Chapter 7: Product Differentiation

A1. Firms meet only once in the market.
Relax A2. Products are differentiated.
A3. No capacity constraints.

Timing:
1. firms choose simultaneously their location in the product space,
2. given the location, price competition.

- Spatial-differentiation model
  – Linear city (Hotelling, 1929)
  – Circular city (Salop, 1979)
- Vertical differentiation model
  – Gabszwicz and Thisse (1979, 1980);
- Monopolistic competition (Chamberlin, 1933)
- Advertising and Informational product differentiation (Grossman and Shapiro, 1984)
1 Spatial Competition

1.1 The linear city (Hotelling, 1929)

- Linear city of length 1.
- Duopoly with same physical good.
- Consumers are distributed uniformly along the city, $N = 1$
- Quadratic transportation costs $t$ per unit of length.
- They consume either 0 or 1 unit of the good.
- If locations are given, what is the NE in price?

Price Competition

Maximal differentiation

- 2 shops are located at the 2 ends of the city, shop 1 is at $x = 0$ and of shop 2 is at $x = 1$. $c$ unit cost
- $p_1$ and $p_2$ are the prices charged by the 2 shops.
- Price of going to shop 1 for a consumer at $x$ is $p_1 + tx^2$.
- Price of going to shop 2 for a cons. at $x p_2 + t(1-x)^2$. 
• The utility of a consumer located at $x$ is

$$U = \begin{cases} \bar{s} - p_1 - tx^2 & \text{if he buys from shop 1} \\ \bar{s} - p_2 - t(1 - x)^2 & \text{if he buys from shop 2} \\ 0 & \text{otherwise} \end{cases}$$

• Assumption: prices are not too high (2 firms serve the market)

• Demands are

$$D_1(p_1, p_2) = \frac{p_2 - p_1 + t}{2t}$$

$$D_2(p_1, p_2) = \frac{p_1 - p_2 + t}{2t}$$

• and profit

$$\Pi^i(p_i, p_j) = (p_i - c)\frac{p_j - p_i + t}{2t}$$

• So each firm maximizes its profit and the FOC gives

$$p_i = \frac{p_j + t - c}{2}$$

for each firm $i$
• Prices are strategic complements:

\[ \frac{\partial^2 \Pi^i(p_i, p_j)}{\partial p_i \partial p_j} > 0. \]

• The Nash equilibrium in price is

\[ p_i^* = p_j^* = c + t \]

• The equilibrium profits are

\[ \Pi^1 = \Pi^2 = \frac{t}{2} \]

**Minimal differentiation**

• 2 shops are located at the same location \( x_0 \).
• \( p_1 \) and \( p_2 \) are the prices charged by the 2 shops.
• Price of going to shop 1 for a consumer at \( x \) is

\[ p_1 + t(x_o - x)^2. \]
• Price of going to shop 2 for a consumer at \( x \) is

\[ p_2 + t(x_o - x)^2. \]
• The consumers compare prices.... Bertrand competition
• Nash equilibrium in prices is
\[ p_i^* = p_j^* = c \]

• and the equilibrium profits are
\[ \Pi^1 = \Pi^2 = 0 \]
Different locations

• 2 shops are located at \( x = a \) and of shop 2 is at \( x = 1 - b \) where \( 1 - a - b \geq 0 \).

• If \( a = b = 0 \): maximal differentiation

• If \( a + b = 1 \): minimal differentiation

• \( p_1 \) and \( p_2 \) are the prices charged by the 2 shops.

• Price of going to shop 1 for a consumer at \( x \) is \( p_1 + t(x - a)^2 \).

• Price of going to shop 2 for a consumer at \( x \) is \( p_2 + t(1 - b - x)^2 \).

