Decentralized International Exchange∗

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Abstract

Traditional models of international trade take place within trading environments in which trade is coordinated by a Walrasian auctioneer who instantly matches buyers and sellers in world markets. This assertion seems inconsistent with evidence suggesting that trading arrangements seem to economize on transactions costs associated with international trade. Recognizing these insights, we depart from standard models of international trade by incorporating the difficulties in the coordination of exchange within a two country random matching model of international trade. We capture a motivation for trade through comparative advantage and interpret trade across economies to be associated with an agent’s endogenous choice to participate in foreign markets. Although an individual may decide to search abroad to obtain additional gains from trade resulting from comparative advantage, she is less likely to meet trading partners abroad as search frictions are higher in the foreign country than in the domestic market. Using this framework, we show that international trade is associated with convergence in relative prices across countries. We also show that the extent of market frictions in the foreign country affects relative prices, the volume of international trade, and welfare levels. Due to market participation externalities associated with international trade, we show that a movement from autarky to free trade may actually lower welfare levels. We conclude by examining the possible inefficiencies associated with international trade due to the decentralized nature of exchange and the policy implications of our model.

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1 Introduction

In attempting to understand the fundamental issues concerning the international exchange of goods and services, two central issues emerge: Why does such exchange take place and how it is accomplished? Traditional trade theory focuses on the former issue in examining the determinants of comparative advantage and investigating the effects of policy on prices, trade volumes, and welfare. As for the latter issue, how exchange occurs, standard trade theory typically assumes that exchange takes place in inherently frictionless environments in which trade is coordinated by a fictitious Walrasian auctioneer instantly matching buyers and sellers on world markets. That is, traditional trade theory takes place within a very specific trading environment in which transactions (aside from physical transportation costs) are costless. This assertion, however, seems inconsistent with evidence suggesting that trading arrangements seem to economize on transactions costs associated with international trade.\(^1\)

In contrast, this paper, we incorporate the difficulty of coordination of exchange within a two country model of international trade. Following Diamond (1982), we dispense with the idea that trade (both within and across economies) is coordinated by the Walrasian auctioneer and formally model a matching and bargaining process in which prices are determined through pairwise, random matching between agents. Specifically, we assume that it is more difficult for an individual to meet potential trading partners abroad rather than in the home country. Such an assumption can be motivated for a host of reasons. In traditional gravity models of international trade, countries which share similar language or culture have a higher volume of trade as such commonalities reduce the difficulties in international exchange. Additionally, informational barriers about the foreign market may hinder market participation as described in Casella and Rauch (1998)\(^2\). Non-tariff barriers of many kinds such as complex regulatory codes regarding import standards are selected by policy makers to make international exchange more difficult. As explicit tariff barriers have been reduced in importance, the impacts of such non tariff barriers on trade and welfare are of particular interest.

\(^1\)See, for example, Frankel, Stein, and Wei (1993) who find that trade is significantly higher between countries that share the same language. In addition, Gould (1994) points out evidence of an “immigration-link” effect—immigration flows to the United States are associated with changes in bilateral trade from the immigrants’ home countries. Rauch (1996) and Casella and Rauch (1997) illustrate the role of pre-existing “ties” used to exchange information about foreign markets in trading “networks” rather than trading in decentralized markets where buyers and sellers are more anonymous.

\(^2\)The idea that trade frictions are likely to be lower for agents within their home country has been utilized in other random matching models studying issues in international economics. See, for example, Matsuyama, Kiyotaki, and Matsui (1993), Trejos and Wright (1996), and Casella and Rauch (1998). This also corresponds to a sense in which language barriers inhibit the ability to engage in international trade as suggested by Frankel et al.
We begin in Section 2 by introducing an autarkic version of our model. Consistent with fundamental insights in trade theory, we capture a motivation for trade through comparative advantage and demonstrate how different economic environments lead to differences in autarkic relative prices across countries. In Section 3, we discuss how the possibility of international trade affects the underlying economic environments in both economies. We interpret trade across countries to be associated with an agent’s endogenous choice to participate in the foreign market. That is, an individual can decide to search abroad where she may have a comparative advantage, but is less likely to meet trading partners due to market frictions. For these reasons, the degrees of market participation in each country will affect the underlying matching probabilities and traded quantities and ultimately welfare in the trading equilibrium.

