Forward Markets and Signals of Quality

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Introduction

How information about quality may be conveyed via forward trading

• Market participants
  1. A privately informed monopolist
  2. Speculators
  3. Consumers

• Forward and spot markets
Basic setting

• Monopolist produces commodity of quality q=H or L
• Constant unit cost: $c_H = c > 0$, $c_L = 0$
• Continuum of consumers with unit mass; taste parameter $\theta$ uniformly distributed over $[0, \tilde{\theta}]$
• Unit demand; she gets $\theta q - p$ if she buys quality q and 0 otherwise.
• 2 markets at two different dates: forward market opens before production takes place; spot market opens one period later at the same time production takes place.
• Risk-neutral speculators in forward market and consumers in spot market. Common belief $\mu_0 \in (0,1)$
The model

• Stage1: monopolist privately observes $q$ and proposes $x$ forward contracts to speculators; posterior beliefs of speculators $\mu_f(x)$; speculators announce bids; $f$ equals expected spot price

• Stage2: monopolist sets price $p$; consumer’s posterior beliefs $\mu_s(x, p)$; consumers max expected utilities, given beliefs; monopolist satisfies consumers’ demand and settles forward position
The model, cont’d

• Monopolist’s strategy: \((x_q, p_q(x))\) for type-\(q\) monopolist

• Speculators’ strategy: \(f(x) = \mu_f(x)p_H(x) + (1 - \mu_f(x))p_L(x)\)

Stage 2:

• If \(\mu_s(x, p) = \mu\), the demand faced by the monopolist is
  
  \[ D(p, \mu) = \theta - \frac{p}{q(\mu)} \text{, where } q(\mu) = \mu H + (1 - \mu) L \]

• Monopolist’s problem: \(\max \pi_q(p, \mu) = (f - p)x + (p - c_q)D(p, \mu)\)

• Single crossing property: \(\frac{\partial \pi_L(p, \mu)}{\partial p} < \frac{\partial \pi_H(p, \mu)}{\partial p}\)

Stage 1:

• Monopolist and speculators anticipate \(p_q(x)\).

Monopolist’s profit \(\pi_q(p, \mu) = (f - p)x + (p - c_q)D(p, \mu)\)
Equilibrium definition

Perfect Bayesian equilibrium: \( S^* \equiv (x_L^*, x_H^*, p_L^*(\cdot), p_H^*(\cdot), \mu_f^*(\cdot), \mu_s^*(\cdot)) \) s.t.

1. For \( q=H,L, \quad p_q^*(x) \in \arg \max_{p} \pi_q(p, \mu_s^*(x, p)) \)

2. (i) If \( x_L^* \neq x_H^* \), then for \( q=H,L, \quad \mu_s^*(x_q^*, p_q(x_q^*)) = \mu_f^*(x_q^*) \)
   (ii) If \( x_L^* = x_H^* = x^* \) and \( p_L^*(x^*) \neq p_H^*(x^*) \), then \( \mu_s^*(x^*, p_L^*(x^*)) = 0 \)
   and \( \mu_s^*(x^*, p_H^*(x^*)) = 1 \).

3. For \( q=H,L, \quad x_q^* \in \arg \max_{x} \pi_q(x, \mu_f^*(x)) \)

4. If \( x_L^* \neq x_H^* \), then \( \mu_f^*(x_L^*) = 0 \) and \( \mu_f^*(x_H^*) = 1 \)
   If \( x_L^* = x_H^* \), then \( \mu_f^*(x_L^*) = \mu_f^*(x_H^*) = \mu_0 \)
Benchmark situation: complete information

Spot market:

• Monopolist’s problem: \( \max_{p}(f - p)x + (p - c_q)D_q(p) = \pi^f_q(p; x) + \pi^s_q(p) \)

\[ FOC \Rightarrow p^* \equiv p^m_q(x) = \frac{q(\theta - x) + c_q}{2} \]

Forward market:

• No arbitrage profit: \( f = p^m_q(x) \)

• Monopolist’s problem: \( \max_{x} \pi_q(x) = \pi^s_q(p^m_q(x)) \)

\[ FOC: \quad \frac{d\pi_q(x)}{dx} = -\frac{q}{2} x < 0 \]

\( \Rightarrow x^* = 0 \quad \text{So, monopolist is worse off trading forward.} \)
The spot market

• Type I: \( x_L^* \neq x_H^* \) induce complete information in the spot market: the monopolist can choose \( p_q^m(x_q^*) \).