• Thus there exists an indifferent consumer located at \( \tilde{x} \)

\[
p_1 + t(\tilde{x} - a)^2 = p_2 + t(1 - b - \tilde{x})^2
\]

\[
\Rightarrow \tilde{x} = \frac{p_2 - p_1}{2(1 - a - b)} + \frac{1 - b + a}{2}
\]
and thus the demand for each firm is

\[ D_1(p_1, p_2) = a + \frac{1 - b - a}{2} + \frac{p_2 - p_1}{2(1 - a - b)} \]
\[ D_2(p_1, p_2) = b + \frac{1 - b - a}{2} + \frac{p_1 - p_2}{2(1 - a - b)} \]

The Nash equilibrium in price is

\[ p_1^*(a, b) = c + t(1 - a - b)(1 + \frac{a - b}{3}) \]
\[ p_2^*(a, b) = c + t(1 - a - b)(1 + \frac{b - a}{3}) \]

Profits are

\[ \Pi^1(a, b) = [p_1^*(a, b) - c]D_1(a, p_1^*(a, b), p_2^*(a, b)) \]
\[ \Pi^2(a, b) = [p_2^*(a, b) - c]D_2(b, p_1^*(a, b), p_2^*(a, b)) \]

Product Choice

Timing:
1. firms choose their location simultaneously
2. given the location, they simultaneously choose
prices
• Firm 1 chooses $a$ that maximizes $\Pi^1(a, b) \Rightarrow a(b)$
• Firm 2 chooses $b$ that maximizes $\Pi^2(a, b) \Rightarrow b(a)$
• and then $(a^*, b^*)$.
• What is the optimal choice of location?

$$
\frac{d\Pi^1(a, b)}{da} = \frac{\partial \Pi^1(a, b)}{\partial p_1} \frac{\partial p_1}{\partial a} + \frac{\partial \Pi^1(a, b)}{\partial p_2} \frac{\partial p_2}{\partial a}
$$
where $\frac{\partial \Pi^1(a, b)}{\partial p_1} \frac{\partial p_1}{\partial a} = 0$ (due to envelope theorem)

• We can rewrite

$$
\frac{d\Pi^1(a, b)}{da} = [p_1^*(a, b) - c] \left( \frac{\partial D_1(.)}{\partial a} + \frac{\partial D_2(.)}{\partial p_2} \frac{\partial p_2}{\partial a} \right)
$$

where $\frac{\partial D_1(.)}{\partial a} = \frac{3 - 5a - b}{6(1 - a - b)}$ Demand Effect (DE)

and $\frac{\partial D_2(.)}{\partial p_2} \frac{\partial p_2}{\partial a} = \frac{-2 + a}{3(1 - a - b)} < 0$ Strategic Effect (SE)
Thus, 
\[ \frac{d\Pi^1(a, b)}{da} = [p_1^*(a, b) - c]\left(\frac{-1 - 3a - b}{6(1 - a - b)}\right) < 0 \]

- As \( a \) decreases, \( \Pi^1(a, b) \) increases.

**Result** The Nash Equilibrium is such that there is **maximal differentiation**, i.e. \((a^* = 0, b^* = 0)\)

- 2 effects may work in opposite direction
  - **SE**<0 (always): price competition pushes firms to locate as far as possible.
  - **DE** can be >0 if \( a \leq 1/2 \), to increase **market share**, given prices, pushes firms toward the center.
    - But overall **SE**>**DE**, and they locate at the two extremes.

1.1.1 **Social planner**

- Minimizes the average transportation costs.
- Thus locates firms at 
\[ a^s = \frac{1}{4}, b^s = \frac{1}{4} \]
Result  Maximal differentiation yields too much product differentiation compared to what is socially optimal.
1.2 The circular city (Salop, 1979)

- circular city
- large number of identical potential firms
- Free entry condition
- consumers are located uniformly on a circle of perimeter equal to 1
- Density of unitary around the circle.
- Each consumer has a unit demand
- unit transportation cost
- gross surplus $\bar{\sigma}$.
- $f$ fixed cost of entry
- marginal cost is $c$
- Firm $i$’s profit is
  \[ \Pi^i = \begin{cases} 
  (p_i - c)D_i - f & \text{if entry} \\
  0 & \text{otherwise} 
  \end{cases} \]
- How many firms enter the market? (entry decision)
Timing: two-stage game

1. Potential entrants simultaneously choose whether or not to enter \((n)\). They are automatically located equidistant from one another on the circle.