In Section 4, we adopt some simplifying assumptions and analyze the trading equilibrium in a simplified setting. Focus on studying what we define as a symmetric steady-state trading equilibrium. Upon allowing for trade or economic exchange to take place among agents from different economies, we demonstrate that price equalization may not occur due to increased trade frictions associated with international exchange. As the frictions associated with international trade diminish, however, relative prices across countries converge. We show how the extent of market frictions in the foreign country affects relative prices in both countries. Thus, the volume of international trade is reduced due to higher search frictions in foreign markets. We continue our analysis by investigating the effects of higher search frictions in foreign markets on the volume of trade and welfare of each country. As expected, we show that higher trade frictions in the home country lowers trade volumes and welfare levels in each country. Interestingly, we show that a movement from autarky to trade may actually lower welfare levels.

Finally, in Section 5, we offer some concluding comments and describe additional issues we intend to pursue in future research.

2 Autarky Equilibrium

In this section, we present an autarkic version of the model in order to specify the basics of the underlying economic environment by outlining the structure of tastes, technology, and timing of actions undertaken by agents. We consider an economy in which there are continuum of agents of two types, 1 and 2 where the proportion of type $i$ agents in the economy is given by $\mu_i$ for $i = \{1, 2\}$ and $\mu_1 + \mu_2 = 1$. The types are differentiated in that each type is capable of producing only one of the two completely divisible goods available in the economy. In order to provide a simple motivation for exchange between agents, we impose that agents cannot produce the good that they like to consume. Thus, a type $i$ agent produces good $i$ but only derives utility from the consumption of good $j$ for $i, j = \{1, 2\}$. 


In order to consume, agents must seek out trading partners in the marketplace. The timing of actions undertaken by agents in this pursuit is as follows. First, agents in a decentralized trading environment, randomly encounter potential trading partners where \( \beta \) is the Poisson flow probability of matching with any agent. The probability of matching with an appropriate trading partner, i.e. a type \( i \) agent meets a type \( j \) agent and a double coincidence of wants occurs, additionally depends on the proportion of appropriate partners in the marketplace, \( \mu_i \) for \( i = \{1, 2\} \). When appropriate partners meet, the two agents bargain over the quantities of goods to be exchanged, produce and consume, and then exit the economy to be replaced immediately by an identical pair. In the following sections, we first formally model the bargaining process and then characterize the equilibrium by deriving the expected lifetime utilities, or asset value functions, for the two types of agents as well as the equilibrium quantities exchanged between the types.

2.1 Bargaining Outcomes

Taking as given that two appropriate agents have met in the marketplace (a type \( i \) agent has encountered a type \( j \) agent), they must bargain over the quantities of goods to be exchanged between them. For a given pair of quantities exchanged, \( q_1 \) and \( q_2 \), a type \( i \) agent obtains utility \( u_i(q_j) \) upon consumption of \( q_j \) units of good \( j \) and incurs a cost given by \( c_i(q_i) \) from the production of \( q_i \) units of good \( i \). Both the utility and cost functions are continuous and twice differentiable and satisfy the following restrictions:

\[
\begin{align*}
    u_i'(q_j) &> 0; \ u_i''(q_j) \leq 0; \ c_i'(q_i) > 0; \ c_i''(q_i) \geq 0 \\
    u_i(0) &= c_i(0) = 0; \ u_i'(0) > c_i'(0) = 0
\end{align*}
\]

In addition, the second derivative of the utility function or cost function has a strict inequality and the functions are such that there exists a \( \bar{q} > 0 \) such that \( u_i(\bar{q}) = c_i(\bar{q}) \).

