Sustainable Fiscal Policy with Rising Public Debt-to-GDP Ratios *

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Abstract

In financial and economic policy circles concerned with public debt in developing countries, a rising debt-GDP ratio is interpreted as a signal of overborrowing, warning of debt defaults if strong fiscal corrections are not adopted in time. This paper shows that this interpretation is incorrect. It does so by building a simple model of fiscal policy in which upward-sloping debt paths are observed even though the probability of default is equal to zero “almost surely”.

JEL classification codes: E62, F34, F37, H63

Key words: public debt, fiscal policy, debt sustainability, debt limits

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1 Motivation

Studies of public debt dynamics focus on the government budget constraint that takes the following form in a non-monetary economy:

\[ b_t = b_{t-1} \mathcal{R}_t + g_t - \tau_t \]

All variables in (1) except the growth-adjusted interest rate \( \mathcal{R}_t \equiv R_t / \Gamma_t \) are measured as GDP ratios, where \( R_t \) and \( \Gamma_t \) are the gross rates of interest and GDP growth, respectively; \( g_t \) represents non-interest fiscal outlays and \( b_t \) stands for the public debt issued at the end of time \( t \). Throughout, it is assumed that \( \mathcal{R}_t \), as well as fiscal revenue \( \tau_t \), are exogenous and stochastic. More specifically, in the model below, the dynamics of \( \mathcal{Q}_t \equiv [\tau_t, \mathcal{R}_t]' \) are governed by a Markov chain \( Q(q_{t+1}; q_t) \).

The literature on public debt sustainability is, in general, silent about the level of debt that is deemed as unsustainable. A first glance at the empirical evidence is not helpful either: the median public debt-GDP ratio among countries that defaulted on their public debt during the last 30 years is about 50 percent; 35 percent of the defaults happened at debt ratios of less than 40 percent, and around 30 percent of the defaults happened at debt ratios equal to or above 80 percent (IMF, 2003a, Ch.III).

The lack of a benchmark level of debt that can be considered as unsustainable leads policymakers and analysts to focus on other aspects of \( b_t \) to assess its sustainability. One of the most common practices is to focus on the dynamics of \( b_t \) and to point out that an upward-sloping time path of \( b_t \) signals unsustainable levels of public debt. For instance, the slope of the debt paths is a key component of a framework elaborated by the IMF for assessing the sustainability of countries’ public debt (see IMF, 2002, 2003b). The sustainability assessments conducted by these international organisms often have far reaching implications in developing countries because the outcome of these assessments serves as the basis for advice on macroeconomic policies. As documented by IMF (2003b), debt sustainability analyses conducted by investment houses and risk-rating agencies also focus on the short-
term dynamics of public debt to gauge the public debts’ repayment probabilities.

Seeking to identify unsustainable debt ratios, recent contributions by Mendoza and Oviedo (2006a, 2006b) propose to adopt the notion of the natural debt limit (NDL) of the precautionary-savings literature (see Aiyagari, 1994) to pinpoint the maximum sustainable level of public debt that a country can support. As in that literature, levels of debt that exceed the NDL are inconsistent with solvency (i.e. debt repayment) in all states of nature. Mendoza and Oviedo (2006a) suggest defining the NDL on public debt, $\phi$, by considering the worst state in the Markov chain $Q$, as well as the minimum level of $g_t$, say $g_{\text{min}}$, that can be considered as socially or politically tolerable; concretely, the NDL arises from solving equation (1) for the maximum level of debt consistent with $g_{\text{min}}$ and the worst state in $q$:

$$
\phi \equiv \min \left( R_t^{-1}(\tau_t - g_{\text{min}}) \right); \quad (2)
$$

A country that never borrows more than $\phi$ is able to honor its debt services even under the most adverse scenarios. In contrast, if $b_t > \phi$, there are states of the world with non-zero probabilities in which it is impossible to repay the stock of public debt without reducing $g_t$ below $g_{\text{min}}$. This would happen, for example, after a sufficiently long sequence of adverse realizations of $q_t$.

2 A Model of Sustainable Fiscal Policy

Consider a country whose fiscal authority seeks to provide a smooth path of fiscal outlays without incurring too large a debt and without incurring large primary fiscal deficits $d_t \equiv g_t - \tau_t$, while insuring the provision of the minimum level of expenditures $g_{\text{min}}$. This can be the goal, for instance, of an economy interested in satisfying the Maastricht fiscal convergence criteria that requires that the primary deficits and debt ratios of prospective member countries of the European Union to not exceed 3 and 60 percent of GDP, respectively.