2. Price competition given these locations.

**Price Competition**

- Equilibrium is such that all firms charge the same price.
- Firm \(i\) has only 2 real competitors, on the left and right.
- Firm \(i\) charges \(p_i\).
- Consumer indifferent is located at \(x \in (0, 1/n)\) from \(i\)

\[
p_i + tx = p + t\left(\frac{1}{n} - x\right)
\]

- Thus demand for \(i\) is

\[
D_i(p_i, p) = 2x = \frac{p + \frac{t}{n} - p_i}{t}
\]

- Firm \(i\) maximizes its profit

\[
(p_i - c)D_i(p_i, p) - f
\]
• Because of symmetry $p_i = p$, and the FOC gives

$$p = c + \frac{t}{n}$$

**How many firms?**

Because of free entry condition

$$(p_i - c)\frac{1}{n} - f = 0 \Rightarrow \frac{t}{n^2} - f = 0$$

which gives

$$n^* = \left(\frac{t}{f}\right)^{\frac{1}{2}}$$

and the price is

$$p^* = c + (tf)^{\frac{1}{2}}$$

• **Remark**: $p - c > 0$ but profit $= 0$....

• If $f$ increases, $n$ decreases, and $p - c$ increases.

• If $t$ increases, $n$ increases, and $p - c$ increases.

• If $f \to 0$, $n \to \infty$ and $p \to c$ (competitive market).

• Average transportation cost is

$$2n \int_0^{\frac{1}{2n}} xtdx = \frac{t}{4n}$$
• From a social viewpoint

\[ Min_n [nf + 2n \int_{0}^{\frac{1}{2n}} xtdx] \]

\[ \Rightarrow n^s = \frac{1}{2} n^* < n^* \]

**Result**  The market generates **too many firms**.

• Firms have too much an incentive to enter: incentive is stealing the business of other firms.

• Natural extensions:
  – location choice
  – sequential entry
  – brand proliferation
1.3 Maximal or minimal differentiation

• Spatial or vertical differentiation models make important prediction about business strategies

   Firms want to differentiate to soften price competition
   In some case: maximal product differentiation.

• Opposition to maximal differentiation
  – Be where the demand is (near the center of linear city)
  – Positive externalities between firms (many firms may locate near a source of raw materials for instance)
  – Absence of price competition (prices of ticket airline before deregulation)
2 Vertical differentiation


Timing: two-stage game
1. Simultaneously choice of quality.
2. Price competition given these qualities.

- Duopoly
- Each consumer consumes 0 or 1 unit of a good.
- $N = 1$ consumers.
- A consumer has the following preferences:

$$u = \begin{cases} 
\theta s - p & \text{if he buys the good of quality } s \text{ at price } p \\
0 & \text{if he does not buy}
\end{cases}$$

where $\theta > 0$ is a taste parameter.

- $\theta$ is uniformly distributed between $\underline{\theta}$ and $\overline{\theta} = \underline{\theta} + 1$; density $f(\theta) = 1$; cumulative distribution $F(\theta) \in [0, \infty)$
- $F(\theta)$: fraction of consumers with a taste parameter $< \theta$. 
• 2 qualities $s_2 > s_1$
• Quality differential: $\Delta s = s_2 - s_1$
• Unit cost of production: $c$
• Assumptions
  A1. $\bar{\theta} > 2\theta$ (insure demand for the two qualities)
  A2. $c + \frac{\theta - 2\theta}{3} \Delta s < \theta s_1$ (insure that $\frac{p_1}{s_1} < \theta$)
• Firms choose $p_1$ and $p_2$

