ECON 415
Firms, Markets and Industry Structure

Homework assignment # 5: answers

Exercise 1 A discriminating monopoly sells in two markets. Assume that no arbitrage is possible. The demand curve in market 1 is given by \( p_1 = a - bq_1 \). The demand curve in market 2 is given by \( p_2 = a - dq_2 \), where \( a, b \) and \( d \) are all positive parameters. We denote the monopoly’s aggregate production by \( Q \) where \( Q = q_1 + q_2 \). Assume that the monopolist produces each unit at a cost of \( c \). Answer the following questions:

1. Calculate the profit-maximizing output level that the monopoly sells in each market. Calculate the price charged in each market. Calculate the monopoly’s profit level. What is the condition on \( c \)? Determine the total Deadweight loss.

2. Suppose that markets 1 and 2 are now open, and all consumers are free to trade and to transfer the good costlessly between the markets. Thus, the monopoly cannot longer price discriminate and has to charge a uniform price denoted by \( p \), \( p = p_1 = p_2 \). Find the profit maximizing value of \( p \). Determine the new total Deadweight loss. Compare the two deadweight.

1) Price discrimination is allowed. In each market, the monopolist sets \( MR = MC \).

In market 1

\[
TR_1 = p_1 q_1 = (a - bq_1)q_1
\]

\[
MR_1 = a - 2bq_1
\]

Therefore,

\[
MR_1 = MC
\]

\[
a - 2bq_1 = c
\]

and

\[
q_1^* = \frac{a - c}{2b}
\]

and the price charged in market 1 is thus

\[
p_1^* = \frac{a + c}{2}
\]

The profit in market 1 is

\[
\Pi_1^* = (p_1 - c)q_1
\]

\[
= \frac{(a - c)^2}{4b}
\]
In market 2,

\[ TR_2 = p_2q_2 = (a - dq_2)q_2 \]

\[ MR_2 = a - 2dq_2 \]

Therefore,

\[ MR_2 = MC \]

\[ a - 2dq_2 = c \]

and

\[ q^m_2 = \frac{a - c}{2d} \]

The price charged in market 2 is thus

\[ p^m_2 = \frac{a + c}{2} \]

The profit in market 1 is

\[ \Pi^m_2 = (p_2 - c)q_2 \]

\[ = \frac{(a - c)^2}{4d} \]

Condition on \( c \): \( a - c > 0 \) to insure positive quantities.

What are the DWL? (graph)

In market 1,

\[ DWL_1 = \frac{1}{2} \left( \frac{a + c}{2} - c \right) \left( \frac{a - c}{b} - \frac{a - c}{2b} \right) \]

\[ = \frac{(a - c)^2}{8b} \]

In market 2,

\[ DWL_2 = \frac{1}{2} \left( \frac{a + c}{2} - c \right) \left( \frac{a - c}{d} - \frac{a - c}{2d} \right) \]

\[ = \frac{(a - c)^2}{8d} \]

The total DWL is

\[ DWL_1 + DWL_2 = \frac{(a - c)^2}{8b} + \frac{(a - c)^2}{8d} \]

\[ = \frac{(a - c)^2}{8bd}(d + b) \]

2) Uniform pricing.

Demand in market 1 is

\[ q_1 = \frac{a - p_1}{b} \]
Demand in market 2 is 
\[ q_2 = \frac{a - p_2}{d} \]

Aggregate demand is 
\[ Q = q_1 + q_2 = \frac{a - p}{b} + \frac{a - p}{d} \]
\[ Q = \frac{(b + d)}{bd}(a - p) \]

The inverse demand function is 
\[ p = a - \frac{bd}{b + d}Q \]

Therefore, \( TR \) and \( MR \) are 
\[ TR = pQ = (a - \frac{bd}{b + d}Q)Q \]
\[ MR = a - 2\frac{bd}{b + d}Q \]

Set \( MR \) equal to \( MC \) 
\[ a - 2\frac{bd}{b + d}Q = c \]
\[ Q_u = \frac{(b + d)}{2bd}(a - c) \]

and price is 
\[ p_u = \frac{a + c}{2} \]

The new \( DWL \) is 
\[ DWL = \frac{1}{2}(p_u - c)(\frac{b + d}{bd}(a - c) - Q_u) \]
\[ = \frac{(a - c)^2}{8bd}(d + b) \]

Problem 3 page 131
This is an example of second degree price discrimination, involving multipart pricing. Since the consumers are heterogeneous and the company cannot identify who is who, it offers multiple pricing policies and let the consumers self select.

Problem 4 page 132
a) if the club owner cannot price discriminate, she will consider the aggregate demand, which is 
\[ Q = (18 - 3P) + (10 - 2P) \]
\[ = 28 - 5P \]
The corresponding inverse demand function is
\[ P = \frac{28 - Q}{5}, \]
the total revenue is
\[ TR = P(Q)Q = \frac{28 - Q}{5}Q \]
and the marginal revenue is
\[ MR = \frac{dTP}{dQ} = \frac{28}{5} - \frac{2}{5}Q. \]
Equate MR with MC, that is
\[ \frac{28}{5} - \frac{2}{5}Q = 2 \]
\[ Q = 9 \]
and therefore
\[ P = \frac{19}{5} \]

Her profit without price discrimination is
\[ \Pi = (P - 2)Q = \frac{9^2}{5} = 16.2 \]

b) If the owner of the club can price discriminate, she will equate \( MR \) and \( MC \) for each group. This is
\[ 6 - \frac{2}{3}Q^S = 2 \Rightarrow Q^S = 6 \text{ and } P^S = 4 \]
\[ 5 - Q^A = 2 \Rightarrow Q^A = 3 \text{ and } P^A = 3.5 \]
Hence, total profit with price discrimination is
\[ \Pi^S + \Pi^A = (4 - 2) \times 6 + (3.5 - 2) \times 3 = 16.5 \]
Therefore her profit is higher with price discrimination.

c) In this scenario, she can practice two-part pricing. For each group, the number of token will be equal to quantity demander at price $2, which is the \( MC \) of a drink.

Number of tokens for students: \( 18 - 3 \times 2 = 12 \), and for adults: \( 10 - 2 \times 2 = 6 \).
Now for each group, the cover charge should equal the \( CS \) received at the given number of tokens. That is, cover charge for a student
\[ \frac{1}{2}(6 - 2) \times 12 + 2 \times 12 = 48 \]
and cover charge for an adult
\[ \frac{1}{2}(5 - 2) \times 6 + 2 \times 6 = 21 \]
Therefore her profits are
\[ \Pi = TR - TC = \frac{1}{2}(6 - 2) \times 12 + 2 \times 12 + \frac{1}{2}(5 - 2) \times 6 + 2 \times 6 - 2 \times 12 = 33 \]