Fiscal Solvency and Macroeconomic Uncertainty in Emerging Markets: The Tale of the Tormented Insurer

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Abstract

Governments in emerging markets often behave like a “tormented insurer,” who would like to smooth government outlays given the randomness of public revenues in an imperfect world where the only public debt instrument is a non-state-contingent bond denominated in units of tradable goods. How can a fiscal authority tell if the stock of public debt is consistent with fiscal solvency in this environment? This paper proposes a quantitative framework that aims to answer this question. The model is used to quantify the dynamics of public debt implied by the competitive equilibrium of a two-sector small open economy subject to exogenous shocks to income given tax and expenditure policies. A government committed to repay must not borrow above the “natural” debt limit set by the annuity value of the “crisis” level of the primary balance, set by the minimum levels of public revenues and outlays. This limit, and the likelihood that the government may hit it along an equilibrium path, are determined jointly by tax and expenditure policies and by endogenous and exogenous variables outside the government’s control. Moreover, liability dollarization (i.e., the mismatch between debt denominated in units of tradables and revenues that are largely collected in units of nontradables) implies that endogenous fluctuations of the real exchange rate alter the variability of public revenues and outlays and thus affect the government’s ability to issue and service debt.

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1 Introduction

A central question in fiscal policy debates is whether the observed net stock of the government’s financial liabilities is consistent with fiscal solvency considerations - that is, consistent with the requirement to equate the present values of total government revenues and outlays. If it is, the observed debt-output ratio is commonly referred to as “sustainable.” If it is not, the fiscal position is judged to be unsustainable and in need of policy correction. In short, the goal of public debt sustainability analysis is to determine whether the government is living “within its means” and to facilitate the assessment of corrective policy measures when this is not the case.

The methodologies for evaluating fiscal sustainability that are most favored in policymaking institutions are based on (a) steady-state debt-output ratios implied by the stationary, growth-adjusted government budget constraint, or (b) econometric tests of the intertemporal government budget constraint. Interest in the latter is partly motivated by the fact that while the steady-state analysis illustrates the level of debt that can be supported in the long-run equilibrium of a world without uncertainty, in practice the key issue is to assess public debt sustainability at a particular point time (possibly far from steady state) and in a world where a variety of shocks can affect the government’s ability to place and service debt.

Unfortunately, tests of the intertemporal government budget constraint fall somewhat short from delivering an effective methodology to make these assessments. Their main objective is to test whether the hypothesis that the fiscal solvency condition holds in a country’s historical time-series data. However, these tests fail to connect the observed underlying sources of macroeconomic uncertainty affecting the economy with the observed dynamics of public debt and therefore they are unable to provide short- and long-run forward-looking measures of public debt sustainability.

The importance of incorporating uncertainty considerations into public debt sustainability analysis is clearly reflected in two striking empirical observations. First, as Figure 1 shows, countries with lower coefficients of variation in the ratios of public revenues to GDP support higher debt-output ratios on average. An unconditional panel regression suggests that an increase of 1 percent in the volatility of revenues reduces the mean debt-output ratio by 3 percentage points. Second, as Figure 2 shows, countries with lower GDP volatility support higher average debt-output ratios. Countries with a standard deviation of GDP

\[1\] This criterion of sustainability is different from the requirement that public debt plans formulated by the government in their strategic interaction with the private sector be free from time inconsistency. The literature examining this issue from the perspective of the theory of dynamic games refers to public debt plans that satisfy this requirement as “sustainable.”
growth in excess of 3 percent cannot support debt-output ratios higher than 50 percent. The samples in these figures are small because of serious limitations of cross-country databases on fiscal data. Yet, these observations clearly suggest that the stochastic nature of the environment in which governments operate must be taken into account in estimating sustainable debt ratios.

These considerations are particularly important for emerging markets. As the report by the International Monetary Fund (2003a) shows, emerging economies display significantly higher coefficients of variation in public revenues and larger cyclical fluctuations in economic activity than industrial countries. Moreover, the response of emerging economies to macroeconomic shocks also differs from that of industrial countries because of the financial frictions that emerging economies confront in world capital markets. The possibility of “Sudden Stops” to capital inflows and the syndrome of “liability dollarization” that affect these economies influence public debt sustainability analysis. Because of liability dollarization, emerging markets’ public debt instruments are typically issued in hard currencies but largely leveraged on public revenues generated in the non-tradable-goods sector. In this situation, as Calvo, Izquierdo, and Talvi [2003] showed, a foreign or domestic shock that triggers a “Sudden Stop” can force a large reversal of the current account and a collapse of the relative price of nontradable goods, and the latter can compromise the ability to service public debt and result in sharp declines in sustainable debt-output ratios.

The aim of this paper is to propose a quantitative framework for assessing public debt sustainability that takes into account these elements of uncertainty and financial market imperfections. The starting point is the same from which both the long-run approach to fiscal sustainability and the intertemporal tests started: the budget constraint of a government credibly committed to repay. The framework proposed here differs in that it models explicitly the mechanism by which different macroeconomic shocks affect the behavior of the government and the private sector. The framework is based on a model of a two-sector small open economy where the output of the tradables and nontradables sectors are subject to exogenous random shocks. The government sets tax and expenditure policies and, since it suffers of the syndrome of “liability dollarization,” it issues debt denominated in units of tradable goods. Tax rates and government outlays are policy choices, but tax revenues and thus the financing needs of the public sector are endogenous outcomes that depend on variables beyond the control of the fiscal authority (such as the tax bases, the realizations of the shocks, and the equilibrium relative price of nontradables). Government expenditure decisions are modeled as aiming to provide a smooth flow of government outlays to the private sector.

This stochastic framework makes explicit the operational implications of the govern-
ment’s commitment to repay under uncertainty. In particular, a credible commitment to repay implies that the government must be able to repay regardless of the realization of public revenues drawn at any point in time. As a result, the government imposes on itself a “natural” debt limit analogous to those that households adopt typically in models of incomplete markets and income uncertainty (see Aiyagari [1994] and Huggett [1993] and the analysis of optimal taxation with non-state-contingent public debt by Aiyagari, Marcet, Sargent, and Sepala [2001]). This debt limit is given by the annuity value of the difference between the worst realization of public revenue and the minimum level of outlays that the government can commit to adjust to in a state of “fiscal-crisis” (defined as a long sequence of realizations of the lowest level of public revenue, which by definition can occur with non-zero probability). If the government were to borrow above this natural debt limit, its commitment to repay would not be credible because it cannot guarantee that it can repay at all times – in particular, it clearly would not be able to repay in a state of fiscal crisis.

The debt limit is a key part of this framework but in most cases it is not the same as the equilibrium or sustainable level of public debt. The latter is determined by the government budget constraint taking into account the endogenous behavior of tax bases and the price of nontradables along a stochastic equilibrium path, and the tax and expenditure policies. Thus, the computation of the stochastic equilibrium dynamics of the economy and the assumptions adopted for tax and expenditure policies are also central to the analysis.

One option to model taxes, debt and expenditures would be to consider optimal government policy in the traditional sense of Ramsey optimal taxation problems (by choosing optimal state-contingent rules for debt and tax rates for a given random process of government purchases as in Aiyagari et al. [2001]). In contrast, the guideline followed here is to adapt the model to the reality of emerging economies where government outlays tend to be inflexible and public revenues have important components exogenous to the government (commodity export revenues, for example), or where tax policy deviates sharply from the predictions of optimal taxation theory (as illustrated by the procyclical nature of fiscal policy in developing countries documented by Talvi and Vegh [2000]). Hence, the framework proposed here assumes that the government fixes non-state-contingent tax rates and seeks to provide a smooth level of government outlays in “normal” times, in which it may need to issue debt but the natural debt limit is not binding. On the other hand, in times of “fiscal crisis” the debt limit binds and the government adjusts outlays to a fixed lower level.

The natural debt limit and dynamics of sustainable public debt depend on how the above tax and expenditures policies are set. For example, if the government has no flexibility to reduce outlays during a fiscal crisis, the commitment to repay requires setting the “smoothed” level of government outlays equal to the worst realization of public revenues, so
as to equate the annuity values of the inelastic outlays and the worst realization of revenues. In this case, no positive amount of public debt is sustainable because the natural debt limit is zero. At the other extreme, if outlays could be cut to zero in a fiscal crisis, the debt limit would be equal to the annuity value of the worst realization of tax revenue. This would yield the highest natural debt ceiling that the government could reach for a given stochastic process of tax revenue. Thus, governments that can complement a pledge to commit to repay with commitments to undertake significant expenditure cuts during a fiscal crisis face a higher natural debt limit and hence are allowed to borrow more.

The exogenous macroeconomic uncertainty coming from shocks to domestic income and the world-interest rate also plays a crucial role. Countries that have more volatile tax revenues face lower debt ceilings and are able to borrow less because their worst realization of public revenues is lower, for given tax and expenditure policies. The effects of “liability dollarization” identified by Calvo, Izquierdo, and Talvi [2003] are also at play. In particular, if the relative price of nontradable goods falls when the government hits its debt limit, the debt limit itself can feature an endogenous magnifying effect that tightens the debt limit further (since the value of tax revenues in units of tradable goods can fall with the relative price of nontradable goods). Through this mechanism, fluctuations of the real exchange rate can have important effects on the model’s predictions for sustainable public debt ratios.

In summary, this paper develops an approach to study public debt dynamics and fiscal solvency that views the government as a “tormented insurer” operating in a largely imperfect and uncertain world. This insurer seeks to provide insurance to society by keeping government outlays smooth given the uncertainty of public revenues, but the financial markets are incomplete and hence do not allow the insurer to diversify away idiosyncratic risk. Faced with this situation, the insurer practices self-insurance and seeks to determine the optimum liability position that smooths government outlays as much as possible while not exposing the government to the risk of becoming insolvent. Thus, the insurer’s goal is to design a state-contingent plan for adjusting its non-state-contingent debt obligations so as to smooth outlays as much as possible while respecting the debt limit that ensures that it can repay its obligations.

