Exercise 1: Cournot Equilibrium. The demand is \( P = 100 - 2Q \) where \( Q = Q_1 + Q_2 \), \( Q_1 \) is the number of bottled waters produces and sold per month by Sweetwater, and \( Q_2 \) is the number of bottled waters produced and sold per month by Goodwater. \( MC = 2 \).

The program of firm 1 is

\[
\max_{Q_1} \{ \Pi_1(Q_1, Q_2) = P(Q_1, Q_2)Q_1 - 2Q_1 \} \\
\Rightarrow \max_{Q_1} (100 - 2(Q_1 + Q_2))Q_1 - 2Q_1
\]

The FOC

\[
\frac{\partial \Pi_1(Q_1, Q_2)}{\partial Q_1} = 100 - 4Q_1 - 2Q_2 - 2 = 0
\]

Solution is:

\[
Q_1 = \frac{49}{2} - \frac{1}{2}Q_2 \tag{1}
\]

This is the best response of firm 1 to quantity \( Q_2 \). Because firms are symmetric, same best response function for 2:

\[
Q_2 = \frac{49}{2} - \frac{1}{2}Q_1 \tag{2}
\]

Then plug (1) in (2) gives:

\[
Q_2^c = \frac{49}{3}
\]

Thus

\[
Q_1^c = \frac{49}{3}
\]

The price is

\[
P^c = 100 - 2Q = 100 - 4 \times \frac{49}{3} = 34.667
\]

The profit of each firm is

\[
\Pi_1(Q_1^c, Q_2^c) = P^cQ_1^c - 2Q_1^c = 533.56.
\]

Same profit for firm 2.

Exercise 2:

1. (a) Prisoners’ Dilemma.
   (b) The Nash Equilibrium of the game is (Left, Left) and the associated payoffs are (50, 50).
   (c) If they decide to collude they can both get a higher payoff (70, 70). So it would be collectively rational to choose (right, right) but this is not an equilibrium. Each player has too strong an incentive to cheat and deviates from this strategy.
Problem 4 page 225 (Pepall, Richards and Norman, 2005)

To determine my best response function, I equate my marginal revenue with my marginal cost

\[ 200 - 4Q_1 - 2Q_2 = 8 \]

\[ Q_1 = \frac{1}{4}(200 - 2Q_2 - 8) \]

Since my rival and I are identical,

\[ Q_1^* = Q_2^* = Q^* \]

\[ \frac{1}{4}(192 - 2Q^*) = Q^* \]

\[ Q^* = 32 \]

Price is

\[ P = 72 \]

and my profit is

\[ \Pi = (72 - 8)32 - 1500 = 548 \]

Note that because of the fixed cost, there are two other asymmetric equilibria. At each, one form produces its monopoly output and the other produces none. We assume that in this case, a symmetric equilibrium is more reasonable than an asymmetric equilibrium.

Problem 5 page 225-226 (Pepall, Richards and Norman, 2005)

Assume that I am firm 1. To determine my best response function I equate my \( MR \) with my \( MC \),

\[ 290 - \frac{2}{3}Q_1 - \frac{1}{3}(\sum_{i=2}^{14} Q_i) = 50 \]

\[ Q_1 = \frac{3}{2}(240 - \frac{1}{3}(\sum_{i=2}^{14} Q_i)) \]

Since my rival and I are identical, \( Q_1^* = Q^* \) for all \( i \). Therefore,

\[ Q^* = \frac{3}{2}(240 - \frac{1}{3}(\sum_{i=2}^{14} Q^*)) \]

\[ Q^* = 48 \]

and the price is

\[ P = 290 - \frac{1}{3}(14)(48) = 66 \]

My profit is

\[ \Pi = (66 - 50)48 - 200 = 568 \]