A Definition of 'More Systemic Production Risk' with Some Welfare Implications

by

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Abstract

Producers are subject to similar production risks, and so their outputs are likely positively correlated. This is believed to be among the main impediments to the creation of viable production risk insurance markets, is a major motive for such stabilization schemes as the holding of buffer stocks, and is a prominent motive for private storage activities. Taking a state contingent perspective, this paper provides a definition of systemic risk. We then show that an increase in this risk is both necessary and sufficient to ensure that any risk averse insurer is worse off. For risk-neutral producers, and for a variety of market structures, we trace the effects of an increase in systemic risk through to impacts on production and on welfare measures. Overall, expected welfare generally falls under more systemic risk but either of expected producer surplus or expected consumer surplus may rise. We also show that our definition of systemic risk has relevance in other areas of economics, such as the incentive to incur R&D expenditures.

JEL classification: D8, G1, Q1

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1. Introduction

Systemic risk is a major problem in many sectors. While not confined to developing countries, these countries are particularly vulnerable because of their strong dependence on primary agricultural food and fiber commodities such as coffee, cocoa, jute, and sisal. Although systemic risk originating on the demand side or through an unstable macroeconomic environment can be significant contributors to overall systemic risk, in the case of developing countries the problem often originates on the supply side.

Comparative advantage in producing primary commodities is typically due to favorable climatic and soil endowments. Favorable agronomic conditions may be confined to a few regions in the world. And so a weather, disease, or political shock in one region may have a systemic impact on world markets. Exacerbating the problems facing these countries are comparatively poor physical and risk management infrastructures that generally find support in these countries. Temporal markets tend to be poorly integrated due to waste in storage and to incomplete forward markets. Spatial markets tend to be poorly integrated due to unreliable and high cost modes of transporting goods and information.

Economic policies with intent to remedy the problem have included buffer stocks (Newbery and Stiglitz, 1981), commodity cartels (Cave and Salant, 1995), and storage market interventions (Gardner and López, 1996). The main concern of this article is not, directly at least, with policy prescription to remedy the problem. Rather, the intent is to provide a deeper understanding of what constitutes systemic production risk and how it impacts incentives and welfare measures. The point of departure in our analysis is a data-level specification of 'more systemic risk'. We then apply this definition to demonstrate that it implies increases in the equilibrium premium rates that should pertain in an insurance market. After endogenizing production decisions, market equilibrium is developed for a variety of competition structures. To conclude, we show that our notion of 'more systemic risk' also has implications for firm-level incentives to gamble on product development investments.

2. Data Structures

Our analysis is based on a quite arbitrary multivariate output distribution, which we describe in the form of a data structure. Let there be *n* equi-probable states of nature and *m* production units. The *j*th state output in the *r*th production unit is given as . The *data structure* is described as the data presented as an array of state-contingent vectors of output observations across production units,

(1)

Here the *r*th vector describes the (marginal) yield distribution across all states for the *r*th production unit.[2] This data structure, which we denote by *S*, may also be read coordinate-wise to identify each of the *n* realizations, .

Without loss of generality, for convenience we re-organize the data structure. Identify as the state of nature with the least production among the observations for unit 1. Of course, the corresponding observation for units 2 through *m* must be re-named accordingly. That is, if is renamed as then must be renamed as . Then identify as the second least element of the observations for unit 1 and re-name the corresponding observation for unit accordingly. Continue in this manner until . The statistical attributes of the data structure's multivariate distribution are invariant to this procedure. An example is the re-ordering of the bivariate data structure as . Because these data structures are equivalent, we do not distinguish between them.

Of interest to us are operations on the data structure that will alter the value of a function of *S* in a determinate way. Suppose, in our example, that the 7 in unit 2 of the bivariate structure

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[2] Non-uniform weightings might be handled through vector replications. However, replication fails in accommodating non-rational weightings. As will become apparent later, the assumption of equi-probability allows us to increase alignment in the data while preserving all statistical attributes of the marginals.

above is interchanged with the 3 to give the new data structure . Then, while the marginals have been preserved, the data structure has been fundamentally altered. Indeed, the new data structure would seem more aligned and so possessed of more non-diversifiable (i.e., systemic) risk. For a bivariate data structure, we engage in the data structure operation whereby for and then map . This operation clearly increases the alignment within the data structure. One might say that it increases the covariability among the data.