The paper documents the results of implementing the stochastic debt sustainability framework for the case of Mexico. In a limiting case where the government fixes its expenditures unless its debt grows over natural debt limit, the quantitative analysis shows that, if the government is assumed to cut total outlays by 4 percentage points of GDP in times of fiscal crisis, the model yields a natural debt limit (at 0.5) slightly above the observed average debt-to-GDP ratio (0.459 for the period 1990-2002). This natural debt limit is very sensitive to small variations in the variability of tax revenues, the world interest rate and
the size of the cuts in government outlays. A mean-preserving spread that increases the variability of tax revenues by 1/2 of a percentage points cuts the natural debt limit to 18 percent of output.

The stochastic simulations show that, starting from debt ratios below 40 percent of GDP, the economy takes more than 20 quarters to hit the natural debt limit on average (in fact, for initial debt ratios below 0.30 percent the limit never binds when shocks take average values). However, these averages hide the fact that from the same initial conditions there are non-zero-probability paths in which adverse shocks, and the endogenous response of the economy to these shocks, cause the government to hit its debt limit and fall into a fiscal crisis in six quarters or less.

When a fully dynamic general equilibrium model is used to study the mechanism by which macroeconomics shocks affect the determination of the sustainable public debt, results indicate that the long-run mean of the stock of public debt is about 7.87% of the ratio observed in Mexico (which is 45.9%). The sharp contrast between the actual and long-run mean of the debt ratio reflects the precautionary saving behavior of a government committed to deliver a minimum of government expenditures and to repay its debt even during a period of fiscal crisis. The general equilibrium model also shows that the mean value of the sustainable debt decreases with the degree of risk aversion and that a reduction of 10% in output volatility increases the mean value of the sustainable debt in about 7.5%.

The paper proceeds as follows. The next section surveys the existing methods to evaluate public debt sustainability and compares them with the framework proposed in this paper, and documents key stylized facts of public debt and fiscal revenue ratios that motivate the use of stochastic methods to study debt dynamics and fiscal solvency. Section 3 considers a basic one-good variant of the model in which public revenues are driven by an exogenous stochastic process and the problem of liability dollarization is not present. Section 4 presents the complete model. Section 5 discusses the calibration and the quantitative predictions of the model. Section 6 contains final remarks.

2 A Review of Public Debt Sustainability Analysis

This section provides a short review of the different methods that have been proposed for studying public debt sustainability. The aim is not to survey the literature thoroughly but to highlight the central differences between the existing methods and the framework that this paper develops as well as to motivate.2 The section ends documenting the major

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2For a survey, see Chalk and Hemming [2000] or IMF [2002] and IMF [2003]
differences across industrial and developing countries regarding their fiscal variables to serve as a motivation to the models of sections 3 and 4.

The starting point of all the methods for calculating sustainable public debt-output ratios is the period budget constraint of the government. In an economy in which output grows at an exogenous gross rate $\gamma$ in the long run, the government budget constraint can be written as follows:\textsuperscript{3}

$$\gamma b_{t+1}^{g} = b_{t}^{g} R_{t} - (t_{t} - g_{t})$$

(1)

In this expression, $b_{t+1}^{g}$ is the ratio of public debt issued by the end of period $t$ and maturing at $t+1$ as a share of date $t+1$ output, $b_{t}^{g}$ is the ratio of maturing public debt to output at date $t$, $R_{t}$ is the gross real interest rate on public debt, $t_{t}$ is the ratio of total government revenue to output, and $g_{t}$ represents the ratio of total government outlays (current purchases plus transfers) to output. Thus $t_{t} - g_{t}$ is the primary fiscal balance as a share of output.

The methodologies for computing sustainable debt ratios differ in the manner in which they use the above constraint to assess whether observed debt ratios are consistent with the fiscal solvency condition that follows from solving the constraint forward. The solvency condition states that the present value of the primary balance must be equal to the interest and principal on the outstanding debt as of the initial date in which solvency is being evaluated.

### 2.1 Long-Run Methods.

The long-run methods for assessing public debt sustainability are based on long-run, perfect-foresight results that transform the government’s budget constraint from an accounting identity into an equation that maps the steady-state primary balance into a sustainable debt-output ratio (see Buiter [1985]). Thus, this method defines the sustainable debt-output ratio as the value that it attains at steady state, when the primary balance has also attained its long-run equilibrium (see Buiter [1985], Blanchard [1990], and Blanchard et al. [1990]). Given the budget constraint (1), the steady-state debt output ratio satisfies the following condition:

$$b^{g} = \frac{t - g}{R - \gamma}$$

(2)

where variables without time subscripts correspond to steady-state values. In policy applications, condition (2) is interpreted either as an indicator of the “permanent” value (or growth-adjusted annuity value) of the primary balance-output ratio that is needed to

\textsuperscript{3}As noted earlier, at the highest level of generality, this constraint is merely an accounting identity that relates all the flows of government receipts and payments to the change in public debt
stabilize the debt-output ratio at a target level, or as an indicator of the “sustainable” debt-output ratio consistent with the permanent primary balance-output ratio.

An important shortcoming of the long-run approach to debt sustainability analysis is that it fails to recognize that the “long run” is a theoretical construct. In the short run, governments face a budget constraint that does not reduce to the simpler formula of the long-run analysis. In a world without uncertainty in which the economy grows gradually to a stationary state, there can be temporarily high debt ratios, or temporarily large primary deficits, that are perfectly consistent with government solvency. Furthermore, incurring in such temporarily high debt or deficits could be optimal from a tax-smoothing perspective (see for example the quantitative simulations of the effects of tax reforms in Mendoza and Tesar [1998]). On the other hand, in a world with uncertainty, there can be sufficiently adverse realizations of the primary balance such that a public debt ratio allowed for in a deterministic long-run environment can be too large to be repayable in the short run. Thus, a country that keeps its public debt-output ratio at the level that corresponds to the long-run stationary state can make serious mistakes (by, for example, not borrowing enough to fully exploit the benefits of economic reforms or borrowing too much to prevent being unable to repay).

2.2 Intertemporal Methods.

The realization of these flaws in the long-run calculations led to the development of methods that test whether the intertemporal government budget constraint holds in the data. These methods shifted the focus from analyzing stationary debt-output ratios to studying the time-series properties of the fiscal balance. The aim was to test whether these properties are consistent with the conditions required to satisfy the government’s solvency condition. This condition serves as a means to link the short-run dynamics of debt and the primary balance with the long-run solvency constraint of the government.

In their original form (see Hamilton and Flavin [1986] ), the intertemporal methods aimed to test whether the data can reject the hypothesis that the condition ruling out Ponzi games on public debt holds. This condition states that at any date \( t \), the discounted value of the stock of public debt \( t + j \) periods into the future should vanish as \( j \) goes to infinity:

\[
\lim_{j \to \infty} \prod_{k=0}^{j} \left[ \gamma_{t+k}/R_{t+k} \right] \gamma b_{t+1+j} = 0.
\]

In other words, the debt-output ratio cannot grow faster than the growth-adjusted gross interest rate in the long run. When this no-Ponzi-game (NPG) condition holds, the forward
solution of eq. (1) implies that the present value of the primary fiscal balance (as a share of output) is equal to the interest and principal on the outstanding debt-output ratio. Thus, the existing public debt-output ratio is deemed “sustainable” because the government is able to honor it overtime. The survey by Chalk and Hemming [2000] provides a detailed review of the literature on empirical tests of this hypothesis.

By their nature, these intertemporal-budget-constraint methods introduced elements of uncertainty into public debt sustainability analysis, but mostly in an indirect manner. Uncertainty was introduced mainly as a source of statistical error in hypothesis testing. Some of the tests focused on the above NPG condition in expected value while others considered intertemporal optimality conditions to reformulate the test as an orthogonality condition. The orthogonality condition states that the sequence of expected growth-adjusted real interest rates used to discount the “terminal” debt stock must match the intertemporal marginal rate of substitution in private consumption at equilibrium:

$$\lim_{j \to \infty} \prod_{k=0}^{j} E_{t} \left[ \frac{\beta^{t+1+j} u'(c_{t+1+j}) b_{t+1+j}}{u'(c_{t}) b_{t}} \right] = 0$$

where $\beta$ is the growth-adjusted discount factor and $u'(c_{t})$ is the marginal utility of consumption as a share of output.

Bohn [1998] provides a very useful alternative interpretation of the intertemporal methods that reduces to testing whether or not the primary balance responds positively to increases in public debt. Under his approach, if the primary balance-output ratio and the debt-output ratio are stationary time-series processes, the following regression can be used to test for sustainability:

$$s_{t} = \rho b_{t}^{\alpha} + \alpha Z_{t} + \epsilon_{t}$$

where $s_{t}$ is the ratio of the primary fiscal balance over GDP, $\epsilon_{t}$ is a well-behaved error term, and $Z_{t}$ is a vector of determinants of the primary balance other than the initial stock of public debt. Bohn [1998] estimates the equation above including the cyclical variations in U.S. GDP and a measure of “abnormal” government expenditures as elements of $Z_{t}$. A positive coefficient $\rho$ indicates that the primary balance displays a linear response that is both positive and systematic to increases in debt. By imposing this property on the budget constraint (1), one can show that $\rho > 0$ is sufficient to ensure that the intertemporal government budget constraint holds. Hence, $\rho > 0$ is Bohn’s measure of fiscal sustainability.\(^4\)

\(^4\)Bohn [1998] found strong evidence in favor of $\rho > 0$ in U.S. data. In addition, Chapter 3 of the publication by the IMF [2003] shows results of the application of this test for a sample of industrial and developing countries. The results indicate that the sustainability condition holds for industrial countries and for developing countries with low debt ratios, and it fails for developing countries with high debt ratios.
2.3 Stochastic Methods and Methods with Financial Frictions.

