ is $\Delta_1 \cap S_1$ where $S_1 = \{(s_d, c_2)/s_d \geq s'_1(c_2)\}$.

- For $(s_d, c_2) \in \Delta_2$ the optimal level of royalty is $r^n$, and thus, we compare $\Pi_1^d(r^n, R^n)$ and $\Pi^m$. $\Pi_1^d(r^n, R^n) > \Pi^m$ if $s_d < s_2''(c_2)$ and $s_d > s'_2(c_2)$ where $s_2''(c_2) = \frac{1}{105s_m}(s_m^2 + c_1^2 - \sqrt{\Delta})$ and $s'_2(c_2) = \frac{1}{105s_m}(s_m^2 + c_1^2 + \sqrt{\Delta})$ where $\Delta = 5^2(c_1^2 - s_m^2)^2 - 12^2s_m^2(c_2 - c_1)^2 > 0$. We can easily check that $s_m > s_2''$ and thus the only condition that must be satisfied is $s_d > s_m$. Thus, the set of parameters for which the optimal per unit royalty is $r^n$ is $\Delta_2 \cap S_2$ where $S_2 = \{(s_d, c_2)/s_d \geq s'_2(c_2)\}$.

- For $(s_d, c_2) \in \Delta_3$ the optimal level of royalty is $r_0^2$, and thus, we compare $\Pi_1^d(r_0^2)$ and $\Pi^m$. $\Pi_1^d(r_0^2) > \Pi^m$ if $s_d < s_3''(c_2)$ and $s_d > s'_3(c_2)$ where $s_3''(c_2) = \frac{1}{28s_m(1-\kappa^2)}(4ks_m(c_2 - c_1) + (s_m^2 + c_1^2) - \sqrt{\Delta})$ and $s'_3(c_2) = \frac{1}{28s_m(1-\kappa^2)}[(4ks_m(c_2 - c_1) + (s_m^2 + c_1^2) + \sqrt{\Delta})$ with $\Delta = 20s_m^2\kappa^2c_1^2 - 32s_m^2\kappa^2c_1c_2 - 8s_m^3\kappa c_1 - 8s_m\kappa c_1^3 + 16s_m^2\kappa^2c_2^2 + 8s_m^2\kappa c_2 + 8s_m\kappa c_2c_1^2 + s_m^4 - 2s_m^2c_1^2 + c_1^4 > 0$. Thus, the set of parameters for which the optimal royalty rate is $r_0^2$ is $\Delta_3 \cap S_3$ where $S_3 = \{(s_d, c_2)/s_d \geq s'_3(c_2)\}$. 

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Licensing for free, \( s_m \leq 3c_1 \)

Optimal per unit royalty function of \( s_d \)
Optimal per unit royalty function of $c_2$

Optimal technology sharing
Optimal Technology Sharing

No Technology Sharing

No production

Optimal technology sharing