When there are more than two production units in the data set, then operations on data structures that preserve marginals and yet assuredly increase the covariability within the data structure are somewhat more involved. The problem is that an operation that increases alignment among a pair of units may decrease alignment among a different pair. To facilitate exposition, we assume that . For , let the data structure be . The data structure is preserved when all the are re-permuted in the same manner. Thus, the structures of *A* and *B* are equivalent where . It is clear from *B* that and are not all that well aligned with in the sense that the 2 and 5 in are ordered inversely with the 1 and 4 in while the 1 and 4 in are also ordered inversely with the 2 and 3 in . The level of covariability in the data would surely increase if *simultaneously* the 2 and 5 were interchanged in and the 2 and 3 in were interchanged. This gives .

The algorithm can be repeated on data structure *C*. Specifically, data structure covariability would surely be increased if the 4 and 3 in were interchanged when the 4 and 1 in were interchanged. This gives . At this point the marginal for unit 1 is perfectly aligned with the marginal for unit 2, and we only need to bring in line with both. This is done with operations that first give rise to , and then .

Now if is a function of the data structure and if it were believed that, say, the function should increase in the level of systemic risk, then we would expect that . But before studying impacts on functions, we will first formalize the data re-structuring algorithm. Denote by the rearrangement of vector such that the *j*th ordinate is the *j*th least element in the vector. That is, if then . The following concept is a variant by Kim and Proschan (1994) of a definition by Boland and Proschan (1988);

Definition 1 For a given pair of vectors with *n*-dimensional vector arguments and , define if there exists a permutation π of such that for each . Define if and only if there exists a finite number, *t*, of elements such that

(a) and , and

(b) for each there exists a pair of coordinate indices, *c* and *d* with , such that may be obtained from by interchanging the *c* and *d* coordinates of *every* vector such that where is the *v*th coordinate in vector (such an operation of obtaining from is said to be a *basic rearrangement*).

Data structure is said to be possessed of more systemic risk than *S* if .[3] We are now in a position to apply the concept of increased systemic risk. Note that the rearrangement preserves all ordinal attributes of the data structure. Suppose that . Then let the coordinates of, say, and its permutation undergo the coordinate-wise map for increasing. Then the relation continues to pertain after the mapping.

3. Insurance Problem

An insurance company possessed of von-Neumann & Morgenstern utility function with , the class of increasing and concave utility functions, writes a contract on each of the *m* production units whereby net cash flows from the *r*th production unit are exclusive of premium. Here is assumed to be increasing so that high outputs are good news for the insurer, and the *r*th contract

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[3] Boland and Proschan (1988) name the relation as the multi-variate arrangement increasing order. When , the definition reduces to the one applied by Epstein and Tanny (1980).

premium is . Averaging over states of nature, we have insurer expected utility

(2)

There is a benchmark utility level representing opportunity costs available to insurance companies. The must adjust to ensure that the reference utility is achieved, for otherwise insurance companies will exit or enter the market. The question we ask is: what sorts of alterations in data structure *S*, while preserving marginals, will reduce insurer expected utility and so require an increase in the levels of premiums demanded? For then some risk prospects may demur and the markets will become less viable. Of course, insurer preference for less variability in pooled payouts is central to systemic risk.

Proposition 1 For any pair of data structures, and , all insurers will demand lower aggregate premium, , under than if and only if .

Proof Please see Appendix A.[4]

The main insight that the result provides concerns how interrelations, across marginals, among realizations in a given pair of states of nature must be transferred (i.e., rearranged) in order to assuredly impact the variability of the aggregate when the aggregation is across production units. Any effort to rearrange realizations in a pair of states must be such that all are similarly ordered after the rearrangement. Consequently, a simple reduction in correlations among every pair of the random variables is insufficient to ensure the consequence in Proposition 1 unless additional structure, such as normality, is imposed on the multi-variate distribution.

[4] A rudimentary proof of the main idea in the proposition has been provided in Boland and Proschan (1988).

4. Market Equilibrium

In this section we consider the equilibrium effects of an increase in systemic risk on some of the most widely studied market structures when producing agents are risk-neutral, are fully informed about the nature of demand and the decisions faced by other agents, and form rational expectations.