Recent developments in public debt sustainability analysis follow two strands. One strand incorporates elements of the financial frictions that have played an important role in recent emerging-markets crises. In particular, public debt in many emerging markets displays “liability dollarization” (i.e., debt is denominated in foreign currency or indexed to the price level but leveraged on public revenues that depend to a large extent on prices, incomes and expenditures of the nontradables sector). As a result, abrupt changes in domestic relative prices that are common in the aftermath of a large devaluation, or a “Sudden Stop” to net capital inflows, can alter dramatically standard long-run calculations of sustainable debt ratios and render levels of debt that looked sustainable in one situation unsustainable in another. Calvo et al. [2003] evaluate these effects for the Argentine case and show that large changes in the relative price of nontradables alter significantly the assessments obtained with standard steady-state sustainability analysis.

The second strand emphasizes the fact that governments, particularly in emerging markets, face significant sources of aggregate uncertainty as they try to assess the patterns of government revenue and expenditures, and hence the level of debt that they can afford to maintain. From the perspective of these stochastic methods, measures of sustainability derived from the long-run approach or the intertemporal analysis are seen as being of limited use for governments that hold large stocks of debt and face large shocks to their revenues. The key question for these governments is not whether their debt is sustainable at a deterministic steady state, or whether in the sample of their recent or historical past the NPG condition holds. The key question is whether their current debt-output ratio is sustainable given the current domestic and international economic environment and its immediate future prospects.

The existing stochastic methods for assessing fiscal sustainability propose alternative strategies for dealing with macroeconomic uncertainty, although these strategies follow non-structural or reduced form representations of the process that drives the dynamics of public debt. For instance, a method proposed at the IMF by Barnhill Jr. and Kopits [2003]) incorporates uncertainty by adapting the value-at-risk (VaR) principles of the finance industry to public debt instruments to estimate the probability of a negative net worth position for the government. A second method recently considered for country surveillance at the IMF (see International Monetary Fund (2003b)) modifies the long-run method to incorporate variations to the determinants of sustainable public debt in the right-hand-side of equation (2). This method is also used to examine the short-term debt dynamics that result from different assumptions about the short-run path of the variables that enter the govern-
ment budget constraint (1) in deterministic form.\textsuperscript{5} The same IMF publication proposes a stochastic simulation approach that computes the probability density function of possible debt-output ratios. The IMF’s stochastic simulation model, like the VaR approach, is based on a non-structural time-series analysis of the macroeconomic variables that drive the dynamics of public debt (particularly output growth, interest rates, and the primary balance). The difference is that the stochastic simulation model produces simulated probability distributions based on forward simulations of a vector-autoregression model that combines the determinants of debt dynamics as endogenous variables with a vector of exogenous variables. The distributions are then used to make assessments of sustainable debt in terms of the probability that the simulated debt ratios are greater or equal than a critical value.

Xu and Ghezzi [2002] developed a third stochastic method to evaluate sustainable public debt. Their method computes “fair spreads” on public debt that reflect the default probabilities implied by a continuous-time stochastic model of the dynamics of treasury reserves in which exchange rates, interest rates, and the primary fiscal balance follow Brownian motion processes (so that they capture drift and volatility observed in the data). Default occurs when treasury reserves are depleted, and thus debt is deemed unsustainable when the properties of the underlying Brownian motions are such that the expected value of treasury reserves declines to zero (which occurs at an exponential rate).

Although the stochastic methods described above make significant progress in incorporating macroeconomic uncertainty into debt sustainability analysis, they are not robust to the Lucas critique since they follow from a non-structural representation of the determinants of the public debt dynamics. Whereas this is not a serious limitation when these methods are used for an ex-post evaluation of fiscal solvency conditions, it can be a shortcoming for a forward-looking analysis that requires a framework for describing how equilibrium prices and allocations, and hence the ability of the government to raise revenue and service debt, adjust to alternative tax and expenditure policies or other changes in the environment.

On the contrary, the framework proposed in this paper provides an explicit dynamic general equilibrium model of the mechanism by which macroeconomic shocks affect government finances and yields estimates of sustainable public debt that are robust to the Lucas critique. The framework determines sustainable debt ratios consistent with a commitment to repay rather than with the exposure to negative net worth or depletion of treasury reserves. This framework also takes into account elements of the financial frictions strand of fiscal sustainability models by incorporating the real-exchange-rate effects identified by Calvo et al. (2002).

\textsuperscript{5}For example, deterministic debt dynamics up to 10 periods into the future are computed for variations of the growth rate of output of two standard deviations relative to its mean.
2.4 Empirical Regularities of Public Debt and Revenues.

A key question that debt sustainability methods aim to answer is: why sustainable debt ratios for emerging markets often turn out to be significantly smaller than for industrial countries? If the economic principles on which stochastic methods are based hold in the data, one would expect to find systematic differences in the stochastic features of government revenues and public debt. A formal cross-country analysis of these differences is beyond the scope of this paper, but in the following paragraphs we document the major differences across industrial and developing countries in the characteristics of fiscal variables.\footnote{This review is largely a summary of the facts documented in Chapter 3 of the report by the IMF (2003a).}

A comparison of the mean ratios of public revenue to GDP using data for the period 1990-2002 for 47 industrial and developing countries shows that industrial countries generate significantly larger revenue-GDP ratios on average (see Figure 3). In addition, coefficients of variation show that revenue-output ratios are significantly more stable in industrial countries than in developing countries (see Figure 4). As illustrated in Figure 1 in the Introduction, an unconditional scattered diagram shows that countries with lower coefficients of variation in revenue-output ratios generally support higher mean debt-output ratios. The report by IMF (2003a) shows that the same is true for countries with higher mean revenue-GDP ratios.

The IMF (2003a) report went deeper into the characteristics of the tax structures across countries and found major differences in the averages and coefficients of variation of effective tax rates. The report shows estimates of the averages and the coefficients of variation of effective direct and indirect tax rates for a subset of industrial and developing countries for the period 1970-2000, computed using a simplified version of the methodology proposed by Mendoza et al. [1994]. Mean effective tax rates in industrial countries exceed those of developing countries by large margins. The differences in mean effective income tax rates are particularly striking. Industrial countries collect on average more than 30 percent of the total annual flow of payments to factors of production in taxes, while developing countries outside Eastern Europe collect less than 15 percent. From an accounting perspective, this wide gap in mean effective tax rates could reflect smaller statutory tax rates in developing countries, but it also reflects the lower “yields” of the tax systems in developing countries because the effective tax rates are measured in terms of what is actually paid in each tax relative to the relevant tax base.

The differences in the volatility of effective tax rates across industrial and developing countries are also staggering. Coefficients of variation of effective direct and indirect tax
rates in large industrial countries are below 4 percent, whereas those for developing countries are in a similar range only in the case of Chile. In general, developing countries display coefficients of variation in excess of 7 percent and 6 percent in direct and indirect tax rates respectively, and they can be as high as 22 percent for direct tax rates and 17 percent for indirect tax rates.

In summary, developing countries are significantly handicapped in their ability to raise government revenues on average and they also face much higher volatility in their revenue base. The stochastic model of public debt sustainability proposed in this paper predicts that these two characteristics of developing countries, combined with structural rigidities in their ability to adjust public expenditures, play a key role in explaining why emerging economies should be expected to sustain lower ratios of public debt to GDP than industrial countries.

3 A First Approximation: Exogenous, Stochastic Public Revenues

The starting point of the methodology proposed in this paper is the same premise of the government’s credible commitment to repay assumed in traditional methods of debt sustainability. The implications of this premise in an environment with uncertainty are easier to characterize in a basic model in which the government receives a stochastic stream of revenue each period. This section examines this basic case as a building block for the analysis of the next section.

A sustainable public debt policy under uncertainty is defined as one for which the government can credibly commit to always be able to repay in all states of nature. The commitment is credible only in the sense that the debt policy satisfies this “ability to pay” criterion because the government is assumed to be otherwise committed in an intertemporal sense. However, as argued later, the basic model of this section could be made compatible with a willingness to pay criterion based on credit-market participation constraints for non-contingent debt instruments.

A government credibly committed to service its debt under any state of nature must take into account the probabilistic processes and policy variables that determine the dynamics of the primary balance. In particular, the commitment requires the government to impose on itself a “natural” debt limit by which it cannot borrow more than the amount of debt it could service in the worst-case scenario that we label a state of “fiscal crisis.” The state of fiscal crisis is the one at which the fiscal authority arrives after experiencing a
long sequence of the worst realization of public revenues (that is, if public revenues were to remain “almost surely” at their lowest possible level). In addition, in a fiscal crisis the government is assumed to have the flexibility to adjust its outlays to a minimum level. This state of “fiscal crisis” has non-zero probability of occurring even in the long run (although it could be a very low probability) as long as there are non-zero transition probabilities of moving across all realizations of public revenues. In this environment, the government knows that from today’s perspective, there is a chance that it can end up in a fiscal crisis at some future date (after a long sequence of draws of the worst realization of revenues and with expenditures adjusted down to their minimum level). Therefore, to credibly commit to repay, it must not hold more debt than it could service in a fiscal crisis.

The above notion of sustainability requires an explicit setup describing the probabilistic dynamics of the components of the primary balance. On the revenue side, the probabilistic process driving public revenue reflects the uncertainty affecting tax rates and tax bases. In emerging markets, this process has two components. One component is the combined result of domestic tax policy and the endogenous response of the tax bases to this policy and the underlying shocks driving business cycles. The second component is largely exogenous to tax policy and reflects the nontrivial effects of fluctuations in commodity prices and exports on public revenues. In Mexico, for example, although oil exports are less than 15 percent of total exports, oil-related revenues still represent more than 1/3 of public sector revenue. On the expenditure side, government outlays adjust largely in response to policy decisions, but the manner in which they respond varies widely across countries. In emerging markets in particular, there is a tendency for fiscal policy to be procyclical, so that expenditures tend to contract during downturns.