Although many trade determination rules may be adopted, we assume that quantities exchanged in each match are consistent with the symmetric Nash bargaining protocol. This solution is equivalent to that arising from an alternating offers bargaining game as the amount of time between offers becomes negligible.\(^3\)

We assume that agents may search for alternative trading partners while bargaining which implies that the threat point of the type \( i \) agent in the Nash bargaining problem is his expected lifetime utility in the marketplace denoted \( \bar{V}_i \) taken as given in this stage. That is, if the terms of the bargain are not acceptable, the type \( i \) agent can reenter the marketplace and attempt to meet another trading partner and receive expected lifetime utility of \( \bar{V}_i \). The Nash bargaining problem

\(^3\)See Trejos and Wright (1995).
is then given by:

$$\max_{q_1, q_2} \Pi = [\tilde{u}_1(q_1, q_2) - \nabla_1] [\tilde{u}_2(q_1, q_2) - \nabla_2]$$  \hspace{1cm} (1)

Subject to the participation constraints:

$$\tilde{u}_i(q_i, q_j) - \nabla_i \geq 0 \text{ for } i, j \in \{1, 2\}$$  \hspace{1.5cm} (2)

where \(\tilde{u}_i(q_i, q_j) \equiv u_i(q_j) - c_i(q_i)\) denotes the net utility received by a type \(i\) agent in the exchange for \(i, j = \{1, 2\}\).

For an interior solution in which the participation constraints do not bind, the first order conditions are easily derived as:

$$\Pi_{q_i} = -c'_i(q_i) [\tilde{u}_j(q_j, q_i) - \nabla_j] + u'_i(q_i) [\tilde{u}_i(q_i, q_j) - \nabla_i] = 0 \text{ for } i, j = \{1, 2\}$$  \hspace{1.5cm} (3)

(3) then determines the quantities exchanged as functions of the parameters of the cost and utility functions - traditional determinants of comparative advantage - as well as the agents’ threat points, \(q_i (\nabla_i, \nabla_j)\) such that:

$$\frac{\partial q_i}{\partial \nabla_i} < 0 ; \frac{\partial q_i}{\partial \nabla_j} > 0 ; \frac{\partial \tilde{u}_i(q_i, q_j)}{\partial \nabla_i} > 0 ; \frac{\partial \tilde{u}_i(q_i, q_j)}{\partial \nabla_j} > 0$$

### 2.2 Asset Value Functions

We now consider the evolution of the expected lifetime utilities or asset value functions of agents evolve given the outcome of the bargaining process described above. We begin by deriving the asset value functions of agents at time \(t\) taking the quantities exchanged \((Q_{1t}, Q_{2t})\) as given. In a discrete period of time, \(\Delta\), the probability that a producer of good \(i\) meets exactly one good \(j\) producer is approximately \(\beta \mu_j \Delta\). When such a match occurs, the type \(i\) agent receives \(\tilde{u}_i(Q_{it+\Delta}, Q_{jt+\Delta})\) and then exits the economy and is replaced immediately. The probability that he will meet more than one good \(j\) producer will be given by \(\sigma(\Delta)\), where \(\lim_{\Delta \to 0} \sigma(\Delta)/\Delta = 0\). Finally, the probability that he does not meet an appropriate trading partner is then approximately \((1 - \beta \mu_j \Delta)\). In such a case, the type \(i\) agent continues on to the next period with an expected lifetime utility of \(V_{it+\Delta}\). We may thus define the asset value function for a type \(i\) agent at time \(t\) for given values of \(Q_{1t}\) and \(Q_{2t}\) as:

$$V_{it} = \frac{1}{1 + r \Delta} \left\{ \beta \mu_j \Delta(\tilde{u}_i(Q_{it+\Delta}, Q_{jt+\Delta})) + (1 - \beta \mu_j \Delta)V_{it+\Delta} + \sigma(\Delta) \right\}$$  \hspace{1.5cm} (4)

where \(r\) is a time discount parameter identical for both types of agents. We restrict our attention to steady state equilibria in which the quantities exchanged and thus the value functions will be constant over time. The value function for
a type \( i \) agent in such an equilibrium is then obtained by collecting terms in (4) and taking the limit of both sides of as \( \Delta \to 0 \) which provides:

\[
V_i(Q_i, Q_j) = \delta_i(\beta, \mu_j, r)\tilde{u}_i(Q_i, Q_j)
\]