4.1 Perfect Competition

For the data structure described in (1) above a price-taking producer faces the *j*th state price, , where is the *j*th state aggregate output, and where is a thrice continuously differentiable and decreasing function. We suppose that the producer controlling the *r*th production unit is possessed of a production technology giving rise to increasing and (weakly) convex cost function where is a state invariant location shifter of the firm's output vector, i.e., where the vectors are written horizontally. The state-contingent outputs are translated along the unit vector so that .[5] The expected profit maximizing firm will then solve

(3)

and the first-order conditions under perfect competition are

(4)

Denoting by the *i*th derivative of inverse demand, some analysis yields the following[6]

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[5] Chambers and Quiggin (1997, 2000), in their studies of a single firm, develop a state contingent theory that admits a more general stochastic production technology.

[6] Throughout this paper, interior solutions are assumed for first-order conditions. Also, as with most studies involving consumer surplus under uncertainty, our treatment ignores income effects in the demand function. The limiting conditions under which this treatment is strictly valid are provided in Stennek (1999).

Proposition 2 Let where . Then, under perfect competition,

a) expected production increases (decreases) if is convex (concave),

b) expected producer surplus decreases if ,

c) expected consumer surplus increases if ,

d) expected welfare decreases.

Proof Please see Appendix B.

Part a) has a straight-forward interpretation; if the map decreases mean price then effort devoted to production falls and, because of the ray-shifting nature of the stochastic technology, output in each state of nature will fall. While it is true that map implies an increase in the variance of aggregate output, an arbitrary increase in the variance of will not necessarily give rise to the consequences in a), or for that matter, b), c), or d).

Part b) assures that if demand function curvature adheres to the bounds , then producers are worse off on the average. Concavity of revenue, , ensures that expected revenue falls even with output fixed.[7] The problem is exacerbated if the inverse demand function is convex so that an increase in systemic risk elicits a larger level of firm effort. This drives revenue down further by depressing the price level. Notice, we have not ruled out the possibility that some producers gain under the conditions in part b). In particular, those whose production tends to be counter-systemic may gain.

Part c) places tighter sufficient conditions such that expected consumer surplus increases. If the conditions in c) are satisfied, then the conditions in b) are satisfied and producers lose out on the average from an increase in systemic risk. When , then mean output increases with the level

[7] As shown in footnote 12 of Caplin and Nalebuff (1991), concave is equivalent to convex, where demand function is decreasing.

of systemic risk. But consumers may not benefit on the average because implies that incremental consumer surplus tapers off at large levels of market output. The upper bound ensures that incremental consumer surplus does not taper off too much. Observe that inverse elasticity of demand, which we denote as , equals . It assuredly decreases under , as required by the condition. So the condition requires that the absolute value of the elasticity falls with an increase in market output.

As to part d), suppose that the context is a crop market and map arises from a shift to a systematically more risky variety. Suppose, in addition, that average yield is somewhat higher under the new variety. By continuity, it is conceivable that overall welfare falls even though the risk-neutral price-taking farmers gain from adopting the new variety. Returning to a situation where marginals are fixed, that welfare will decrease under the map should be intuitive if one considers the standard deterministic problem for a decreasing demand function and an increasing, convex industry cost function. Then . It seems reasonable then that increased riskiness in aggregate output, as reflected in a Rothschild and Stiglitz (1970) mean-preserving spread, will decrease the expectation over welfare in perfect competition. It is readily shown that our notion of increase in systemic risk, , implies a mean-preserving spread in .[8] An immediate consequence of part d) is that expected producer surplus and expected consumer surplus cannot both increase. Part c), therefore, provides sufficient conditions such that expected producer surplus decreases. However, these conditions are not as lax as those in b).

4.2 Multi-Plant Monopoly

We turn now to the case of a monopolist where each of its *m* plants is subject to output risk. This risk may be due to bottlenecks or strikes, or it might be due to input randomness. The firm is posed with the problem

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[8] See Appendix A, where the assertion is effectively demonstrated.

(5)

The first-order conditions are

(6)

where it is assumed that so that non-market disposal of output never occurs. Further, it is assumed that the monopolist's marginal revenue curve is downward sloping, , so that any interior is assuredly unique.