The basic model of this section assumes for simplicity that public revenues follow a Markov chain with a known vector of discrete realizations and a known, non-degenerate transition probability matrix. The lowest realization of revenues is denoted as $t$. The government aims to keep its outlays constant at a positive level $g$ as long as it has access to debt markets. Otherwise, if the natural debt limit binds, government outlays are reduced to $g$. The world interest rate is kept constant for simplicity. In this environment, a credible commitment to repay implies that the public debt ratio must satisfy this constraint:

$$b_{t+1}^g \leq \phi = \frac{t - g}{R - \gamma}$$

Hence, $\phi$ is the “natural” debt limit on the public debt-GDP ratio. The limit will tend to be lower for governments that have (a) higher variability in tax revenues (for example, if the Markov chain is symmetric and obeys the rule of simple persistence, the absolute value
of $t$ can be written as a multiple of the standard deviation of public revenues and hence lower values of $t$ reduce $\phi$), (b) less flexibility to adjust government outlays, and (c) lower growth rates or higher real interest rates.

By eq. (1) and the above debt constraint, if the government starts with sufficiently low debt at date 0 and the realization of the revenue-output ratio is $t$, the government will finance the constant higher level of $g$ by increasing $b_{t+1}$. In an example with $\gamma = 1$ and zero initial debt, it is straightforward to show that if the government keeps drawing the minimum realization of revenue in the following dates, it will take at most the $T$ periods that satisfy the following equation for the government to hit the debt limit:

$$\sum_{i=0}^{T} R^i \leq \frac{g - g}{g - t}$$

(4a)

Thus, in this example the highest number of periods that the government can access the debt market (if revenues remain “almost surely” at their minimum) depends on the ratio of the excess of “normal” government spending over its minimum level relative to the excess of normal spending over the minimum level of revenues. At any date in which the debt ratio starts at $\phi$ and the realization of tax revenues is $t$, the budget constraint and the debt constraint imply that debt remains at $\phi$ and $g = g$. Hence, in this example, the government uses debt to keep its outlays as smooth as possible (at the level $g$) given its capacity to service debt as determined by the volatility of its tax revenues reflected in the value of $t$.

The credibility of the announcement setting the level of $g$ is an important part of the commitment to repay that drives this model. The ability to issue debt and the credibility of the announcement that government outlays will be cut in a fiscal crisis depend on each other because a government with a credible commitment to major expenditure cuts can borrow more and hence, everything else the same, this government faces a lower probability to be called to act on its commitment.

The condition defining the natural debt limit has a similar form as the long-run sustainability condition in (2). However, the implications of the two conditions for debt sustainability are very different. The long-run condition can easily identify as sustainable a debt-output ratio that is unsustainable once uncertainty of the determinants of the fiscal balance and a credible commitment to repay the debt are taken into account. Consider the case of two governments with identical long-run averages of tax revenue-output ratios at 20 percent. The tax revenue-output ratio of government A is very stable, with a standard deviation of 1 percent relative to the mean, while that of Government B has a standard deviation of 5 percent relative to the mean. If the distributions of tax revenue-output ratios
are Markov processes with \( t \) set two-standard-deviations below the mean, the probabilistic model would compute the sustainable debt ratio for A using a value of \( t \) of 18 percent, while for B it would use 10 percent. The long-run method yields the same debt ratio for both governments at 20 percent, using their common 20 percent average tax revenue-output ratio. In contrast, the probabilistic model would find that debt ratio unsustainable for both governments and would produce a sustainable debt-output ratio for B that is significantly lower than that for A.

Another important difference between the stochastic method proposed here and the long-run approach is how the two view the sustainable debt ratios. In the long-run analysis, the debt ratio is viewed as either a target to which a government should be forced to move to, or as the anchor for a target primary balance-GDP ratio that should be achieved by means of a policy correction. In contrast, the natural debt limit only sets the maximum level of debt, not the equilibrium debt policy that is sustainable (i.e., consistent with the commitment to fiscal solvency) along an equilibrium path for the economy as a whole. The debt limit does play a central role in determining both the equilibrium path and the sustainable debt, but it is not the model’s measure of sustainable debt. Depending on the probabilistic and policy assumptions driving taxes and expenditures, a country can exhibit levels of debt lower than \( \phi \) most of the time, and may take a very long time on average to enter a state of fiscal crisis or even never arrive at it.

Table 1 presents illustrative calculations of natural debt limits for emerging economies under alternative assumptions about the variability of public revenue-output ratios, the level of \( g \), and the world interest rate. To facilitate comparisons with the numerical simulations applied to the case of Mexico later in the paper, the growth rate, the mean public revenue-output ratio, and the mean ratio of total government outlays to GDP are set to the values computed from Mexican data (3.7, 22.9 and 21.7 percent respectively – see Section 4 for details). The data reported in International Monetary Fund (2003a) show that these Mexican figures are in the range of those that are typical for emerging economies. Case 1 in the Table shows natural debt limits for a “low risk” environment in which the real interest rate is 6.5 percent. Case 2 considers a “high risk” environment in which the interest rate is 10 percent. The public revenue-output ratio is assumed to follow a discrete, symmetric Markov process with a minimum realization \( t \) set two standard deviations below the mean. The Table shows natural debt limits for coefficients of variation in public revenue ranging from 4.4 to 13.1 percent and for commitments to expenditure cuts during fiscal crises of 2 to 8 percentage points of GDP. Scenarios that yield negative debt limits are reported as zeros, since negative debt limits indicate that in those cases the government cannot borrow.

Coefficients of variation of public revenue-output ratios in excess of 4 percent are very
common in emerging economies (see IMF (2003a)). For a “low-risk” emerging market with these characteristics, Table 1 indicates that the government would need to commit to fiscal cuts of at least 4 percentage points of GDP in order to attain debt limits that include observed average debt-GDP ratios. Moreover, natural debt limits are very sensitive to modest changes in the volatility of revenues and the commitment to expenditure cuts. Economies with coefficients of variation in public revenue in excess of 6.5 percent and commitments to expenditure cuts of 2 percent of GDP cannot sustain positive public-debt output ratios with a credible commitment to repay. On the other hand, if these economies can reduce the volatility of the public revenue-output ratio to 4.3 percent and/or make credible commitments to larger expenditure cuts of 4 percentage points of GDP or more, their natural debt limits would rise sharply.

The results in Table 1 can also be used to explain why the governments of large industrial countries can in general sustain higher debt ratios than those of emerging economies. Large industrial countries exhibit coefficients of variation in public revenues ranging between 2 and 4 percent, whereas the coefficients of variation in developing countries exceed 5 percent in general and are above 8 percent for several middle-income emerging countries like Argentina, Brazil, Korea, Indonesia and Mexico (see IMF (2003a)). Moreover, the gap between interest rates and growth rates is smaller for industrial countries, as they pay negligible country risk premia. These factors imply that, from the perspective of the model, the governments of industrial countries are capable of making credible commitments to repay higher levels of debt-GDP ratios, and hence if macroeconomic conditions require it they are able to borrow more than the governments of emerging countries.

As explained earlier, the debt limit is an important part of the analysis but in most situations does not correspond to the model’s predicted sustainable debt ratio. The model provides information on the short- and long-run dynamics of sustainable debt ratios that satisfy the government budget constraint and the debt limit along an equilibrium path. These results are easy to illustrate using this section’s basic setup with exogenous, random government revenue. This numerical exercise is again calibrated to Mexican data in order to facilitate comparisons with the results of the more general model of the next section that is also calibrated to Mexico. The simulations are conducted at a quarterly frequency. The growth rate is set to the quarterly-equivalent of Mexico’s annual average GDP growth rate over the period 1972-2000, 3.7 percent. The world real interest rate is set to be consistent with the annual real interest rate of 6.5 percent widely used in Real-Business-Cycle models of industrial countries. Public revenues follow a Markov process with the same mean, standard deviation and first-order autocorrelation of Mexico’s government revenue-GDP ratio over the period 1990-2002 (0.229, 0.185 and 0.601 respectively). The GDP share of
total government outlays (current purchases plus transfer payments) in normal times is set to match the difference between the mean public revenue-GDP ratio, 0.217, and the mean public debt-GDP ratio, 0.459, over the same sample period. The minimum value of the ratio of government outlays to GDP is set to obtain a natural debt limit of 0.5, which is near the maximum value of the debt-GDP ratio observed in the data (0.549 in 1998). This implies setting the minimum outlays-GDP ratio in a fiscal crisis at 83.5 percent of the same ratio in normal times.

The simulations consider a grid of initial public debt-GDP ratios that spans the interval from 0 to 0.5 (which is the natural debt limit). The short-run dynamics of sustainable debt can be traced from any initial public debt ratio in this interval. The long-run distribution of the public debt ratio is unique and invariant to initial conditions, so the mean and variance of the sustainable debt-GDP ratio in the long run is independent of initial conditions.

Figure 5 shows average and "extreme" estimates of the number of periods before hitting a fiscal crisis (i.e., before the natural debt limit binds) for different initial debt ratios. From each initial condition at present, there are different stochastic paths that public debt, revenues and outlays can follow in the future, and each of these paths features a different number of periods to hit a fiscal crisis. The figure reports the mean and the mean plus two standard deviations of this measure of time to a fiscal crisis (the latter is referred to as the "extreme" estimate). Depending on initial conditions, there are scenarios in which a fiscal crisis never occurs (particularly for low initial debt ratios). In these cases, the measure in the vertical axis goes to infinity and hence they are ignored in Figure 5.

