Suppose the monopolist offers 2 entry fees, unit price combinations. One is targeted towards the low demanders and the other is targeted towards the high demanders. Let \((F_1, p_1)\) be the entry fee and unit price combination paid by the low demanders and \((F_2, p_2)\) be the entry fee and unit price combination paid by the high demanders. Now from the discussion in the text, it follows that \(F_1 = \frac{1}{2}(12 - p_1)^2\). Also, the monopolist needs to adjust \(p_2\) so that the high demanders do not buy the combination intended for the low demanders. Therefore it follows that

\[
\frac{1}{2}(16 - p_2)^2 - F_2 = \frac{1}{2}(16 - p_1)^2 - \frac{1}{2}(12 - p_1)^2
\]

\[
F_2 = \frac{1}{2}(16 - p_2)^2 - \frac{1}{2}(16 - p_1)^2 + \frac{1}{2}(12 - p_1)^2
\]

Thus the profit of the monopolist is given by

\[
\Pi_1 = N_h[(p_2 - 4)(16 - p_2) - F_2] + N_l[(p_1 - 4)(16 - p_1) + \frac{1}{2}(12 - p_1)^2]
\]

Differentiate \(\Pi_1\) with respect to \(p_1\) and \(p_2\), and equate each expression to zero to obtain

\[
p_1 = 4 \left[1 + \frac{N_h}{N_l}\right]
\]

\[
p_2 = 4
\]

Substituting the optimal prices into the profit expression, we obtain the maximum profit the monopolist can earn by serving both groups,

\[
\Pi_1 = N_h\left[72 - \frac{1}{2}(12 - 4 \frac{N_h}{N_l})^2 + \frac{1}{2}(8 - 4 \frac{N_h}{N_l})^2\right] + N_l\left[4 \frac{N_h}{N_l}(8 - 4 \frac{N_h}{N_l}) + \frac{1}{2}(8 - 4 \frac{N_h}{N_l})^2\right]
\]

If the monopolist serves only high demanders only, its profit will be

\[
\Pi_2 = N_h(16 - 4)^2 = 72N_h
\]

Therefore the monopolist serves both groups if

\[
\Pi_1 \geq \Pi_2
\]

It can be verified (use a program such as maple) that \(\Pi_1 \geq \Pi_2\) if \(N_h \geq N_l\).
There is a unique Nash Equilibrium, where firm 1 chooses A and firm 2 chooses C.

b) Note that A is a dominant strategy for firm 1. Therefore, even if firm 1 can commit before firm 2, the answer does not change.

• Chapters 9, 10 and 11:

a. Bertrand paradox: each firm sets the price at MC, i.e., \( p = 2 \). Quantities are \( Q_1 = Q_2 = 2 \) as the aggregate quantity is \( Q = 5 - P/2 = 4 \).

b. Cournot Equilibrium

Firm 1 solves

\[
Max_{Q_1} \Pi_1 = (10 - 2Q_1 - 2Q_2 - 2)Q_1
\]

FOC

\[
10 - 4Q_1 - 2Q_2 - 2 = 0
\]

and therefore the best response function of firm 1 is

\[
Q_1(Q_2) = \frac{8 - 2Q_2}{4} = 2 - \frac{1}{2}Q_2
\]

do the same thing for firm 2 and therefore the best response function of firm 2 is

\[
Q_2(Q_1) = 2 - \frac{1}{2}Q_1
\]

Solve the system (+ graph) gives \( Q_1 = Q_2 = \frac{4}{3} \)

The price is

\[
P = 10 - 2\frac{4}{3} - 2\frac{4}{3} = \frac{14}{3} = 4.6667
\]

Profit for each firm is

\[
\Pi_1 = \Pi_2 = (\frac{14}{3} - 2)\frac{4}{3} = \frac{32}{9} = 3.5556
\]

c. Stackelberg

Backward induction. Firm 2 chooses its best response function

\[
Q_2(Q_1) = 2 - \frac{1}{2}Q_1
\]

Plug that in the profit of firm 1

\[
\Pi_1 = (10 - 2Q_1 - 2(2 - \frac{1}{2}Q_1) - 2)Q_1
\]

and solve the maximization program

\[
Max_{Q_1} \Pi_1 = (4 - Q_1)Q_1
\]

FOC gives

\[
4 - 2Q_1 = 0
\]
and therefore

\[ Q_1 = 2 \]

and

\[ Q_2 = 1 \]

Price is

\[ P = 10 - 2 - 4 = 4 \]

and profit for each firm is

\[ \Pi_1 = (4 - 2)2 = 4 \]
\[ \Pi_2 = (4 - 2)1 = 2 \]

**d.** Compare all of the results and comment...