Proposition 3 Let where . Let be concave, (i.e., marginal revenue is decreasing) and assume that non-market disposal is never optimal. Then, under multi-plant monopoly,

a) expected production increases (decreases) if ,
b) expected producer surplus decreases,
c) expected consumer surplus increases if . It decreases if ,
d) expected welfare decreases if .

Proof Please see Appendix C.

Note, if then aggregate output rises under all states with under both perfect competition and monopoly. If , then both fall. If remains between these bounds, then the impact of a change in systemic risk on aggregate output may not be the same across the two market structures. Notice that, in contrast to part b) of Proposition 3, we cannot rule out the possibility that expected producer surplus increases as systemic risk increases under perfect competition, even if is concave. Concerning welfare, in contrast to perfect competition if elicits larger output across all states then it is conceivable that expected welfare under monopoly will rise due to consumer gains. This is because monopoly leads to under-production.

4.3 Cournot Oligopoly

Perfect competition and monopoly are at extreme poles of the market competitivity spectrum, and this facilitates any analysis that exploits symmetry. Essentially, symmetry at the market level and symmetry at the firm level coincide for these market structures because asymmetries arising from the opportunity for strategic interactions are absent at both poles. We will study the intermediate case of Cournot oligopoly, and here it is readily shown that there can be more symmetry at the market level than at the firm level. While each Cournot firm faces a similar problem, the problem is distinctly its own.

Let the *r*th of *m* Cournot oligopolists seek to

(7)

so that the system of first-order conditions is

(8)

Of course for each state and firm, i.e., the non-disposal condition, is required if (8) is to be valid. Unlike the two earlier market contexts, the conditions in (8) are not symmetric in the . Nor are they symmetric in the . Even if marginal costs are constant and firm-invariant, then firms do not behave symmetrically because firm-level output will vary directly with .

Now let the marginal costs be constant, at value but not necessarily firm-invariant. Dropping the firm subscript and then summing across firms in (8) yields

(9)

In (9) the only output level that matters is market level output. As observed by Bergstrom and

Varian (1985), Salant and Shaffer (1999), and others seeking to understand the role of cost structures in determining market equilibrium, equation (9) demonstrates that firm-level asymmetries can be 'papered over' at the market level. The (almost) symmetry expressed in equation (9) permits insights into how the market reacts to a change in the level of systemic risk.[9]

Proposition 4 Let where . Let be decreasing, and let firm marginal costs be constant but not necessarily firm invariant. Then, under Cournot oligopoly,

a) expected production increases (decreases) if is convex (concave),

b) expected consumer surplus rises if , while it falls if .

Suppose, further, that marginal costs are common across firms. Then

c) expected producer surplus rises if , and it falls if ,

d) expected welfare falls if .

Proof Please see Appendix D.

Additional conditions are required to establish consequences for producer surplus and welfare because it is not clear what impact an increase in systemic risk will have on the output of a particular firm. At least as far as random output goes, the movement from oligopoly to duopoly does generate additional exploitable symmetry. In particular, in contrast to the multivariate context, if the distribution of one marginal becomes better aligned with the (only) other marginal then systemic risk assuredly increases. If the two firms face common marginals and a common convex cost function, then there is complete symmetry at the agent level even though the agents autonomously solve their problem. And so each firm will produce the same amount of output.

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[9] The analysis to follow assumes that the requisite stability conditions are satisfied. See Dixit (1986) for the relevant conditions.

Moving from an environment of linear costs to convex costs and from oligopoly to duopoly, we can assert

Proposition 5 Let where . In duopoly ( ), let firm cost functions be common and convex, and let firm marginals be common. If is decreasing then

a) parts a) and b) of Proposition 4 pertain,

b) expected producer surplus falls if , and it rises if ,

c) expected welfare falls if .

Proof Please see Appendix E.

Our discourse thus far has established that systemic production risk has an adverse impact on expected welfare in market equilibrium. However, there exist situations in which more systemic production risk is likely desirable. We complete our analysis by studying one such case.

5. Incentives to Innovate

In circumstances where a decision-maker has the option to discard a bad draw, then more systemic risk may be desirable because it amplifies the upside potential of a risk while having little effect on the downside risk. Suppose that a firm seeks to develop a better mousetrap or crop seed variety. It has identified *m* product criteria that are important to acceptance in the marketplace. These criteria might include flood and drought tolerance, disease resistance, and protein levels in the case of a seed variety. The company is contemplating a lump-sum investment of magnitude *I* to test a conjecture on improving its product, and believes that the state space is comprised of *n* equiprobable states of nature over these traits. The multi-variate trait distribution can be described by the data structure given in (1) where is the *j*th state realization of the *r*th trait. All traits are defined to be good traits so that larger values are preferred.