Figure 5 shows that with the calibrated parameters, initial public debt ratios of 25 percent or less never lead to a state of fiscal crisis. The mean time to a fiscal crisis is high (at 24 quarters or more) for initial debt ratios below 40 percent, but it declines rapidly to less than 10 quarters for initial debt ratios around 45 percent. Moreover, even though it takes long to hit a fiscal crisis on average with a low initial debt ratio, there are sequences of adverse realizations of public revenue within the two-standard-deviations boundary that lead to a fiscal crisis much quicker. This is illustrated by the extreme measure of the time to a fiscal crisis for an initial debt ratio of 30 percent. While the average time to a fiscal crisis is above 40 quarters, the two-standard-deviation scenario predicts a fiscal crisis in less than 8 quarters. These large differences between the mean and extreme times to a fiscal crisis for low initial public debt ratios are an striking illustration of the importance of uncertainty in analyzing public debt sustainability.

Figure 6 illustrates how this simplified model with exogenous random public revenues could be used to forecast the dynamics of sustainable debt ratios. The graph shows the conditional mean and extreme forecast of sustainable debt-output ratios at different future
dates for four initial debt ratios (10, 20, 30 and 40 percent). Notice that whereas with an initial debt ratio of 0.3, the debt ratio increases rapidly until it reaches the debt limit, with an initial debt ratio of 0.1, the mean debt ratio increases very slowly and takes much longer to hit the debt limit. However, the fact that for a large range of initial values of public debt the model predicts that debt-to-GDP ratio will reach the debt limit while for some other initial values the debt ratio never hits that limit raises a word of caution regarding the accuracy of the debt-dynamics forecasts produced by this simplified model. The nature of the problem is clear in Figure 7, which shows a sample of simulated time series of the public debt-output ratio starting from an initial condition of 10 percent. The simulated series either grow to hit the debt limit or decline until the debt is fully retired (the model assumes that when public debt becomes negative the fiscal surplus is rebated to the private sector in a lump sum).

This striking result motivates the scrutiny of the important elements of fiscal sustainability analysis left out in the basic model as well as the reconsideration of the model assumptions. First, the government expenditure rule that indicates that government outlays should be kept constant except when doing so threatens the fiscal solvency is too rudimentary to become an effective insurance mechanism for the government and could be dominated by many conceivable rules capable of observing the commitment to fiscal solvency. Second, as the existent methods to evaluate fiscal sustainability do, the model of this section assumes that public revenues are entirely driven by an exogenous random process. Third, by assumption, the model rules out the possibility of default by sovereign debtors. The model examined in the next section makes some progress in addressing the first two limitations and some of the implications of default risk are considered next.

A straightforward manner to introduce default risk is to think of the case of a lender interested in designing a credit contract that enforces the government’s “participation constraint.” This constraint requires the government to always find it preferable (from the point of view of its own payoff) to fulfill its financial obligations than to go on default. A risk-averse lender with a constant-relative-risk-aversion payoff function would want the participation constraint to hold at all times, since the lender is not willing to accept the risk of not being paid. If non-contingent public debt is the only financial instrument available, the lender would manage the risk of default by comparing the government’s payoff under the credit relationship and under financial autarky and imposing a limit on government borrowing. This limit would be the smallest debt for which the government is indifferent between defaulting and repaying across all possible realizations of the random variables that determine the government’s payoff.
4 The Stochastic General Equilibrium Model

This section generalizes the approach to model public debt dynamics and fiscal solvency developed in the last section to the case of a two-sector dynamic, stochastic general equilibrium model of a small open economy. Households derive utility from the consumption of both private and government goods, which are represented by composites of tradable and nontradable goods. The specification of preferences is such that the marginal utility of private (public) consumption is independent of government (private) consumption. Households collect government transfers and draw stochastic endowments of tradable and nontradable goods each period. Endowment incomes are taxed by the government at a pre-determined, time-invariant tax rate. Households have access to a world credit market of one-period real bonds issued at a world-determined real interest rate in units of tradable goods. They can also buy bonds issued by the domestic government. These bonds are denominated in units of tradable goods, even though an important fraction of government revenues originate in the nontradables sector (that is, the government suffers from liability dollarization).

The economy grows at an exogenous gross rate $\gamma$. This common trend is shared by the consumption allocations, sectoral endowments and the balance of trade. In the short run, the economy displays fluctuations around this trend induced by the stochastic shocks to the sectoral endowments. The analysis focuses on the deviations from trend and hence all variables are detrended by expressing them as ratios relative to aggregate output in units of tradable goods (the price of tradables is the model’s numeraire). Detrended variables are denoted by lowercase letters.

The government aims to provide a smooth flow of public expenditures over time, which is chosen endogenously to maximize an explicit formulation of the government’s objective function. The role of the tormented insurer (i.e., the desire to smooth public outlays) that the government had in the basic model of the previous section is recovered by assuming that, because the government sets expenditures to maximize private utility, the payoff function of the government takes a similar “smoothing-inducing” form as the private utility function. In a world with incomplete markets, this specification of preferences gives the government an incentive to engage in precautionary saving, retiring debt when revenues are high and issuing debt when revenues are low. The commitment to expenditure cuts in a fiscal crisis is recovered by assuming that the government has target minimum levels (or “basic needs”) of public expenditures in tradable and nontradable goods. This is modelled via a Stone-Geary-type utility index that includes basic needs in government expenditures.

The stochastic process driving government revenues has an exogenous stochastic element (the endowment shocks), but it is also affected by the endogenous equilibrium stochastic
process of the relative price of nontradable goods (i.e., the real exchange rate). Non-state-contingent public and private bonds are the only financial assets available to domestic agents so that markets for contingent claims are incomplete.

The incompleteness of financial markets and the assumed commitment to repay debts induces public and private precautionary saving effects, as households and the government try to self-insure against the non-diversifiable risk of very low consumption if a long series of adverse income shocks occurs. These effects lead them to impose on themselves natural debt limits. The government’s natural debt limit is determined jointly by the properties of the equilibrium processes of tax revenues, government outlays, the relative price of nontradables and the interest rate.

4.1 The Private Sector

The objective of the representative household is to choose sequences of consumption and bond holdings so as to maximize the expected lifetime utility:

$$\max_{\{c_t^T, c_t^N\}_{t=0}^{\infty}} E \left[ \sum_{t=0}^{\infty} \beta_h(c_t) \left( \frac{c_t^{1-\sigma} + g_t^{1-\sigma}}{1 - \sigma} \right) \right]$$ (5)

where $c_t$ and $g_t$ are CES composites of private and public goods defined by the following functions:

$$c_t = c(c_t^T, c_t^N) = [\omega_h (c_t^T)^{-\eta} + (1 - \omega_h)(c_t^N)^{-\eta}]^{-1/\eta}$$

$$g_t = g(g_t^T, g_t^N) = [\omega_g (g_t^T - g_T)^{-\eta} + (1 - \omega_g)(g_t^N - g_N)^{-\eta}]^{-1/\eta}$$

$g_T$ and $g_N$ represent basic needs of government consumption in tradables and nontradables respectively.

In eq. (5), $\sigma$ is the coefficient of relative risk aversion, and $\beta_h$ is the discount factor function that depends on the consumption composite function:

$$\beta_h(c_t) = (1 + c_t)^\psi$$

The parameter $\psi > 0$ defines the elasticity of the private subjective discount factor with respect to consumption and induces stationarity in the consumption and asset dynamics. This specification of the discount factor indicates the existence of an impatience effect, as the discount factor rises with the value of the consumption composite. However, atomistic

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7 Notice that although government transfers are equal to a fixed amount of nontradable goods, the cost of providing this transfers varies with the value of the real exchange rate.
households do not internalize the effect of consumption on the impatient effect when they take their consumption decisions.\(^8\)

In the composite functions of public and private goods, \(\omega_g\) and \(\omega_h\) are the CES weighting factors and \(\eta\) determines the elasticity of substitution between consumption of tradable and nontradable goods, \((1/(1 + \eta))\), which is assumed to be the same for private and public expenditures.

The maximization indicated in (5) is subject to the following flow budget constraint for \(t = 0, \ldots, \infty\):

\[
c_t^T + p_t^N c_t^N + \gamma b_{t+1} \leq (1 - \tau)(y_t^T + p_t^N y_t^N) + R b_t + p_t^N \ tr
\]

This constraint equates the sources and uses of household income. The uses of income in the left-hand-side of the equation consist of purchases of tradable goods, nontradable goods and bonds all valued in units of tradables (\(p_t^N\) is the relative price of nontradables in units of tradables). The household chooses an aggregate bond position \(b_{t+1}\) which is composed of domestic government bonds, \(b_g^{I+1}\), and international bonds, \(b_I^{I+1}\), (i.e., \(b_{t+1} = b_g^{I+1} + b_I^{I+1}\)). Without loss of generality, the government is assumed to issue its bonds at the same terms that domestic households face in the international bond market. Hence, domestic households are indifferent between both types of assets and the two pay the same world-determined gross real interest rate \(R\). The right-hand-side of (6) represents the household after-tax income which has three components. First, stochastic endowment income collected from the tradable and nontradable sectors, \(y_j^T\) for \(j = T, N\), which is taxed by the government at the income tax rate \(\tau\). Second, payments of interest and principal on total bond holdings, \(R b_t\). Third, transfer payments from the government \(p_t^N \ tr\), which are set at a fixed level in units of nontradables and thus have a value in units of tradables that moves together with the real exchange rate, \(p_t^N\).