Price competition (given qualities)
• There exists an indifferent consumer: $\tilde{\theta} = \frac{p_2 - p_1}{s_2 - s_1}$.
  – A consumer with $\theta \geq \tilde{\theta}$ buys the quality 2 ($\theta \geq \frac{p_2 - p_1}{s_2 - s_1}$). The proportion of consumers who will buy good of quality 2 is $F(\tilde{\theta}) - F(\theta)$.
  – A consumer with $\theta < \tilde{\theta}$ and $\theta \geq \theta > \frac{p_1}{s_1}$ buys low quality 1. So the proportion of consumers who will buy good of quality 1 is $F(\frac{p_2 - p_1}{s_2 - s_1}) - F(\theta)$.
  – if $\theta < \frac{p_1}{s_1}$ no purchase.
• Then demands are
\[ D_1(p_1, p_2) = \frac{p_2 - p_1}{\Delta s} - \theta \]
\[ D_2(p_1, p_2) = \bar{\theta} - \frac{p_2 - p_1}{\Delta s} \]

• Each firm maximizes its profit
\[ \max_{p_i} (p_i - c)D_i(p_i, p_j) \]

• The reaction functions are
\[ R_1(p_2) = p_1 = \frac{1}{2}[p_2 + c - \theta \Delta s] \]
\[ R_2(p_1) = p_2 = \frac{1}{2}[p_1 + c + \bar{\theta} \Delta s] \]

• The Nash equilibrium is
\[ p_1^* = c + \frac{\bar{\theta} - 2\theta}{3}\Delta s \quad (A2.) \]
\[ p_2^* = c + \frac{2\bar{\theta} - \theta}{3}\Delta s > p_1^* \]

• Demands are
\[ D_1^* = \frac{\bar{\theta} - 2\theta}{3} \quad (A1.) \]
\[ D_2^* = \frac{2\bar{\theta} - \theta}{3} \]
• and profits

\[ \Pi^1(s_1, s_2) = \frac{(\theta - 2\theta)^2}{9} \Delta s \]

\[ \Pi^2(s_1, s_2) = \frac{(2\theta - \theta)^2}{9} \Delta s \]

• High quality firm charges a higher price than low quality firm.

• High quality firm makes more profit.

**Choice of quality**

• Simultaneous choice of quality.

• Quality is costless.

• \( s_i \in [\underline{s}, \overline{s}] \), where \( \underline{s} \) and \( \overline{s} \) satisfy A2.

• Each firm chooses \( s_i \) that maximizes its profit \( \Pi^i(s_i, s_j) \)

• \( s_1 = s_2 \) cannot be an equilibrium, as they can do better if \( s_1 \neq s_2 \) (profit increases)

• If \( s_1 < s_2 \), as \( \partial \Pi^i(.) / \partial \Delta s > 0 \) both firms make more profit if more differentiation.

• Firm 1 reduces its quality towards \( \underline{s} \), firm 2 increases its quality towards \( \overline{s} \).
2 Nash equilibrium in quality: \( \{ s_1^* = \underline{s}, s_2^* = \overline{s} \} \) and \( \{ s_2^* = \underline{s}, s_1^* = \overline{s} \} \)

**Result**  The equilibrium is such that there is **maximal differentiation**.

⇒ Firms try to relax the price competition through product differentiation.

- If sequential entry – the first chooses \( \overline{s} \) and the second chooses \( \underline{s} \). (unique NE)
- But then race to be first...

- Even if quality is costless to produce, the low quality firm gains from reducing its quality to the minimum (because it softens price competition).

- **Difference with location model:**
  - if A1. does not hold anymore: only one firm makes profit in the market.
  - A “low” low quality cannot compete with a high quality,
  - A “high” low quality trigger tough price competition.
3 Monopolistic competition

- Chamberlin (1933)
- Dixit and Stiglitz (1977), Spence (1976)
- Monopolistic competition corresponds to the following industry configuration:
  - each firm faces a downward sloping demand,
  - each firm makes no profit (free entry condition),
  - the price of one firm does not affect the demand of any other firm
- No strategic aspect;
- Too many of too few products?

