War of attrition

- 2 firms are fighting for a price
- Value: $\Pi_m$
- Cost: $f$
- Discount rate: $r$
- Decision: when to stop; dropping date
- Each firm drops out according to a Poisson process
- $x_i$: (instant) proba of stopping for $i = 1, 2$
- $\tau_i$: date at which firm $i$ drops

$$\tau_i \sim Exp(x_i)$$

$$f_i(t) = x_i e^{-tx_i}, \ i = 1, 2$$

$$E\tau_i = 1/x_i$$

$$\tau = \min\{\tau_1, \tau_2\} \sim Exp(x_1 + x_2)$$
• Expected profit of firm 1

\[ E\Pi_1 = E\int_0^\tau (-f)e^{-rt}dt + E\int_\tau^{+\infty} p(\Pi_m - f)e^{-rt}dt \]
\[ = -f\frac{1 - Ee^{-r\tau}}{r} + \frac{\Pi_m - f}{r}pEe^{-r\tau} \]

• \( Ee^{-r\tau} \)?

• \( p \)?

\[ Ee^{-r\tau} = \int_0^{+\infty} e^{-rt}f(t)dt \]
\[ = \frac{x_1 + x_2}{x_1 + x_2 + r} \]

\[ p = \text{prob}(\tau_2 < \tau_1) \]
\[ = \int_0^\infty \int_t^\infty f_2(t)f_1(u)du dt \]
\[ = \frac{x_2}{x_1 + x_2} \]
Thus

\[ E\Pi_1 = \frac{-f + \frac{\Pi_m - f}{r}x_2}{x_1 + x_2 + r} \]

- Symmetric equilibrium: \( x_1 = x_2 \) and

\[ E\Pi_1 = \frac{-f + \frac{\Pi_m - f}{r}x}{2x + r} \]

- At each date, a firm is indifferent between dropping or not

\[ \frac{-f + \frac{\Pi_m - f}{r}x}{2x + r} = 0 \]

\[ x = \frac{fr}{\Pi_m - f} \]