Problem 1 page 244 (Pepall, Richards and Norman, 2005)
a) At equilibrium
\[ p_1^* = p_2^* = 10 \]
assuming that if both firms charge the same price, then the consumers buy from the lower priced firm.

b) The higher cost firm makes zero profit, whereas the lower cost firm’s profit is
\[ (p_1^* - c_1)q_1 = (10 - 6)(5000 - 200 \times 10) = 12000 \]
c) No this outcome is not efficient.

Problem 2 pages 244
a) Note that the inverse demand function is
\[ P = 30 - \frac{Q}{3}. \]
The Cournot quantities are
\[ q_1^* = \frac{30 - 2 \times 15 + 10}{3 \times \frac{1}{3}} = 10 \]
\[ q_2^* = \frac{30 - 2 \times 10 + 15}{3 \times \frac{1}{3}} = 25 \]
The market price is
\[ P = 30 - \frac{10 + 25}{3} = 18.3333 \]
Profit of firm 1
\[ (18.3333 - 15) \times 10 = 33.333 \]
Profit of firm 2
\[ (18.3333 - 10) \times 25 = 208.25 \]

b) At a Bertrand equilibrium, \( p_1^* = p_2^* = 15 \), assuming that if both firms charge the same price, the the consumers buy from the lower priced firm.

Total sale = 90 - 3 \times 15 = 45.
Firm 1 sells zero and earns zero profit. Firm 2 sells 45 units and earns \((15 - 10) \times 45 = 225\)

c) Yes, the outcome will change. The two lower cost firms will charge $10 and share the market equally.
d) The answer may change depending on how much premium the consumers are willing to pay for the green balls endorsed by Tiger Woods.

**Problem 1 page 263**

a) Firm 2 chooses its quantity to maximize

\[ \Pi_2 = (1000 - 4q_1 - 4q_2)q_2 - 20q_2 \]

FOC is

\[ \frac{d\Pi_2}{dq_2} = 0 \]
\[ 1000 - 4q_1 - 8q_2 - 20 = 0 \]

and therefore the best response of firm 2 is

\[ q_2(q_1) = \frac{980 - 4q_1}{8} \]

Now firm 1 chooses its quantity to maximize

\[ \Pi_1 = (1000 - 4q_1 - 4q_2(q_1))q_1 - 20q_1 \]
\[ \Pi_1 = (1000 - 4q_1 - 4\frac{980 - 4q_1}{8})q_1 - 20q_1 \]
\[ \Pi_1 = \frac{1}{2}(980 - 4q_1)q_1 \]

FOC is

\[ \frac{d\Pi_1}{dq_1} = 0 \]
\[ \frac{980}{2} - 4q_1 = 0 \]

and therefore

\[ q_1^* = \frac{980}{8} = 122.5 \]

Firm 2 produces

\[ q_2^* = \frac{980 - 4 \times 122.5}{8} = 61.25 \]

b) There is no non-negative \( c \) such that the leader and the follower have the same market share. To see, consider \( c = 0 \). The leader’s quantity is 120, whereas the follower’s quantity is less than 120. As \( c \) increases, the market share of the leader goes up and the market share of the follower goes down.

**Problem 3 pages 264**

a)
b) The strategy of splitting the money is never an equilibrium since once the game reaches the node where player 2 has to decide, the optimal strategy for player 2 is to take the entire $4. Because player 1 knows this will be the outcome when player 2 has to choose, player 1 will always choose the strategy "grab" and the outcome will be "Grab" with player 1 getting $1 and player 2 getting nothing.
c) It is clear that in the third stage player 1 will choose to keep all the money. Thus we can eliminate this choice from the tree, and consider the new game with the final node removed. This will give

```
          Player 1
          /   \
     wait  grab
          /     /
        Player 2 Player 2
   Take it all share wait
   0  4          2  2          8  0
```

It is now obvious that player 2 will choose to take it all when he has to choose. Thus we can eliminate this node from the tree and replace it with the payoffs to both players when player 2 chooses to take it all. This will give

```
          Player 1
          /   \
     wait  grab
          /     /
        Player 2 Player 2
   0  4          1  0
```

It is now clear that player 1 will grab the money at the initial node and the final payoff will be (1,0).