Endowment incomes are random processes defined as perturbations of exponential support around mean values:

\[
y_t^T = y^T \exp(e_t^T); \quad y_t^N = y^N \exp(e_t^N)
\]

The exponential shocks are Markov processes with known vectors of realizations (I values for \(e^T\), J values for \(e^N\), where I and J are positive integers). Define \(\bar{e}\) as a discrete-valued random vector that includes the joint realizations of endowment shocks (i.e., the \(I \times J\) pairs \((e_{ih}^T, e_{ij}^N)\), for \(i=1, \ldots, I, j=1, \ldots, J\), that represent all the combinations of the possible realizations of each shock). Hence, \(\bar{e}\) is a column vector with \(s = I \times J\) rows. The joint

\(^8\)This is equivalent to assuming that the discount factor depends on the aggregate and not the individual consumption ratio.
states included in \( \bar{e} \) follow a Markov process defined by \((\bar{e}, P, \pi_0)\), where \(P\) is an \(s \times s\) state-transition probability matrix of moving from each joint state represented by a pair in \(s\) to another joint state in \(s\) in one period and \(\pi_0\) is the initial probability vector.

Households choose optimal plans for bond holdings, consumption of tradables and consumption of nontradables, taking as given endowment incomes, prices and fiscal policy (which sets the value of public consumption of tradable and nontradable goods given a pre-determined tax rate and level of transfers). The first-order conditions of the households’ maximization problem are the budget constraint in equation (6) and the following conditions

\[
\left(1 - \frac{\omega_h}{\omega}\right) \left(\frac{c_t^T}{c_t^N}\right)^{1+\eta} = p_t^N \tag{8a}
\]

\[
[c(c_t^T, c_t^N)]^{-\sigma} c_{t+1}(c_t^T, c_t^N) = \beta_h(c_t) E_t \left\{ [c(c_{t+1}, c_{t+1}^N)]^{-\sigma} c_{t+1}(c_{t+1}^T, c_{t+1}^N) R \right\} \tag{8b}
\]

where \(c_{t+1}(c_t^T, c_t^N)\) is the derivative of the CES aggregator with respect to the consumption of tradable goods. These two conditions have straightforward interpretation. Eq. (8a) equates the marginal rate of substitution of tradable for nontradable goods to the relative price of nontradable goods (i.e., the real exchange rate). Eq. (8b) is the consumption Euler equation for tradable goods that equates the marginal cost and benefit of an additional unit of savings in foreign or domestic government bonds. The household is indifferent between the two financial instruments, but at equilibrium, the composition of the portfolio is well-defined because the supply of government debt is limited.\(^9\)

### 4.2 The Public Sector: Government Taxes, Debt and Outlays

As before, the government’s credible commitment to repay requires it to respect a natural debt limit by which it must not borrow more than it could repay in a state of “fiscal crisis”. In contrast with the basic model of Section 3, the realization of tax revenues and the cost of total government outlays in units of tradables are now endogenous variables that depend on the equilibrium relative price of nontradable goods. The government chooses levels of public expenditures and a debt policy so as to maximize the “governments contribution” to

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\(^9\)The domestic government sets its supply of debt so that it crosses the demand from households that is infinitely-elastic at the world interest rate at the level of debt it needs to sell.
private utility,\(^\text{10}\)
\[
\max_{\{g^T_t, g^N_t, b^T_{t+1}\}} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta(g_t) \left( \frac{c_{t+1}^{1-\sigma} + g_t^{1-\sigma}}{1-\sigma} \right) \right] \tag{9}
\]
subject to a flow budget constraint for \(t = 0, \ldots, \infty\),
\[
\gamma b^g_{t+1} = b^g_t R + y^T_t + p^N_t (y^N_t + tr) - \tau (y^T_t + p^N_t y^N_t) \tag{10}
\]
In the objective function (9), \(\beta(g_t)\) is the government’s discount factor function, which is analogous to the private sector discount function. The government’s discount factor is given by \(\beta(g_t) = (\psi_g + g_t)^{-\psi}\). The term \(\psi_g > 1\) is introduced to guarantee that the elasticity of the government discount factor with respect to the composite \(g_t\) is equal to the elasticity of the private-sector discount factor with respect to the composite \(c_t\). For \(\psi > 0\), this function guarantees the stationarity of public expenditures and the public debt.

Government transfers \(tr\) are introduced to capture payments for welfare and entitlement programs that in actual economies are a significant fraction of total government outlays and pertain mainly to the nontradables sector. Since welfare programs are also the most inflexible component of government outlays, they are modelled as a fixed quantity of nontradable goods (this assumption is also in line with the aim of the government to act as a “social insurer”).

Government expenditures are managed along with the debt policy to guarantee a smooth path of public expenditures. Adjustments in purchases of nontradable goods affect the equilibrium relative price of nontradables, and hence they are critical for determining the value of tax revenues and the cost of total outlays in units of tradables. The government optimality conditions for \(t = 0, \ldots, \infty\) are the budget constraint (10) and the following conditions:
\[
\left(1 - \frac{\omega_g}{\omega_g}\right) \left(\frac{g^T_t - g^T}{g^N_t - g^N}\right)^{1+\eta} = p^N_t \tag{11a}
\]
\[
[g(g^T_t, g^N_t)]^{-\sigma} g^\tau(g^T_t, g^N_t) = \beta(g_t)E_t \left[ g(g^T_{t+1}, g^N_{t+1}) \right]^{-\sigma} g^\tau(g^T_{t+1}, g^N_{t+1}) R \tag{11b}
\]
Condition (11a) dictates that, in every period, government consumption expenditures should be allocated between tradable and nontradable goods not only following market incentives, as summarized by \(p^N_t\), but also observing the economy’s basic needs of both types of goods,
i.e. $g^T$ and $g^N$. Operationally, for a given level of $g^T_t$ ($g^N_t$), raising the value of $g^T$ ($g^N$) increases the marginal utility of tradable (nontradable) government expenditures. Furthermore, the existence of basic needs in a world of incomplete financial markets supplies an additional precautionary saving motive. This is because the requirement to satisfy basic needs induces the government to reduce the stock of public debt so as to avoid situations where providing for basic needs would imply raising the stock of debt above the level that could be repaid if the government experiences a long sequence of the worst realizations of public revenues. The expenditure Euler equation (11b), which is the efficiency condition of the optimal public-debt management, implicitly incorporates the additional precautionary saving motive at introducing the basic needs in the functional form of the expenditure aggregate function $g$.

A government committed to repay its debt and to provide for basic needs will not borrow above a natural debt limit $\phi$ set by the annuity value of the “catastrophic” level of the primary fiscal balance that incorporates these basic needs. The value of $\phi$ is the maximum stock of debt that the government can repay at all times, even during a state of fiscal crisis where the government draws a long sequence of the lowest realization of its tax revenues. In the model of this section, the natural debt limit arises from equation (10) and is equal to:

$$\phi = \frac{1}{R - \gamma} \times \min \left[ \tau \left( y_t^T + p_t^N y_t^N \right) - \left( g^T_t + p_t^N (g^N_t + tr) \right) \right]$$

(12)

Whereas the Markovian shocks detailed in (7) and the specification of the basic needs permit to know the bounds of the realizations of the endowments and expenditures, the value of $\phi$ is endogenous and depends on the value of the real exchange rate.\textsuperscript{11}

A critical element of this model is that the sustainable level of public debt differs from the natural debt limit and arises as the result of the optimal government debt-management policy and the equilibrium dynamics of the economy. The forward-looking nature of the government along with the concavity of its objective function (9) will induce the government to hold a sustainable stock of debt that insure itself against the state of nature where it can “just” provide for the basic needs. Therefore, the sustainable level of debt depends, among other things, on the the volatility of the income endowments, the value of the basic needs, the degree of risk aversion of the government, and the equilibrium allocations of the private sector.

\textsuperscript{11}Notice that, other things equal, whereas $\phi$ raises with $p^N$ if $(\tau \min(y^N_t)) > g^N$, $\phi$ falls with $p^N$ if $(\tau \min(y^N_t)) < g^N$.
4.3 Market Clearing Conditions and the Competitive Equilibrium

The market clearing conditions for tradable and nontradable goods are:

\[ y^T_t = -b^I_t R_t + \gamma b^I_{t+1} + c^T_t + g^T_t \]  
\[ y^N_t = c^N_t + g^N_t \]

(13a)

(13b)

The competitive equilibrium is defined by sequences of allocations \( \{c^T_t, c^N_t, b^I_t, b^I_{t+1}, b^I_{t+1}, g^T_t, g^N_t\}_{t=0}^\infty \), and a sequence of real exchange rates \( \{p^N_t\}_{t=0}^\infty \), such that: 1) households solve their constrained lifetime utility maximization choosing the allocations \( \{c^T_t, c^N_t, b^I_t\}_{t=0}^\infty \) so that eqs. (8) and (6) hold, taking as given an initial stock of assets \( b_0 \), the stochastic processes driving the sequence of endowments \( \{y^T_t, y^N_t\}_{t=0}^\infty \), the sequence of relative prices \( \{p^N_t\}_{t=0}^\infty \), and the fiscal policy characterized by \( tr \) and the sequences of government expenditures \( \{g^T_t, g^N_t\}_{t=0}^\infty \). 2) the government carries out its optimal insurance policy by choosing sequences of expenditures \( \{g^T_t, g^N_t\}_{t=0}^\infty \) and public debt \( \{b^I_t\}_{t=1}^\infty \) so as to maximize the objective function (9), subject to the sequence of flow budget constraints (10), given the initial stock of public debt \( b^I_0 \), the stochastic processes driving the sequences of endowments \( \{y^T_t, y^N_t\}_{t=0}^\infty \), and the sequence of real exchange rates \( \{p^N_t\}_{t=0}^\infty \). 3) goods and financial markets clear, .i.e. eqs. (13) hold in every period \( t \), for \( t = 0...\infty \).