The product, if released, will endure in the marketplace for one time period. The profit function, gross of the investment cost, for the period is given by where the *j*th state revenue is . The existing, or benchmark, gross product profit is . As the product will only be brought on the marketplace if it generates larger gross profit than the benchmark, the *j*th state net benefit from the investment is , and expected net benefit is

(10)

Now suppose that is supermodular.[10] Then too is supermodular. Work by Boland and Proschan (1988) shows that whenever .[11] And so an increase in systemic covariation among the product traits increases the incentive to innovate. Here an increase in the systemic 'risk' carries the benefit of increasing the likelihood that the gamble will be 'in the money' and so will be adopted to replace the status quo product design.[12]

6. Conclusion

This paper has provided a non-parametric definition of systemic risk that preserves all statistical attributes of the marginals and yet pertains regardless of the relative contributions of the individual random variables to the aggregate. We have applied the concept to study the viability of insurance markets and also to study the effects of systemic risk on equilibrium expected levels

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[10] That is, for twice continuously differentiable functions, all second-order cross derivatives are non-negative.

[11] See Proposition 2.5 a) in Boland and Proschan (1988).

[12] Athey (2000) presents the conditions under which whenever with , . But her work did not generalize the analysis to data structures in .

of welfare measures for some of the most widely studied market structures. Then we demonstrated how the order could be applied to study R&D activities.

The economic tools we have developed are quite likely to have applications in many other contexts, such as the theories of search, auctions, and portfolio allocation decisions. Given that the methods have already yielded some insights into equilibrium actions in imperfectly competitive markets when the consequence of the actions are random, and also into incentives for firms to engage in R&D activities, it would seem natural to extend the analysis to study the incentives to innovate when a small number of technology providers compete.

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Appendix A

Proof of Proposition 1. Fixing , and for an otherwise arbitrary pair of data sets , we show that if and only if . Given this result, the value of must then rise to re-establish equilibrium. First, we provide two definitions pertaining to the nature of the insurer's problem. We then use these definitions to demonstrate the result. Letting for data structure *S*, note that is symmetric and concave in the vector . These properties suffice to ensure it is possessed of the convenient Schur-concavity property in the . Majorization preserves order for such functions (see pp. 67–68 of Marshall and Olkin, 1979).

Definition A1. For vectors and , denote the respective k^{th} largest components as and . Write if a) , and
b) . Then vector is said to *majorize* vector .

Definition A2. A *Schur-concave* function decreases under the majorization order, i.e., if then whenever is a Schur-concave function.

Now, viewing Definition 1 in the text, we see that the partial sums must satisfy . This is because after a basic rearrangement, conducted on cordinate indices *c* and *d*, all *c*th and *d*th coordinates will be similarly ordered. The effect on the partial sums is to better sort the ordinates. Formally, it must be true that the following order pertain, where by construction. Regarding how these four numbers enter partial sums, there are four cases to be considered.

Case 1.) This is whenever does not enter a sum , . Then *may* enter the sum. And so in this case the partial sums may increase, but will not decrease.

Case 2.) This is whenever only enters a sum , (i.e., does not enter). Then the value of the partial sum *may* increase when is replaced by , but it will not decrease.

Case 3.) This is whenever both and enter a sum , , and is in the partial sum after . Then the partial sum does not change in value because

.

Case 4.) This is whenever both and enter a sum , , but drops out of the partial sum after . Then is replaced by a larger number and the partial sum cannot decrease in value.

In all cases, the partial sums (weakly) increase and so

while . Because is Schur-concave, we have . Now, if there does not exist a finite sequence of basic rearrangements satisfying the relation as then is false for some . It is well-known that a function of the form is Schur-concave if and only if is concave (Hardy, Littlewood, and Pólya). Therefore, we may then construct a concave such that for this pair .

