Intermediate Microeconomics 301
Second Mid-Term
Elements of answer

Exercise 1 [20]: Capital and labor to be perfect complements:

\[ q = f(L, K) = \min\{L, K\} \]

Constant return-to-scale? For \( k > 1 \), \( f(kL, kK) = k \min\{L, K\} \).

Exercise 2 [20]: The firm chooses \( L \) and \( K \) such that

\[ \begin{align*}
\min_{L,K} & \quad 2L + K \\
\text{s.t.} & \quad q = 2KL \\
\end{align*} \]

FOC gives

\[ 2 - \frac{q}{2L^2} = 0 \quad \text{and thus} \quad L^* = \frac{\sqrt{q}}{2} \]

Furthermore

\[ K^* = \frac{q}{2L^*} = \sqrt{q} \]

And the cost function is

\[ C(q) = 2L^* + K^* = 2\sqrt{q} \]

If now \( K = 2 \), the cost function is

\[ C = 2L + 2 \]

where \( L = \frac{q}{2K} \) thus

\[ C(q) = \frac{1}{2}q + 2 \]

Exercise 3 [30]: \( C(q) = 125 + q^2, \ MC = 2q, \ AVC = q \)

1. The individual supply of each firm is \( q \) such that \( p = MC = 2q \) thus \( q = \frac{p}{2} \) for \( p > \min AVC = 0 \).

2. If \( p = $40 \), the profit-maximizing output level for each firm is \( q = 20 \), the total revenue is: \( TR = pq = 40 \times 20 = 800 \) and the profit is \( \Pi = TR - TC = 800 - 125 - 20^2 = 275 \).

Exercise 4 [30]: The inverse demand function is \( p = 60 - Q \) and the supply function is \( Q = p \),

1. The initial equilibrium is such that \( S = D \) and thus \( 60 - Q = Q \), i.e., \( Q^* = 30 \) and \( p^* = 30 \).

2. At the equilibrium price, the consumer’s surplus is \( CS = \frac{60-30}{2} \times 30 = 450 \). The producer’s surplus is \( PS = \frac{30 \times 30}{2} = 450 \) thus the welfare is \( W = 900 \).

3. A tax of $2 is imposed, thus the price paid by consumers becomes \( p' = 31 \) (and producers get \( p = 29 \)) the quantity is \( Q = 29 \) and thus the new CS is \( CS' = \frac{60 - 31}{2} \times 29 = 420.5 \), the producer surplus is \( PS' = \frac{29 \times 29}{2} = 420.5 \). The amount of the tax collected is \( T = 2 \times 29 = 58 \). Thus the welfare is now \( W' = 420.5 + 420.5 + 58 = 899 < 900 \). Or \( \Delta = W' - W = 1 \).