(5)

where \( \delta_i(\beta, \mu_j, r) \equiv \beta \mu_j / (r + \beta \mu_j) \) and

\[
\frac{\partial V_i}{\partial \alpha} = \tilde{u}_i \frac{\partial \delta_i}{\partial \alpha} > 0 \text{ for } \alpha = \{\beta, \mu_j\}
\]

(6)

For given quantities exchanged, the expected lifetime utility of a type \( i \) agent is determined by the term \( \delta_i(\beta, \mu_j, r) \) which characterizes the absolute matching efficiency of the type \( i \) agent. Consider the effect of a marginal increase \( \beta \mu_j \), the ease with which type \( i \) agents are able to meet appropriate partners (type \( j \) agents). The increase can come from either a change in the technology of the matching process, \( \beta \), or from an increase in the proportion of appropriate trading partners in the population, \( \mu_j \). From (4), a marginal increase in \( \beta \mu_j \) has marginal benefit \( \tilde{u}_i(\cdot) \) and marginal cost \( V_{it+\Delta} \). At an interior solution, the net effect is positive on the expected lifetime utility of the type \( i \) agent from the participation constraint, (2).

2.3 Autarkic Equilibrium

We now combine the results of the two preceding sections and characterize the equilibrium for the autarkic model. We restrict our attention to interior solutions in which the participation constraints, given in (2), do not bind and consider solutions to the autarkic equilibrium defined as follows:

**Definition 1:** (Steady-State Equilibrium under Autarky) An (unconstrained) steady-state equilibrium under autarky is a list \( (q_i, Q_i, V_i, V_i) \) for \( i = \{1, 2\} \) satisfying the following conditions: (i) \( q_i = Q_i \) is a solution to the Nash bargaining problem, described by (1) and satisfying the participation constraints in (2) with strict inequality taking \( \nabla_i \) as given; (ii) \( V_i = V_i(Q_i, Q_j) \) satisfies (5) taking \( Q_i \) and \( Q_j \) as given.

In equilibrium, utilizing the equilibrium requirements that \( q_i = Q_i \) and \( \nabla_i = V_i(Q_i, Q_j) \), (3) and (5) provide:

\[
\frac{c'_2(q_2)}{c'_1(q_1)} = \frac{w'_2(q_1)}{w'_1(q_2)} = \left[ \frac{1 - \delta_2}{1 - \delta_1} \right] \left[ \frac{\tilde{u}_2(q_1, q_2)}{\tilde{u}_1(q_1, q_2)} \right] = \delta \left[ \frac{\tilde{u}_2(q_1, q_2)}{\tilde{u}_1(q_1, q_2)} \right]
\]

\[
V_1 = \delta_1 \tilde{u}_1(q_1, q_2) ; \quad V_2 = \delta_2 \tilde{u}_2(q_1, q_2)
\]

First, note that (7) completely determines the equilibrium quantities exchanged. In addition to the parameters of the cost and utility functions, these
quantities are determined by the parameter $\delta$. From (7), it is easily established that:

$$q_1'(\delta) < 0 \; ; \; q_2'(\delta) < 0 \; ; \; p'(\delta) > 0 \; ; \; \tilde{w}_1'(\delta) > 0 \; ; \; \tilde{w}_2'(\delta) < 0$$

(9)

where $p = q_2/q_1$ defines the relative price of good 1 in equilibrium.

$\delta$ is a measure of the relative matching efficiency of the type 1 agent, which is increasing (decreasing) in the absolute matching efficiency of the type 1 (2) agent, $\delta_1$ ($\delta_2$). Using the expressions for $\delta_1$ and $\delta_2$, $\delta_1$ may be expressed as:

$$\delta_1(\mu; r, \beta) = \left( \frac{r + \beta(1 - \mu)}{r + \beta \mu} \right)$$

where $\mu \equiv \mu_1$, the proportion of type 1 agents in the economy. First note that when $\mu = 1/2$, $\delta = 1$, both agents have identical matching efficiencies as the proportions of each type of agent are equal and $r$ and $\beta$ are identical across the two types. Now consider the effect of an increase in $\mu$. As $\mu$ increases type 1 (2) agents become relatively more abundant (scarce) in the population. As a result, the absolute matching efficiency of type 1 (2) agents, $\delta_1$ ($\delta_2$), increases (decreases) - decreasing the relative matching efficiency of type 1 agents and the relative price of good 1 in equilibrium.

