1. Write the equation for the marginal product of capital for each of the following production functions:
   
   a. \( MP_K = 3 \)
   
   b. \( MP_K = 2K^{-0.5}L \)
   
   c. \( MP_K = \frac{5}{2}K^{-0.5}L \)

2. Draw a graph showing a set of isoquants that depict capital and labor to be perfect complements (not substitutable at all) in a production function that exhibits constant return to scale. Be sure to label the input and output levels on the isoquants.

   The isoquants are “L” shaped, indicating perfect complementarity, and for every doubling of inputs, output also doubles. See Figure 1.

3. Two firms currently produce the goods \( q_1 \) and \( q_2 \) separately. Their cost functions are \( C(q_1) = 250 + q_1 \) and \( C(q_2) = 350 + 2q_2 \). By merging, they can produce the two goods jointly with
costs described by the function $C(q_1, q_2) = 450 + q_1 + q_2$. Are there scope economies in this case that would justify the merger?

Using the equation for scope economies given in Section 7.5 of the chapter, scope economies exist if $SC > 0$. In this case, scope economies do exist as the following expression is greater than zero for all values of both outputs.

$$SC = \frac{[250 + q_1 + 350 + q_2 - (450 + q_1 + q_2)]}{450 + q_1 + q_2}$$

(a)

$$AFC = \frac{10}{q}$$
$$MC = 10$$
$$AVC = 10$$
$$AC = \frac{10}{q} + 10$$

See Figure 2.

(b)

$$AFC = \frac{10}{q}$$
$$MC = 2q$$
$$AVC = q$$
$$AC = \frac{10}{q} + q$$
See Figure 3

\[ AFC = \frac{10}{q} \]
\[ MC = 10 - 8q + 3q^2 \]
\[ AVC = 10 - 4q + q^2 \]
\[ AC = \frac{10}{q} + 10 - 4q + q^2 \]

See Figure 4
Total cost, average total cost and average variable cost are

\[ TC = F + 10q - bq^2 + q^3 \]
\[ ATC = \frac{F}{q} + 10 - bq + q^2 \]
\[ AVC = 10 - bq + q^2 \]

(a) \( TC \) and \( ATC \) are positive when \( b < \frac{F}{q^2 + 10/q + q} \). \( AVC \) is positive for \( b < \frac{10}{q} + q \). As \( 10/q + q < F/q^2 + 10/q + q \) we need to have \( b < \frac{10}{q} + q \) to insure that all cost values are positive.

(b) The average cost curve is U-shaped. \( AC \) is minimized at \( \frac{dAC}{dq} = -Fq^{-2} - b + 2q = 0 \).

(c) \( MC \) crosses \( AC \) when the functions are equal. \( MC = AC \) where \( 10 - 2bq + 3q^2 = F/q + 10 - bq + q^2 \), \( MC = AVC \) where \( 10 - 2bq + 3q^2 = 10 - bq + q^2 \).

(d) \( AVC \) is minimized where \( \frac{dAVC}{dq} = 0 \).

\[ \frac{dAVC}{dq} = -b + 2q = 0 \]
\[ b = 2q \]

\( MC = AVC \) where
\[ 10 - 2bq + 3q^2 = 10 - bq + q^2 \]

Substituting \( 2q \) for \( b \) on both sides yields
\[ 10 - q^2 = 10 - q^2 \]

In this case, the production exhibits decreasing returns to scale, resulting in an increasing cost, or upward-sloping long-run cost function.


\[ MP_L/MP_K = (1/2K^{1/2}/L^{1/2})/(1/2L^{1/2}/K^{1/2}) = K/L = w/r \]

In U.S., \( w = r = 10 \Rightarrow L = K \);
In Mexico, \( w^* = 5; r^* = 10 \Rightarrow L = 2K \)
\[ C_{un} = wL + rK = 20K; C_{mexico} = 20K \]
\[ q_{us} = (LK)^{1/2} = K; q_{mexico} = 2^{1/2}K \]
\[ C_{us} = 20q_{us}; C_{mexico} = 14.1q_{mexico} \]

When \( q = 100 \), for U.S., \( C = 2000, L = K = 100 \);
for Mexico, \( C = 1410, K = 70.7, L = 141 \).