ECON 415/515
Economics of Imperfect Competition, Antitrust, and Regulated Industries

Second Mid Term

**Exercise 1:** 2 animals 1 and 2. Strategy space: \( S = \{A, P\} \) Payoffs are such that:

- Each prefers to be aggressive if its opponent is passive, \( \pi_1(A, P) > \pi_1(P, P) \) and \( \pi_2(P, A) > \pi_2(P, P) \).
- and passive if its opponent is aggressive, \( \pi_1(P, A) > \pi_1(A, A) \) and \( \pi_2(A, P) > \pi_2(P, P) \).
- When it is passive, it prefers the outcome when its opponent is passive to that in which its opponent is aggressive, \( \pi_1(P, P) > \pi_1(P, A) \) and \( \pi_2(P, P) > \pi_2(A, P) \).

Thus an example of matrix is

\[
\begin{array}{ccc|cc}
1 & 2 & P & A \\
\hline
P & 5, 5 & 4, 10 \\
A & 10, 4 & 2, 2 \\
\end{array}
\]

2 NE: \((A, P)\), and \((P, A)\).

**Exercise 2:** Cournot model. Firm i maximizes

\[
\max q_1 (\alpha - q_1 - q_2)q_1 - cq_1
\]

and firm 2

\[
\max q_2 (\alpha - q_1 - q_2)q_2 - dq_2
\]

Best response functions are

\[
q_1(q_2) = \frac{\alpha - c - q_2}{2}
\]

\[
q_2(q_1) = \frac{\alpha - d - q_1}{2}
\]

Thus Cournot equilibrium is

\[
q_1^c = \frac{\alpha - 2c + d}{3}
\]

\[
q_2^c = \frac{\alpha - 2d + c}{3},
\]

and the price is

\[
p^c = \frac{\alpha + d + c}{3}.
\]

**Exercise 4:** 2 period: firms 2 and 3 choose their quantities simultaneously. Firm 2 solves

\[
\max q_2 (a - q_1 - q_2 - q_3)q_2 - cq_2
\]
and firm 3 solves

\[ Max_{q_3}(a - q_1 - q_2 - q_3)q_3 - cq_3 \]

for any given quantity \( q_1 \). Denote \( q_1 = \overline{q_1} \). Best responses functions are

\[
\begin{align*}
q_2(q_3) &= \frac{a - c - \overline{q_1} - q_3}{2} \\
q_3(q_2) &= \frac{a - c - \overline{q_1} - q_2}{2}
\end{align*}
\]

The Nash equilibrium is

\[
\begin{align*}
q_2 &= \frac{a - c - \overline{q_1}}{3} \\
q_3 &= \frac{a - c - \overline{q_1}}{3}
\end{align*}
\]

In the first period, firm 1 solves

\[ Max_{q_2}(a - q_1 - \frac{a - c - q_1}{3} - \frac{a - c - q_1}{3})q_1 - cq_1 \]

that gives

\[ q_1 = \frac{a - c}{2} \]

Thus the equilibrium is

\[
\begin{align*}
q_1 &= \frac{a - c}{2} \\
q_2 &= \frac{a - c}{6} \\
q_3 &= \frac{a - c}{6}
\end{align*}
\]