5 A Quantitative Application: The Case of Mexico

This Section explores the quantitative implications of the model using a series of numerical simulations calibrated to the Mexican case. The model is set to a quarterly frequency and the calibration strategy follows closely the calibration to Mexico described in Mendoza [2002].

The calibration exercise is conducted in two stages. First, parameter values are set so that the deterministic, balanced-growth stationary equilibrium of the model matches key characteristics of the Mexican economy reflected in time-series averages from national accounts statistics and fiscal policy and balance-of-payments data. Second, the properties of the Markov processes of endowment shocks are set so as to match the statistical moments of the observed fluctuations of Mexico’s tradables and non-tradables output.

The averages from the data that the model is set to match in the first stage of the calibration are the following:

1. The public debt-GDP ratio is set to the quarterly equivalent of an annual ratio of 45.9...
percent, which is Mexico’s average public debt-GDP ratio over the period 1990-2002 in the data reported in IMF (2003a).

2. The ratio of net foreign assets to GDP is set to the quarterly equivalent of an annual ratio of -35 percent, which is the average for Mexico in the data on net foreign assets constructed by Lane and Lane and Milesi-Ferretti [1999]).

3. The ratio of tradables GDP to nontradables GDP is set to 64.8 percent to match the average for Mexico over the 1988-1998 period. This sample is shorter than for the national aggregates because a consistent set of sectoral national accounts that is required to construct the accounts for the tradables and nontradables sectors is only available from 1988 (for details on the sectoral decomposition of the data see Mendoza [2002]).

4. The sectoral ratios of national accounts measures of consumption, government purchases and investment to GDP in the tradables and nontradables sectors are also set to match Mexican averages over the same period (the ratios are 66.5, 0.092 and 20.0 percent respectively for tradables and 70.8, 14.1 and 15.1 percent respectively for nontradables).

5. The average GDP growth rate from the same source and sample is 3.7 percent per year.

The calibration also sets value for the world real interest rate, the coefficient of relative risk aversion and the elasticity of substitution in consumption of tradables and nontradables. The risk aversion coefficient and the real interest rate are set at values commonly used in quantitative applications of equilibrium business cycle models (σ is set at 2 and the net real interest rate is set at 6.5 percent per year). The elasticity of substitution between tradable and nontradable goods is set to the estimate for developing countries produced by Ostry and Reinhart [1992]. They estimated the elasticity \( \frac{1}{1 + \eta} \) at 0.76 which implies \( \eta=0.316 \). In addition, the deterministic steady state values of \( y^T \) and \( p^N \) are set to 1 as a normalization.

Given the constraints imposed by the average ratios from Mexican data listed in (1)-(6) and the above values of \( \sigma, \eta, R, y^T \) and \( p^N \), the model’s equilibrium conditions at a deterministic steady state produce values for the CES weighing parameters, \( \omega_h = 0.342 \) and \( \omega_y = 2431 \); the income tax rate, \( \tau = 0.239 \); the ratio of transfer payments to households to GDP, \( tr = 0.095 \); the subjective discount factors’s parameters \( \psi = 0.039 \) and \( \psi_g = 1.291 \). In solving for this steady state, investment expenditures are incorporated as lump-sum expenditures so that the model can match the observed consumption and government expenditures shares (even though the model abstracts from capital accumulation decisions).
The second stage of the calibration defines the elements of the joint Markov processes driving shocks to endowment incomes. The two shocks are modeled as zero-mean processes; shocks to the endowment of tradable goods and the world interest rate are set to follow a three-point, symmetric Markov chain following Tauchen (1988). This implies that \( e^T \) and \( e^N \) take three values, with the following properties: \( e^j_1 = -e^j_3 \), and \( e^j_2 = 0 \), \( j = T, N \). Hence, in this specification \( s = 12 \) because there are three realizations of each endowment shock.

The standard deviation and first order autocorrelation of the tradable (nontradable) endowment shock are set to 3.00 (1.74) percent and 0.553 (0.657) respectively. These figures correspond to the standard deviation and first-order autocorrelation of the cyclical component of tradable GDP in Mexico as estimated by Mendoza (2002) using quarterly, seasonally-adjusted data for the period 1980:1-1997:4. For simplicity, shocks to the non-tradables endowment are assumed to be independent of shocks to the tradables endowment. Finally, the basic needs in government expenditures are set to be equal to 50\% of the respective calibrated value of \( g^T \) and \( g^N \), namely \( g^T = 0.0168 \) \( g^N = 0.0428 \).

The model is solved by parameterizing the expectation function following Marcet’s parameterized expectation method. The expectations function is approximated using Chebychev polynomials that depend on the state variables of the model (i.e. the two endowment shocks, the stock of private assets, and the stock of public debt). A detailed description of the solution method is provided in an Appendix available from the authors.

5.1 Results from the Baseline Calibration

The results of the baseline calibration are illustrated in Figure 8 and Table 2. Figure 8 plots the joint marginal limiting probability distribution of the ratio of private total assets to GDP, \( b \), and the ratio of public debt to GDP, \( b^g \). The mean value of \( b^g \) is 7.87 percent, which is about 17 percent of the value of \( b^g \) in a deterministic steady state with identical parameters (which is 45.9 percent). This sharp reduction in the mean value of the public debt ratio between the deterministic and the stochastic solutions reflects the government’s precautionary saving. The reduced debt ratio shows how, given asset market incompleteness and liability dollarization, the volatility of public revenues affects the debt position of a government seeking to smooth its outlays while making a credible commitment to repay its debt and deliver a minimum level of expenditures.

Table 2 shows the first and second moments of the model’s macroeconomic aggregates computed with the limiting distribution plotted in Figure 8. The Table also shows the values of the same variables in the deterministic steady state with identical parameters. By construction, the mean values of the endowments correspond to their deterministic
steady states. However, the mean of aggregate GDP in units of tradables is higher than its deterministic counterpart (which is set to 1) because the mean of the real exchange rate is higher than 1, while the deterministic steady state features \( p^N = 1 \). In turn, the mean price of nontradables is higher than in the deterministic steady state because, as the government reduces its stock of outstanding debt, the government spends less resources on interest payments and has more resources to spend on goods. Given that the supply of nontradable goods is inelastic, the equilibrium value of \( p^N \) is higher than one. The raise in \( p^N \) reduces the private consumption of nontradable goods.

The mean value of the total private assets ratio (which is equal to 0.0477) is equal to 44% of the value of this ratio in the nonstochastic steady state of the model. This is also the result of the precautionary saving effect induced by the household stochastic income; the precautionary-saving-induced portfolio reallocation is, however, superior in the public than in the private sector because the basic needs heighten the government precautionary saving motive. On the other hand, these portfolio reallocations raise the net asset position of the country and reduces the current account deficit. To see why, recall that only the private sector have access to international capital markets; therefore, once households decide their optimal asset position (i.e. the optimal value of \( b^g \)), they offset any change in \( b \) by borrowing or lending in international capital markets.

Table 3 shows the results of a sensitivity analysis that illustrates the dependence of the level of sustainable debt on the deep parameters of the economy. The table reports the results of changing the volatilities of the endowments, the coefficient of risk aversion, and the value of basic needs on the limiting mean and standard deviation of the model variables. Reducing the volatilities of the endowments to 90% of their initial value raises the long-run mean of the sustainable stock of public debt in about 7.6% with imperceptible effects on the allocations of government expenditures. Raising the coefficient of risk aversion from \( \sigma = 2 \) to \( \sigma = 3 \) induces the government to almost fully retire its debt from the market and a similar consequence follows an increment in the level of basic needs from 50% to 60% of the value of both \( g^T \) and \( g^N \) in the non-stationary steady state.

Figure 9 illustrates how the general equilibrium model of section 4 can be used to estimate the dynamics of the stock of sustainable public debt, taking into consideration the equilibrium prices and allocations of the economy. The figure shows the conditional mean of sustainable debt-output ratios at different future dates for three initial debt ratios (-0.10, 0.078 and 0.20). The mean forecasts correspond to the conditional expected value of the public debt ratios implicit in the Markov process of the states variables and describe a low speed of convergence of the debt ratio to its ergodic mean.
Emerging economies seem less able to sustain high ratios of public debt to GDP than industrial countries. At the same time, emerging countries display more volatility in their public revenues and less flexibility in their ability to adjust government outlays. In addition, emerging countries suffer of the syndrome of “liability dollarization” in public debt. That is, they are exposed to large fluctuations in domestic relative prices of nontradable goods to tradable goods while at the same time they leverage public debt denominated in units of tradable goods on the large fraction of revenues they collect from the nontradables sector of their economies.

This environment presents a challenge for a fiscal authority trying to assess whether its tax, expenditures and public debt policies are consistent with the goal of maintaining fiscal solvency. This paper argues that this analysis requires a structural model for mapping the underlying exogenous shocks hitting the economy into endogenous fluctuations in public revenues and relative prices and endogenous dynamics of public debt, and proposes a quantitative methodology for undertaking this analysis and producing estimates of short- and long-run distributions of public debt ratios consistent with fiscal solvency requirements.

The starting point of the method proposed in this paper is the same assumption of a government credibly committed to repay its debt implicit in conventional methods for assessing fiscal solvency (such as the long-run or Blanchard ratios relating public debt to the primary balance and the methods based on econometric tests of the intertemporal government budget constraint). This commitment to repay has important implications in a stochastic environment. In particular, the government imposes on itself a ”natural debt limit” analogous to the endogenous debt limits that households facing non-diversifiable income uncertainty need to satisfy in the literature on savings by heterogenous agents under incomplete markets.