Assume that to satisfy these fiscal policy goals the government implements the policy rules $g_t = \tilde{g}(b_{t-1}, q_t)$ and $b_t = \tilde{b}(b_{t-1}, q_t)$ that minimize the present discounted value of the
infinite-horizon loss function $\mathcal{L}$,

$$\mathcal{L}_0 = \sum_{t=0}^{\infty} \beta^t E_0 \left[ - \log(g_t) + \omega_1 \iota(d_t \geq \bar{d}) + \omega_2 \iota(b_t \geq \bar{b}) \right],$$

subject to: (i) a given $b_{t-1}$; (ii) $b_t \leq \phi$, according to the definition of $\phi$ given in (2); (iii) the flow budget constraint (1); and (iv) a Markov chain $Q(q_{t+1}; q_t)$. In (3), $E_0$ is a conditional expectation operator; $\beta \in (0, 1)$ is a discount factor; $\iota$ is an indicator function equal to 1 if the condition stated in its argument occurs, and equal to 0 otherwise; and $\omega_i \geq 0$, where $i = 1, 2$, are penalty parameters that capture the adverse political consequences arising from “excessive” deficits and/or indebtedness. The parameters $\bar{d}$ and $\bar{b}$ are interpreted as “upper desirable limits” externally imposed on the fiscal authority (as by the Maastricht criteria, for example) on primary deficits and debt ratios. The term $-\log(g_t)$ captures the perceived benefits from government outlays. As in social choice theory, concave preferences over $g_t$ can reflect either that opportunistic policymakers benefit from rents that are proportional to fiscal expenditures or that benevolent policymakers internalize the welfare that residents derive from the consumption of public goods.

The logarithmic function in (3) induces an outlay-smoothing effect similar to the one that helps Mendoza and Oviedo (2006a) to explain why fiscal policy is procyclical in developing countries. The smoothing goal is pursued considering the current and potential future penalties arising from (i) fiscal deficits exceeding $\bar{d}$, or (ii) debt ratios growing beyond $\bar{b}$. How these penalties are weighted against the goal of smoothing outlays depends on the values of the $\omega$’s.

**Calibration**

The foregoing model is calibrated to Costa Rica; data for the period 1985-2005 are used to construct time series of $\hat{q}_t$, the empirical counterpart of $[\tau_t, R_t]'$. To obtain the Markov chain $Q(q_{t+1}; q_t)$ that represents the statistical properties of exogenous state variables in

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2The data sources are the World Bank ($\hat{\Gamma}$); the Costa Rican Ministry of Finance (GDP, $\hat{g}$ and $\hat{R}$, where $\hat{R}$ is computed as the effective interest rate payed by the national government, namely, interest payments$_t$/public debt$_t$); and the Central Bank of Costa Rica ($\hat{\tau}$).
Table 1: Unconditional Moments Implied by the Limiting Distribution of the State Variables

<table>
<thead>
<tr>
<th>Variable ($x$)</th>
<th>$\mu(x)$</th>
<th>$\sigma(x)$</th>
<th>$\rho(x)$</th>
<th>$\rho(x,y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiscal revenue ($\tau_t$)</td>
<td>32.77</td>
<td>1.46</td>
<td>0.63</td>
<td>1.00</td>
</tr>
<tr>
<td>Growth adjusted int. rate ($R_t$)</td>
<td>1.054</td>
<td>2.88</td>
<td>0.42</td>
<td>-0.18</td>
</tr>
<tr>
<td>Public debt ($b_t$)</td>
<td>51.42</td>
<td>6.93</td>
<td>0.99</td>
<td>-0.32</td>
</tr>
<tr>
<td>Non-interest outlays ($g_t$)</td>
<td>29.97</td>
<td>1.70</td>
<td>0.79</td>
<td>0.56</td>
</tr>
<tr>
<td>Primary balance ($-d_t$)</td>
<td>2.81</td>
<td>1.50</td>
<td>0.31</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Notes: $\mu(x)$, $\sigma(x)$, and $\rho(x)$, stand for the mean, standard deviation, and autocorrelation of $x$. $\rho(y,x)$ is the contemporaneous cross correlation between $y$ and $x$. Except for $\mu(R)$, all $\mu(x)$’s, $\sigma(x)$’s are expressed in percentages.

actual data, the results of estimating the VAR(1) system $\hat{q}_t = A + B\hat{q}_{t-1} + e_t$ are passed to the Tauchen’s (1991) algorithm. The algorithm is set to return a Markov chain with nine states, i.e., nine pairs ($\tau, R$), and a transition probability matrix $Q_{9\times9}$ that implies the statistical moments of $\tau_t$ and $R_t$ shown in Table 1.