Thus, differences in autarky prices across countries can arise from differences in cost and preference parameters, as well as the parameters of the matching process which include the technology itself, $\beta$, and the relative abundance of the two agents, $\mu$. Furthermore, the above analysis demonstrates a standard Heckscher-Ohlin result. If we consider two countries, $A$ and $B$, such that the proportion of type 1 agents is higher in Country $A$, $\mu_A > \mu_B$; the autarkic relative price of good 1 will be lower in Country $A$. We now turn to a consideration of a international trading equilibrium.

## 3 International Equilibrium

### 3.1 The Environment

We now consider an environment in which the world is comprised of two countries, $A$ and $B$. In autarky, these two countries are exactly as described in the preceding section and are comprised of continuum of agents of two types each of which produces a unique good $i = \{1,2\}$ and $\mu_{ik}$ denotes the proportion of good $i$ producers in country $k$ in autarky for $i = \{1,2\}$ and $k = \{A, B\}$. In a general setting, cost and preference parameters may differ across all types of agents such

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4 The detailed expressions are provided in the appendix.

5 Note that there now exist four types of agents in the world economy where agents are indexed both by the type of good they produce and their country of origin. Thus, a type “$ik$” agent produces good $i = \{1,2\}$ and is from country $k = \{A, B\}$.
that \( \tilde{u}_{ik}(q_i, q_j) = u_{ik}(q_j) - c_{ik}(q_i) \) denotes the net utility received by a type \( ik \) producer in a trade in which he produces \( q_i \) in exchange for \( q_j \). \(^6\)

When these countries open to international trade, agents of all types from both countries are free to move between the two distinct marketplaces in countries \( A \) and \( B \) in search of trading opportunities. Within each market, the timing of actions undertaken by agents is identical to that of the autarkic model. Agents randomly encounter potential trading partners and when appropriate partners meet, the agents bargain over quantities, produce and consume, and then exit the economy to be replaced by an identical pair. However, the international setting is differentiated from the autarkic setting in two primary ways. First, in the international framework, an additional stage is added to the actions undertaken by agents in that prior to searching for potential partners, agents must first decide in which of the two markets to participate. \(^7\) Secondly, the agents are not equally efficient in matching with potential trading partners in the two markets. Specifically, it is more difficult for agents to search internationally. Letting \( \beta^l_k \) denote the Poisson flow probability of matching for an agent from country \( k \) participating in the country \( l \) market for \( k, l = \{A, B\} \), we impose the following condition:

\[
\beta^l_k = \beta \text{ for } l = k; \text{ and } \beta^l_k = \beta^* < \beta \text{ for } l \neq k
\]

Thus, international exchange is more difficult as agents are simply possess less efficient matching technology in the foreign markets. As discussed in the introduction, these difficulties in international exchange can encompass many features of international trade such as cultural differences, informational differences, and non tariff barriers to trade.

We next complete the description of the in this general setting and define the requirements of the resulting equilibrium by considering the bargaining outcomes, the determination of the asset value functions, and the market participation decisions of agents. Much of the description of the setting is analogous to the autarky equilibrium albeit expanded to account for the more numerous types of interactions which occur. To keep track of these interactions, it is convenient to introduce the following notation.

There are now four types of agents in the environment, \( ik = \{1A, 1B, 2A, 2B\} \). Each type has two potential trading partners and thus four types of potential matches \( \{1A/1B, 2A/2B, 1A/2B, 1B/2B\} \). Furthermore, each of the four types of appropriate matches can potentially occur in either of the two marketplace. To summarize these interactions, we employ the following notation. For any variable

\(^6\)In the following, we reserve the subscripts \( i, j \) to denote different goods i.e. \( i, j = \{1, 2\} \) and the subscripts \( k, l \) to denote different countries of origin i.e. \( k, l = \{A, B\} \).