- First idea (Chamberlin (1933)): too many firms, each produces too little
- In fact not true (Dixit and Stiglitz (1977), Spence (1976))
- 2 effects work in opposite direction:
  - non appropriability of social surplus (too few products)
  - business stealing (too many products)
4 Advertising and Informational product differentiation

- Effect of ad on consumer demand and product differ.
- Ad conveys information on existence and price.
- Information issue can be solved, at some cost, through advertising (search good).
- Monopolistic competition (Butters, 1977)
- Oligopoly (Grossman and Shapiro, 1984)
- Socially too much or too little advertising?
- Firms are differentiated along two dimensions:
  – information,
  – location.
- What is the effect of advertising on the elasticity of individual demands and on the appropriability and business stealing effects?
- Linear-city model
- 2 firms locates at the two extremes
- consumers are distributed uniformly
• $s$ gross surplus
• $t$ transportation cost
• The only way to reach consumers is to send ads randomly.
• Advertising: information about the product, and price.
  – If a consumer receives no ads, he does not buy.
  – If he receives 1 ad he buys from the firm.
  – If he receives 2 ads he chooses the closest firm.
• Fraction of consumers who receive an ad from $i$ is $\phi_i$, $i = 1, 2$
• Consumers located along the segment have equal chances of receiving a given ad.
• Cost of reaching fraction $\phi_i$ is $A(\phi_i) = a\phi_i^2/2$
• Firms choose $p_1$ and $p_2$
• Firms compete for the “common demand”
• Demand for 1 is
  $$D_1 = \phi_1[(1 - \phi_2) + \phi_2\frac{p_2 - p_1 + t}{2t}]$$
  – A fraction $1 - \phi_2$ does not receive ad from 2

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– A fraction \( \phi_2 \) receives at least one ad from 2

- Elasticity of demand at price \( p_1 = p_2 = p \) and \( \phi_1 = \phi_2 = \phi \) is

\[
\varepsilon_1 = -\frac{p_1}{D_1} \frac{\partial D_1}{\partial p_1} = \frac{\phi p}{(2 - \phi)t}
\]

– it is increasing with \( \phi \), so with ad.

- Consider that firms simultaneously choose prices and levels of ads.

- Firm 1

\[
\underset{p_1, \phi_1}{\text{Max}} (p_1 - c) \phi_1 [(1 - \phi_2) + \phi_2 \frac{p_2 - p_1 + t}{2t}] - A(\phi_1)
\]

- FOC are

\[
p_1 = \frac{p_2 + c + t}{2} + \frac{1 - \phi_2 + t}{\phi_2 t}
\]

\[
\phi_1 = \frac{1}{a} (p_1 - c) [(1 - \phi_2) + \frac{\phi_2 (p_2 - p_1 + t)}{2t}]
\]

- Symmetric game

\[
p_1^* = p_2^* = p^*
\]

\[
\phi_1^* = \phi_2^* = \phi^*
\]
• Assume $a \geq t/2$
  
  $p^* = c + (2at)^{\frac{1}{2}}$

  $\phi^* = \frac{2}{1 + (\frac{2a}{t})^{\frac{1}{2}}}$

  $\Pi^1 = \Pi^2 = \frac{2a}{(1 + (\frac{2a}{t})^{\frac{1}{2}})^2}$

• $p^* > c + t$ (price under full information)
  – the price increases with $t$, and with $a$.

• The lower the advertising cost, and the higher the horizontal differentiation, the more the firms advertise.

• Profits are
  – increasing with $t$
  – increasing with $a$ because of 2 effects. If $a$ increases,

  * DE: it induces profit to decrease,

  * SE: it decreases ad, and thus increases informational PD. Firm raises the price.

• The market level of ad can be greater or smaller than the socially optimal level of ad.
– non appropriability of SS (low incentive to ad)
– business stealing (excessive advertising)