□

Appendix B

Proof of Proposition 2. First, we introduce some notation. Write as the rational expectations market equilibrium choices under data structure , while is defined as the *augmented* data structure whereby each state in the *r*th firm's state-contingent vector of outputs is shifted rightward by scalar . Also, we define vector where the are the rational expectations equilibrium state-contingent outputs consistent with data structure . In rational expectations equilibrium for data structure *S*, is consistent with while is the vector of firm actions that is consistent with price vector . This alternative presentation of expected producer surplus is intended to capture feedback in the equilibrium formation process.

For part a), note that is of the same form as in equation (2). If is concave then, from Proposition 1, we know that . Since each is convex, each falls as , i.e., as mean price falls. If is convex, then , and so each rises as .

For part b), and superscripting with *pc* to indicate perfect competition, write expected producer surplus as . Due to optimality, we have . Applying part a), together with , we have . By condition , we have . Combining these inequalities gives the result.

For part c), write expected consumer surplus as

(B1)

Expression (B1) is symmetric in , and convexity pertains if . The latter condition ensures that . Given the convexity of the inverse demand function, arguments in parts a) and b) ensure that, *relative to the action adjusted data structure* , aggregate output undergoes a state-invariant leftward shift of scalar value as actions adjust from rational expectations equilibrium under to rational expectations equilibrium under . And so prices increase in each state of nature, coordinate-wise. As consumer surplus is decreasing in price, we have . And so

(B2)

Turning to part d), write expected welfare as

(B3)

The right-hand expression is symmetric and concave in the . Therefore, . But there are no externalities in the market, and the market supports the firm decision vector that maximizes expected surplus. Thus, . Associate inequalities to verify part d). □

Appendix C

Proof of Proposition 3. Part a) is immediate after noting that equilibrium marginal cost is common across plants. For part b), letting superscripted *mo* denote monopoly, we have expected producer surplus as . For , concavity of yields . And the monopolist can choose any point on the vector of state-contingent inverse demands so that . Part b) follows upon associating inequalities.

For part c), expected consumer surplus is . The function is convex (concave) in the if . If convexity holds then , while the inequality is reversed whenever concavity holds. From part a), aggregate output decreases (increases) in each state *relative to the action augmented data structure,* , as if . And so if , and the inequality in consumer surplus is reversed if . Association of inequalities yields part c).

For part d), write expected welfare in the monopolized market as . As before, is concave in the . And so . From part a), the condition for the action at each plant to increase as is . But the social welfare problem with the solution to condition (6) is that is too small for all . And this is true under data structure as well as . We have coordinate-wise so that . Part d) follows. □

Appendix D

Proof of Proposition 4. For part a), equation (9) readily reveals that the equilibrium level of increases after whenever is convex, while the equilibrium response must decrease whenever the expression is concave. As to part b), Cournot expected consumer surplus, , is convex when . Applying part a), together with the monotonicity of in and of in , we have

(D1)

under conditions . Follow similar logic to demonstrate the other conclusion in part b).

For part c), ensures that aggregate expected revenue increases under , while ensures that mean output falls. Under the additional assumption of common constant marginal costs, this implies that industry-wide costs fall. And so producer surplus, the difference between aggregated expected revenue and aggregated cost, must rise. If both inequalities are reversed, then declines under .

As to part d), if marginal cost is then welfare can be written as . Under , we have due to the concavity of expected welfare in aggregate output. If , then . But there is underprovision of the action in Cournot equilibrium and convexity of the social welfare optimization problem ensures that an aggregate output vector that is smaller in all states than Cournot equilibrium delivers lower expected welfare than the Cournot equilibrium. Thus, and so . □

Appendix E

Proof of Proposition 5. Part a) is immediate. For part b), write as expected producer surplus in duopoly. As before, concavity of aggregate revenue holds if . From part a), expected output increases upon if . Due to the complete symmetry of the problem, both and must then increase under since both have the same value. Denote by and the aggregate choices made in duopoly and monopoly under the given symmetry on cost functions and state space. Compare equations (6) and (8) to conclude that for any data structure . Further, expected producer surplus is locally decreasing in in the neighborhood of . For these reasons, we can write under given . By condition , we also have . Combining these inequalities establishes sufficient conditions under which expected producer surplus falls as . Sufficient conditions under which expected producer surplus rises can be established using the same logic.

Turning to part c), . Under we have, from part a) and the fact that the firms are identical in all ways, that both actions increase as . Because there is under-provision of the action in Cournot duopoly, and part c) follows. □