\(^7\)In the current setting, we impose that agents must choose between the two markets i.e. they cannot search for partners in both simultaneously. A possible extension of this framework is to consider a case in which agents have a fixed endowment of time which they must optimally allocate between searching in the two markets. Thus, the current framework can be interpreted as a case in which the time endowment is indivisible.
of interest, $x_{ik}$ denotes the value of $x$ for a producer of good $i$ from country $k$ in country $l$'s marketplace for $i = \{1, 2\}$ and $k, l = \{A, B\}$. Additionally, some variables of interest will be specific to the types of agents interacting. For such a variable, $x_{ik,jl}^m$ denotes the value of variable $x$ in a match occurring between types $ik$ and $jl$ in country $m$'s marketplace for $i, j = \{1, 2\}$ where $i \neq j$ and $k, l, m = \{A, B\}$.

### 3.2 Bargaining Outcomes

In describing the bargaining outcomes in this setting, we take as given the market participation decisions and expected lifetime utilities of agents and begin from the point at which two appropriate partners have matched in the marketplace of country $m$.

As in the autarkic setting, agents bargain over the quantities to be exchanged in a way which is consistent with the symmetric Nash bargaining protocol. Letting $\overline{\Pi}_{ik}^m$ denote the expected lifetime utility of a type $ik$ agent in country $m$, the Nash bargaining problem associated with such a match is given by:

$$\max_{q_{ik,jl}^m} \Pi_{ik,jl}^m \left[ \tilde{u}_{ik}(q_{ik,jl}^m) - \overline{\overline{u}}_{ik} - (\overline{\overline{u}}_{jl} - \overline{\overline{u}}_{ik}) \right]$$

subject to the participation constraints:

$$\tilde{u}_{ik}(q_{ik,jl}^m) - \overline{\overline{u}}_{ik} \geq 0$$

Where $q_{ik,jl}^m$ is a $(1x2)$ vector of the quantities of good 1 and good 2 exchanged in a match occurring in country $m$ between types $ik$ and $jl$ for all pairs $i, j = \{1, 2\}$ where $i \neq j$ and for all $k, l, m = \{A, B\}$. Thus for the world economy, there are eight such bargaining problems to examine, one for each of the possible pairings of agents in each of the two markets. The two first order conditions are only notationally different from those described by (3) and the comparative statics of the system have similar interpretations - the terms of trade in any given bargain are determined by cost and preference parameters, in addition to the threat points of the two agents in engaged in bargaining, $\overline{\overline{u}}_{ik}$ and $\overline{\overline{u}}_{jl}$.

### 3.3 Asset Value Functions

In deriving the asset value functions for agents of each type in each of the two markets, we take the bargaining outcomes and the market participation decisions of agents as given. $Q_{ik,jl}^m$ denotes the vector of quantities exchanged between agents in a particular match in country $m$ and $\tilde{\mu}_{ik}$ the proportion of type $ik$ agents in country $m$ which will be endogenously determined from the market.

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8That is, a type $ik$ agent has met a type $jl$ agent for $i, j = \{1, 2\}$ where $i \neq j$ and $k, l = \{A, B\}$ and a double coincidence of wants has occurred.
participation decisions of agents. Using a technique identical to described for the autarky case, the asset value function of an agent in country $m$'s market is given by:

$$V_m^i = \frac{\beta^m_{ik} \tilde{u}_{ik}(Q_{ik,jk}^m)}{r + \beta^m_{ik} \tilde{u}_{ik}(Q_{ik,jl}^m)} + \frac{\beta^m_{ik} \tilde{u}_{ik}(Q_{ik,jl}^m)}{r + \beta^m_{ik} \tilde{u}_{ik}(Q_{ik,jl}^m)}$$  \(11\)

$$= \delta^m_{ik,jk} \tilde{u}_{ik}(Q_{ik,jk}^m) + \delta^m_{ik,jl} \tilde{u}_{ik}(Q_{ik,jl}^m)$$  \(12\)

for all $i, j = [1, 2]$ with $i \neq j$ and for all $k, l, m = (A, B)$ with $k \neq l$.

