Exercise 8 page 462 (chapter 2) (i) Only monopoly

- 2 periods $t = 1, 2$
- Consumer’s utility
  \[
  U = \begin{cases} 
  v + s_t - p & \text{if he buys} \\
  0 & \text{otherwise}
  \end{cases}
  \]
- with
  - $s_t = 1$: high quality;
  - $s_t = 0$: low quality.
- $c$: cost of production,
- $c'$: extra cost from supplying high quality,
- unknown quality,
- 2 types of firms:
  - fraction $x$: honest
  - fraction $(1 - x)$: dishonest ($c' > 0$)
- firm knows its type, not the consumers
- Assume: $\delta x > c'$ where $\delta$ discount factor.
Show that an equilibrium can be
- at $t = 1$, whatever its type, the firm charges
  \[ p_1 = v + 1 \]
  and produces high quality;
- at $t = 2$, the firm charges
  \[ p_2 = v + x \]
  and produces high quality only if cheaper.

- **Consumers**: deviation?
  given that the firm is following the equilibrium, each consumer is willing to pay
  - at $t = 1$, $p_1 = v + 1$ for high quality.
  - at $t = 2$, expected valuation is
    \[ x(v + 1) + (1 - x)v = v + x, \]
    the maximum a consumer is willing to pay.

- **Honest firm**:
  - By definition, always produces high quality.
  - Given consumer beliefs, honest firm cannot gain from charging higher prices.
• **Dishonest firm:**
  – at $t = 2$, find optimal to produce low quality.
  – At $t = 1$, they must find it optimal to pool with high quality firms to conceal their types.

\[
(v + 1 - c - c') + \delta(v + x - c) \geq (v + 1 - c) + \delta(v - c)
\]

\[
\Rightarrow \delta x > c'
\]

• If the firm produces a low quality at $t = 1$, consumers know it is dishonest and they are only willing to pay $v$ at $t = 2$.

• Dishonest firm cannot gain by charging higher prices given consumer beliefs.

• **Expected welfare:**
  – in this equilibrium: expected $CS = 0$;
  – Social welfare = Profit

\[
x[(v + 1 - c) + \delta(v + x - c)]
\]
\[
+(1 - x)[(v + 1 - c - c') + \delta(v + x - c)]
\]

\[
\Rightarrow (v - c)(1 + \delta) + x(1 + \delta) + (1 - x)(1 - c')
\]
Exercise 14 page 464 (chapter 3)

(i) Monopolist and competitive fringe. Optimal pricing scheme under a linear tariff? two-part tariff?

- demand: \( q = a - p \)
- \( MC = 0 \)
- competitive fringe: \( p_0 < a \).

- Linear tariff

\[
\begin{align*}
\max_p & \quad p(a - p) \\
\text{s.t.} & \quad p \leq p_0
\end{align*}
\]

thus
\[
p^m = \frac{a}{2}
\]

- 2 cases:
  a. \( p_0 > \frac{a}{2} \): the fringe price is not a binding constraint; \( p^* = \frac{a}{2} \).
  b. \( p_0 \leq \frac{a}{2} \): the fringe price prevents monopolist from charging \( p^m \); and thus \( p^* = p_0 \).
- Two-part tariff

\[
\begin{align*}
\max_p \{ p(a - p) + F \} \\
\text{s.t. } CS(p, F) \geq CS(p_0)
\end{align*}
\]

where

\[
CS(p, F) = \frac{(a - p)^2}{2} - F
\]

and

\[
CS(p_0) = \frac{(a - p_0)^2}{2}
\]

- Binding constraint

\[
F = \frac{(a - p)^2}{2} - \frac{(a - p_0)^2}{2}
\]

and the maximization program becomes

\[
\max_p \{ p(a - p) + \frac{(a - p)^2}{2} - \frac{(a - p_0)^2}{2} \}
\]

FOC gives

\[
p = 0
\]

and thus

\[
F = \frac{2ap_0 - p_0^2}{2}
\]
(ii) Two types of consumers; \( N = 1 \) consumers. Assume \( \frac{a_2}{2} > p_0 > \frac{a_1}{2} \).

- Under third degree price discrimination, monopolist charges 2 prices

\[
\begin{align*}
\max_{p_1, p_2} & \quad xp_1(a_1 - p_1) + (1 - x)p_2(a_2 - p_2) \\
\text{s.t. } & \quad p_i \leq p_0 \text{ for } i = 1, 2
\end{align*}
\]

- FOCs give

\[
\begin{align*}
p_1^* &= \frac{a_1}{2} \\
p_2^* &= p_0 < \frac{a_2}{2}
\end{align*}
\]

- Uniform pricing

\[
\begin{align*}
\max_p & \quad xp(a_1 - p) + (1 - x)p(a_2 - p) \\
\text{s.t. } & \quad p \leq p_0
\end{align*}
\]

FOC gives

\[
p^m = \frac{xa_1 + (1 - x)a_2}{2}
\]
• If $\frac{x a_1 + (1-x) a_2}{2} > p_0$, then
  
  \[ p^{\text{uni}} = p_0 \]

• Would uniform pricing increases welfare?

  \[
  W^d = C S^d_1 + C S^d_2 + \Pi^d \\
  W^{\text{uni}} = C S^{\text{uni}}_1 + C S^{\text{uni}}_2 + \Pi^{\text{uni}}
  \]

  \[ C S^d_1 > C S^{\text{uni}}_1 \text{ because pay } p_0 > p^*_1 \]
  \[ C S^d_2 = C S^{\text{uni}}_2 \text{ because pay the same price } p_0 \]
  \[ \Pi^d > \Pi^{\text{uni}} \text{ because under } d, p_1 = p_2 \text{ does not max } \Pi \]
  
  thus uniform pricing reduces welfare
  
  \[ W^{\text{uni}} < W^d \]

• Robinson-Smalensee result (price discrimination lowers welfare under linear demands if all types buy) does not apply here.

• With a fringe firm, the firm faces a non linear demand curve.
(iii) Second degree price discrimination (no fringe firms anymore but 2 types of consumers)

- Linear price

\[
\max_p \{ p [ x(a_1 - p) + (1 - x)(a_2 - p) ] \} \\
= \max_p \{ p [ a^e - p ] \}
\]

where

\[
a^e = xa_1 + (1 - x)a_2
\]

- Assume that \((a^e)^2 > (1 - x)(a_2)^2\) (both markets are served)

- FOC gives

\[
p = \frac{a^e}{2}
\]

- Two part tariff \(A + \hat{p}q\) where

\[
A = CS(\hat{p}) = \frac{(a_1 - \hat{p})^2}{2}
\]

low demand consumer do not get any surplus.
• Monopolist solves

\[ \text{Max}_p \{ CS(\bar{p}) + \bar{p}[a^e - \bar{p}] \} \]

FOC gives

\[ \tilde{p} = a^e - a_1 = (1 - x)(a_2 - a_1) \]

• And thus

\[ p - \tilde{p} = a_1 - \frac{a^e}{2} \]

Because we assume that the monopolist finds it optimal to serve both markets, then \( p < a_1 \Rightarrow a_1 - \frac{a^e}{2} > 0 \) and thus

\[ p > \tilde{p} \]