As in the Maastritch fiscal criteria, $\bar{d} = 0.03$ and $\bar{b} = 0.60$. The value of $g^{\text{min}}$ is set equal to two standard deviations below the mean in the data (the observed mean value of $g_t$ is equal to 0.29 and $g^{\text{min}} = 0.24$). This value of $g^{\text{min}}$, along with the states of the Markov chain, imply that $\phi = 0.62$. The values of the penalty parameters are $\omega_1 = 8.2 \times 10^{-4}$ and $\omega_2 = 1.5 \times 10^{-4}$; these are the minimum values of the $\omega$’s that reduce to zero the probability that any of the indicator functions in (3) become equal to one in the model’s limiting distribution. Namely, the chosen values of the $\omega$’s guarantee that $b_t \ll \bar{b}$ and $d_t \ll \bar{d}$ almost surely. Note that, as $\bar{b} < \phi$, the economy is always able to honor its public debt and consequently, the fiscal policy and the public debt are always sustainable. Finally, $\beta = 0.946$ is the value of the discount factor that makes the mean value of $b$ in the model consistent with the average value of $b$ in Costa Rica in the last 15 years.

3 Rising Debt Ratios with Sustainable Fiscal Policy

The solution to the constrained loss-minimization problem includes the policy functions $\tilde{g}(b_{t-1}, q_t)$ and $\tilde{b}(b_{t-1}, q_t)$; this solution is found by formulating a discrete-state dynamic pro-
gramming problem. The state space is defined by 500 equidistant values of \( b \in [0.2, \phi] \), along with the 9 states of the Markov chain. The control variables are \( g_t \) and \( b_t \); the pay-off function is the negative of the instantaneous loss function in (3); and the budget constraint (1) and the transition probability matrix \( Q \) are the states’ laws of motions.

Let \( \mathbb{P} \) and \( \pi^\infty \) represent, respectively, the transition probability matrix and the limiting distribution of the states \((b, q)\). Table 1 shows the moment statistics implied by \( \pi^\infty \) and Figure 1a shows the marginal limiting distribution of \( b_t \). The figure makes it clear that the probability that the stock of debt exceeds \( \bar{b} = 0.60 \) is equal to zero, which means that there is no fiscal risk: the government is always able to repay its debt!

Can an economy free from fiscal risk generate short- to midterm increasing paths of public debt ratios? Two computable objects are chosen to answer this question, a set of “forecasting functions” and a fan chart. The forecasting functions of \( b_t \) represent the conditional mean of the debt ratio implied by the model for a given initial state \((b_{-1}, q_0)\). Fan charts, in turn, have been used in country studies of debt sustainability (see for instance Celasun et al., 2006) to represent the results of a large number of simulations of equation (1). Typically, the simulations consider a reaction function of the primary balance with respect to the debt ratio and a distribution of shocks affecting that ratio, both estimated from data on the recent history of the examined country or of a set of similar countries. The starting point of the simulations commonly is the most recently observed debt ratio.

The model’s forecasting functions for five initial debt ratios ranging between 0.2 and 0.6 are shown in Figure 1b; all functions take the fifth state of the Markov chain as the initial exogenous state. The figure shows, for example, that when \( b_{-1} = 0.2 \) \((b_{-1} = 0.6)\), the model predicts time-increasing (time-decreasing) debt ratios. More generally, note that the forecasting functions represent conditional means conditioning on \((b_{-1}, q_0)\); so, as time goes to infinity, all conditional means converge to the unconditional mean of \( b_t, \mu(b) \), shown in Table 1. Therefore, any initial debt ratio such that \( b_{-1} < \mu(b) \) generates time-increasing debt ratios, and any initial debt ratio \( b_{-1} > \mu(b) \) generates time-decreasing debt ratios. Increasing or decreasing debt ratios, however, cannot be interpreted as signals of fiscal policy unsoundness in the modeled economy, where it has been shown that debt default is an impossible event.
It is important to note that the result linking the slope of the time-path of public debt and the value of the initial debt ratio in relation to the unconditional mean of this ratio is not a result particular to the loss function specified in (3). Any fiscal policy that generates a unique unimodal distribution of $b_t$ with a support included in $[\infty, \phi]$ will, in general, lead to the above (essentially statistic) result.

The model version of a fan chart can be generated by simulating the Markov chain $Q$ and by utilizing equation (1), as well as the policy functions $\tilde{g}$ and $\tilde{b}$. Figure 1c shows, for example, the fan chart resulting from 1000 simulations that have the same initial debt ratio $b_{-1} = 0.35$. From lighter to darker, the colored areas show deciles at 20, 40, 60 and
80 percent confidence intervals around the median projection; the chart shows that for the assumed initial debt ratio there is a probability of around 70 percent that this ratio will be rising in the short- to medium term.

The results depicted in Figures 1a to 1c lead to the conclusion of this paper. On average, public debt ratios are expected to rise if the actual debt ratio is below its unconditional mean. Under these circumstances, if an analyst or external reviewer assesses the sustainability of the public debt based on the short-term projected dynamics of the debt ratio, he might advise to adopt unnecessary fiscal policy corrections. These corrections might be unnecessary because increasing debt ratios may have nothing to say about the sustainability of the fiscal policy.

References