Note that given the quantities exchanged and market participation decisions the value functions are determined by the absolute matching efficiencies of the agent, described by the parameters $\delta^m_{ik,jk}$ and $\delta^m_{ik,jl}$. However, the absolute efficiency with which an agent matches can vary across the two potential partners with the proportions of these partners in the population. From (11), if $\mu_{jk}^m > \mu_{jl}^m$, the absolute efficiency of matching for a type $ik$ agent is greater with a partner from country $k$ in country $m$'s marketplace, $\delta^m_{ik,jk} > \delta^m_{ik,jl}$.

### 3.4 Market Participation

Finally, taken the value functions and the terms of trade between agents as given, we consider agents’ decisions with regards to market participation. A type $ik$ agent will choose to participate in the marketplace which will provide him with the highest expected lifetime utility. Defining $\gamma_{ik} \in [0, 1]$ as the probability that an $ik$ agent seeks trading opportunities in country $l \neq k$. The problem faced by a type $ik$ agent may be expressed as:

$$\max_{\gamma_{ik}} \gamma_{ik} V_{ik}^l + (1 - \gamma_{ik}) V_{ik}^k$$  \(13\)

If $V_{ik}^l > V_{ik}^k$, $\gamma_{ik} = 1$ and the type $ik$ agent will choose to leave his domestic market and search for partners abroad. Similarly, if $V_{ik}^l < V_{ik}^k$, $\gamma_{ik} = 0$ and the agent remains in his domestic market. If $V_{ik}^l = V_{ik}^k$, any $\gamma_{ik} \in [0, 1]$ will maximize expected lifetime utility for the agent.

### 3.5 International Equilibrium

We now combine the results of the preceding sections and define the equilibrium in the international setting. Again, we restrict our attention to interior solutions in which the participation constraints do not bind and consider steady state solutions to the international trade equilibrium. As in the autarky case, in equilibrium the decisions of individual agents will be consistent with decisions in the aggregate in an equilibrium and thus $V_{ik}^l = V_{ik}^l$ and $Q_{ik,jl} = q_{ik,jl}$ for all
\(i, j = \{1, 2\}, i \neq j\) and \(k, l, m \in \{A, B\}\). Additionally, the market participation decisions of agents, \(\gamma_{ik}\), will be consistent with the equilibrium proportions of agents in each of the market places, \(\mu_{ik}^m\). In equilibrium, the probability that an individual type \(ik\) agents chooses to seek trading opportunities in country \(m\) for \(m \neq k\) will be equivalent to the proportion of such individuals participating in country \(m\)'s market. The equilibrium values of \(\gamma_{ik}\) will then determine the proportions of agents in the two marketplaces i.e. \(\tilde{\mu}_{jk}^m = \mu_{jk}^m(\Gamma)\) where \(\Gamma = \{\gamma_{ik}\}\). Finally, in a steady state equilibrium, \(\gamma_{ik}\) must be constant over time. For any decision \(\gamma_{ik}\), an interior solution will require \(\gamma_{ik} \left(\bar{V}_{ik} - \bar{V}_{ik}^k\right) = 0\).  

\[\text{Definition 2 : (International Trade Equilibrium)}\] A steady-state equilibrium under international trade is a list \((q_{ik,jl}^m, Q_{ik,jl}^m, \gamma_{ik}, V_{ik}^m, \tilde{\mu}_{ik}^m, \mu_{ik}^m)\) for all \(i, j = \{1, 2\}\) where \(i \neq j\) and \(k, l, m \in \{A, B\}\) such that (i) \(q_{ik,jl}^m = Q_{ik,jl}^m\) solves the Nash bargaining problem given by (10), and satisfies participation constraints taking \(\bar{V}_{ik}^m\) as given. (ii) The value functions, \(V_{ik}^m = \bar{V}_{ik}^m\), satisfy (11) taking \(Q_{ik,jl}^m\) and \(\tilde{\mu}_{jk}^m\) as given; (iii) \(\gamma_{ik} \in [0, 1]\), is consistent with \(\tilde{\mu}_{jk}^m\) and solves (??) such that \(\gamma_{ik} \left(\bar{V}_{ik} - \bar{V}_{ik}^k\right) = 0\) for \(l \neq k\) taking \(\bar{V}_{ik}^m\) as given.