• Type II: \( x_L^* = x_H^* \) (Milgrom and Roberts, 1986)
  Separating spot prices: \( p_H^*(x) \neq p_L^*(x) \)
  Type-L monopolist chooses \( P_L^m(x) \)

• Incentive compatibility:
  \[
  \pi_L(p_H^*(x),1) \leq \pi_L(p_L^m(0,x),0) \quad \pi_L(p_H^*(x),1) \leq \pi_L(p_L^m(x),0)
  \]

• Lemma1: Given \( x \in [0, \theta - c/L] \), there exists a continuum of separating prices such that \( p_L^* = p_L^m(x) \) and \( p_H^*(x) \) satisfies incentive compatibility constraints.
• Riley outcome \[ p_H^* (x) = \begin{cases} p_L (x) \\
 p^m_H (x) \end{cases} \]

• Proposition1: For any given \( x \), the unique pair of prices immune to the intuitive criteria is separating. These Riley prices are \( p_L^R (x) = p_L^m (x) \) and \( p_H^R (x) = \max \{ p_L (x), p_H^m (x) \} \)

• Lemma2: For all uninformative \( x \), \( dp_H^R (x) / dx < 0 \)

• Conclusions so far:

(1) the monopolist will charge \( p_q^m (x) \), if separating in forward market

(2) the monopolist will charge \( p_q^R (x) \), if pooling in forward market
The forward market

Suppose $x_L^* \neq x_H^*$, then

- $\mu_s(x_H^*, p_H(x_H^*)) = \mu_f(x_H^*) = 1$ and $\mu_s(x_L^*, p_L(x_L^*)) = \mu_f(x_L^*) = 0$
- In the spot market, monopolist charges $p_q^*(x_q^*) = p_q^m(x_q^*)$
- In the forward market, $f(x_q^*) = p_q^m(x_q^*)$. Monopolist makes zero arbitrage profit.
- $\pi_L^s(p_L^m(x), 0)$ is decreasing in $x \Rightarrow x_L^* = 0$ and $x_H^* > 0$
- Incentive compatibility constraints.

A1: $\mu_s(x_H^*, p) = 1$ and $\mu_s(x_L^*, p) = 0$ for all $p$.

Proposition 2: Under A1, there exists no forward separating equilibrium.

Conclusion: Information is not revealed in forward market.
The forward market, FPE

- Forward quantity: \( x^* \equiv x_L = x_H \)
- Speculators’ beliefs: \( \mu_f(x^*) = \mu_0 \)
- Monopolist charges \( p^R_q(x^*) \) in the spot market.
- Forward price: \( f(x^*) = \mu_0 p^R_H(x^*) + (1 - \mu_0) p^R_L(x^*) \)

Proposition 3: There exists a FPE in which the monopolist
(i) Sells pooling quantity \( x^* \) at the forward price \( f(x^*) \), and
(ii) Charges separating prices as stated in proposition 1.

- Forward sales are uninformative and spot prices reveal quality
Conclusions

• A monopolist will benefit from forward trading when he is able to signal the quality of his product by distorting his price upward.

• Forward trading can’t directly reveal the quality, but it reduces the signaling cost compared to the cost incurred without forward trading and contributes to more efficient quality signaling in the spot market.

• Total welfare is enhanced: cost of separation reduced for type-H monopolist and lower price paid by consumer.