The natural limit on public debt states that the government must not borrow more than it could repay if public revenues were to remain at their lowest and the cost of borrowing at its highest ”almost surely.” This situation is labelled a state of ”fiscal crisis.” Since governments in emerging markets often behave like insurers that wish to do their best to keep outlays smooth in the face of the volatility of revenues and borrowing costs, the model proposed in this paper is designed so the government aims to smooth its outlays by issuing public debt as long as the natural debt limit does not bind. When access to the debt market is limited and the economy arrives at a state of fiscal crisis, the government adjusts outlays to minimum shares of GDP. Thus, the natural debt limit is jointly determined by the variance of the exogenous random shocks hitting the economy, the minimum levels of
outlays the government can commit to undertake, and the endogenous equilibrium relative price of nontradable goods that determines the value of revenues and outlays in the units in which public debt is issued.

The natural debt limit is a central piece of the methodology for assessing fiscal sustainability proposed in this paper, but it is generally not the same as the equilibrium or “sustainable” debt ratio. The latter is obtained from the law of motion of public debt implied by the government budget constraint, and is thus influenced by the exogenous random processes of shocks to income and the world interest rate, the rules setting government outlays for normal and “crisis” times, and the equilibrium stochastic process of the relative price of nontradable goods. The resulting short- and long-run distributions of public debt can feature dynamics in which public debt does hit the debt limit with non-trivial probability in the long run, but they can also predict dynamics of public debt that very rarely hit the debt limit. These distributions are used to produce conditional and unconditional forecasts of debt ratios, time series simulations of public debt paths for given initial conditions, and estimates of the average “duration” of periods of unconstrained access to public debt markets for alternative initial conditions of the public debt-GDP ratio.

The paper shows the results of applying this methodology to the Mexican case and briefly compares these results with other estimates of sustainable public debt ratios. This is done first for a simple case of a one-sector model in which public revenues as a whole are treated as an exogenous random process, and then the full model is solved using a calibration set to mimic key empirical regularities of the Mexican economy. The quantitative results confirm that considering the stochastic elements of the economic environment that drive fluctuations in public revenues and relative prices is critical for assessments of public debt sustainability. In particular, the results show that if the model calibrated to Mexico is started from a low public debt-GDP ratio under 30 percent, there are large differences in the measure of “duration” of public debt market access depending on future realizations of income shocks similar to those observed in Mexican data. On the “average” path implied by the forecasting functions of the model, the average time to a fiscal crisis is above 40 quarters. In a path in which income remains two-standard-deviations below trend, however, the government is predicted to hit its natural debt limit in less than 8 quarters. On average one could expect Mexico to go for as long as 10 years without hitting its public debt limit. But if there are two bad years of income in a row, the government can find its day of reckoning much sooner.

The above results are in sharp contrast with the predictions of conventional methods for calculating sustainable public debt. In the Mexican data, the sustainable debt ratio predicted by the long-run deterministic approach would be 45 percent of GDP (this is the ratio of the difference between average tax revenue minus average total outlays divided by
the difference between the real interest rate minus the average growth rate). As calibrated, the stochastic model yields a natural debt limit on Mexican public debt of 50 percent, which is above this estimate of sustainable debt from the long-run approach. In this benign case the, sticking to the 45 percent estimate would prevent the government from fulfilling its insurer role efficiently (i.e., there are states in which it could exploit the extra 5 percentage points of public debt before hitting the debt limit to keep outlays smooth while fulfilling its commitment to repay). However, slight modifications to consider larger shocks to income or weaker commitments to adjust outlays reduce the natural debt limit below 45 percent, and in these cases aiming to practice a fiscal policy that assumes a debt ratio that can be as large as 45 percent of GDP would be a serious mistake.

References


IMF. Assessing sustainability. SM/02/166, 2002.


Table 1. Estimates of the Natural Limit on the Public Debt-GDP Ratio

<table>
<thead>
<tr>
<th>Standard deviation of t</th>
<th>Coefficient of variation of t</th>
<th>Adjustment in g (percentage points of GDP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>Case 1.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.010</td>
<td>4.37</td>
<td>17.86</td>
</tr>
<tr>
<td>0.015</td>
<td>6.55</td>
<td>0.00</td>
</tr>
<tr>
<td>0.020</td>
<td>8.73</td>
<td>0.00</td>
</tr>
<tr>
<td>0.025</td>
<td>10.92</td>
<td>0.00</td>
</tr>
<tr>
<td>0.030</td>
<td>13.10</td>
<td>0.00</td>
</tr>
<tr>
<td>Case 2.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.010</td>
<td>4.37</td>
<td>7.94</td>
</tr>
<tr>
<td>0.015</td>
<td>6.55</td>
<td>0.00</td>
</tr>
<tr>
<td>0.020</td>
<td>8.73</td>
<td>0.00</td>
</tr>
<tr>
<td>0.025</td>
<td>10.92</td>
<td>0.00</td>
</tr>
<tr>
<td>0.030</td>
<td>13.10</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: The lowest realization of the public revenue-GDP ratio is set two standard deviations below E(t).
Table 2: Stochastic general equilibrium model: value of variables in the non-stochastic steady state and along equilibrium paths.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Non-stoch. value</th>
<th>Long-run mean</th>
<th>Long-run std. dev. (%)</th>
<th>Relative Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>( GDP )</td>
<td>1.0000</td>
<td>1.0511</td>
<td>2.192</td>
<td>1.000</td>
</tr>
<tr>
<td>( y^T )</td>
<td>0.3932</td>
<td>0.3934</td>
<td>1.181</td>
<td>0.539</td>
</tr>
<tr>
<td>( y^N )</td>
<td>0.6068</td>
<td>0.6069</td>
<td>1.054</td>
<td>0.481</td>
</tr>
<tr>
<td>( c^T )</td>
<td>0.2615</td>
<td>0.2719</td>
<td>0.441</td>
<td>0.201</td>
</tr>
<tr>
<td>( c^N )</td>
<td>0.4296</td>
<td>0.4204</td>
<td>0.935</td>
<td>0.426</td>
</tr>
<tr>
<td>( p^N )</td>
<td>1.0000</td>
<td>1.0845</td>
<td>4.465</td>
<td>2.037</td>
</tr>
<tr>
<td>( b )</td>
<td>0.1090</td>
<td>0.0477</td>
<td>6.502</td>
<td>2.967</td>
</tr>
<tr>
<td>( b^g )</td>
<td>0.4590</td>
<td>0.0787</td>
<td>2.134</td>
<td>0.974</td>
</tr>
<tr>
<td>( b^I )</td>
<td>-0.3500</td>
<td>-0.0310</td>
<td>7.058</td>
<td>3.220</td>
</tr>
<tr>
<td>( g^T )</td>
<td>0.0361</td>
<td>0.0414</td>
<td>0.037</td>
<td>0.217</td>
</tr>
<tr>
<td>( g^N )</td>
<td>0.0856</td>
<td>0.0949</td>
<td>0.150</td>
<td>0.068</td>
</tr>
<tr>
<td>( tb )</td>
<td>0.0170</td>
<td>0.0014</td>
<td>1.223</td>
<td>0.558</td>
</tr>
<tr>
<td>( ca )</td>
<td>-0.0052</td>
<td>-0.0005</td>
<td>1.164</td>
<td>0.531</td>
</tr>
</tbody>
</table>

Table 3: Stochastic general equilibrium model: sensitivity analysis

<table>
<thead>
<tr>
<th>Var.</th>
<th>Baseline Calibration</th>
<th>Endowment Volatility</th>
<th>Risk Aversion</th>
<th>Basic Needs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( GDP )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y_T )</td>
<td>Mean = 3.00</td>
<td>Mean = 1.74</td>
<td>Mean = 2.00</td>
<td>Mean = 0.017</td>
</tr>
<tr>
<td>( y^N )</td>
<td>Mean = 2.70</td>
<td>Mean = 1.56</td>
<td>Mean = 3.00</td>
<td>Mean = 0.020</td>
</tr>
<tr>
<td>( c_T )</td>
<td>Mean = 0.043</td>
<td>Mean = 0.051</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c^N )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p^N )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b^g )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b^I )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( g^T )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( g^N )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( tb )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( ca )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: \( \sigma_{y^T} \) is the standard deviation of endowment \( j = T, N \); \( \sigma \) is the coefficient of risk aversion; \( g^j \) represents the basic needs of good \( j = T, N \).
Figure 1. Mean Debt-GDP Ratios and Coefficients of Variation of Public Revenue-GDP Ratios
(IMF 1990-2002)

\[ y = -3.0322x + 80.256 \]

\[ R^2 = 0.2249 \]
Figure 2. Mean Debt-GDP Ratios and Standard Deviation of GDP Growth (IMF-GFS 1991-2001)

\[ y = -7.4156x + 70.885 \]

\[ R^2 = 0.1236 \]
Figure 7. Average Public Revenue-GDP Ratios: 1990-2002
Figure 8. Coefficients of Variation of Public Revenue-GDP Ratios: 1990-2002

![Bar chart showing the coefficients of variation of public revenue-GDP ratios for various countries from 1990 to 2002.](chart.png)
Figure 5: Notes: Time to reach the borrowing constraint in the simplified model

![Chart showing time to reach borrowing constraint](image)

Notes: For each starting value of the debt-GDP ratio, the bars show the average numbers of quarters it takes the debt ratio to hit the maximum debt limit.

Figure 6: Forecast of debt-to-GDP ratios in the simplified model

![Chart showing forecast of debt-to-GDP ratios](image)

Notes: The first bar of each pair of bars in each graph represents the expected value of the debt-to-GDP ratio and the second bar is the mean plus 2 times the standard deviation of the forecasted ratio.
Figure 7: Time series simulations of public debt-GDP ratio in the simplified model

// Notes: Initial value of the debt-GDP ratio is equal to 0.1. Each simulated time series draws realizations of the Markov process

Figure 8: Limiting joint marginal probability distribution of private assets and public debt
Figure 9: Forecast functions for three starting values of public debt-to-GDP ratio