Instructions. To obtain credit, you must give arguments to support your answer. The numbers in brackets are the numbers of points the questions are worth.

ECON 415: solve exercises 1 [35], 2 [30] and 3 [35].
ECON 515: solve exercises 1 [30], 2 [20], 3 [30] and 4 [20].

Exercise 1: Two animals fight over some prey. Each can be passive or aggressive. Each prefers to be aggressive if its opponent is passive, and passive if its opponent is aggressive. When it is passive, it prefers the outcome when its opponent is passive to that in which its opponent is aggressive. Formulate this situation as a strategic game and find all its Nash equilibria in pure and mixed strategies.

Exercise 2: In a Cournot’s model of oligopoly with two firms, the payoff function of each firm $i$ is

$$\pi_i(q_1, q_2) = q_i P(q_1 + q_2) - C_i(q_i)$$

where $P(.)$ is the inverse demand function and $C_i(q_i)$ is the cost function of firm $i$. Find the Nash equilibrium (equilibria?) of the associated strategic game when the inverse demand function is linear, given $P(Q) = \alpha - Q$ if $Q \leq \alpha$ and $P(Q) = 0$ if $Q > \alpha$, and the cost function of firm 1 is $C_1(q_1) = cq_1$ and the cost function of firm 2 is $C_2(q_2) = dq_2$. (carefully present the step in your argument: determine the optimization program for each firm, derive the best response function for each firm, draw the best response functions.)

Exercise 3: Three oligopolists operate in a market with inverse demand given by $P(Q) = a - Q$, where $Q = q_1 + q_2 + q_3$ and $q_i$ is the quantity produced by firm $i$. Each firm has a constant marginal cost of production, $c$, and no fixed cost. The firms choose their quantities as follows: (1) firm 1 chooses $q_1 \geq 0$; (2) firms 2 and 3 observes $q_1$ and then simultaneously choose $q_2$ and $q_3$, respectively. What is the subgame-perfect outcome?