The general setting just described allows for an enormous richness in terms of the interactions between the various agents in different settings. In the current paper, we are interested in isolating the welfare effects brought about through the interactions between the market participation decisions of agents and the traditional determinants of comparative advantage such as differences in costs across countries. In order to make the analysis tractable, in the following section we consider a simplified version of the model, consistent with the general setting.

4 A Simple Case

4.1 Assumptions

In order to obtain tractability in the analysis of the international equilibrium, we adopt the following simplifying assumptions. First, we adopt the assumption that while the proportions of agents in autarky as well as parameters of the cost and utility functions may differ across countries, they differ in a symmetric manner. Specifically, we assume that for \(i, j = \{1, 2\}\) and \(k, l = \{A, B\}\) with \(i \neq j\) and \(k \neq l\):

\[
\mu_{ik} = \mu_{jl}, \quad u_{ik}(q) = u_{jl}(q), \quad \text{and} \quad c_{ik}(q) = c_{jl}(q)
\]

\(^9\)This condition will fail only if \(\gamma_{ik} > 0\) and \(\bar{V}_{ik}^l > \bar{V}_{ik}^k\). \(\gamma_{ik} = 1\) will not be consistent with an interior solution and \(\gamma_{ik} \in [0, 1]\) However, this cannot be consistent with an interior solution to the international equilibrium as it implies that trading opportunities are strictly better in the foreign market yet some proportion of agents choose to remain in the domestic market.
The two countries are thus mirror images of one another. This assumption greatly simplifies the framework in that we can completely describe the international equilibrium from the standpoint of one marketplace. Next, we adopt specific functional forms for the cost and utility functions. We assume that costs are quadratic and may vary across agents and countries while utility functions are specified as linear and identical for all agents in both countries and thus for all:

\[ u_{ik}(q) = q; \quad c_{ik}(q) = c_{ik}q^2 \]

Finally, we provide a motivation for international exchange. Recall from the analysis of the autarky equilibrium that differences in autarkic relative prices can be generated in one of two ways: through differences in the relative matching efficiencies of the types and/or through differences in the cost parameters. We choose the latter and impose that \( \mu_{ik} = 1/2 \) for \( i = \{1, 2\} \) and \( k = \{A, B\} \) and specify cost parameters for agents as:

\[ c_{1A} = c_{2B} \equiv \overline{c} < \overline{c} \equiv c_{2A} = c_{1B} \]

Under these assumptions, in this simple setting, it is straightforward to derive the autarkic terms of trade in the two countries. Utilizing (7) and (8) provides the autarkic relative price of good 1 in the two economies as:

\[ p_{AUT}^A = \left( \frac{q_1}{q_2} \right)_{AUT}^A = \left( \frac{c}{\overline{c}} \right)^{1/3} < 1 < \left( \frac{\overline{c}}{c} \right)^{1/3} = p_{AUT}^B \]

The efficient producers in both countries thus receive a lower relative price than their inefficient counterparts in the opposite country and capture our notion of comparative advantage. Upon opening to trade, the variations in relative price provide an incentive for type 1A and 2B agents to move to markets B and A respectively in search of improved trading opportunities. However, these differences in autarkic relative prices are only a necessary condition for international exchange to occur. As it is more difficult to match with potential partners abroad, i.e. \( \beta < \beta^* \), \( \beta^* \) must be sufficiently high so that the cost of accepting the reduced matching efficiency will not outweigh the potential gains from trade. We will assume that this condition holds and will formalize this notion in the following section. Type 1A and 2B agents will thus have an incentive to search for partners in international market places and using the symmetry assumptions, a portion \( \gamma \equiv \gamma_{1A} = \gamma_{2B} > 0 \) of such agents will choose to participate in foreign markets. By contrast, type 2B agents from home (foreign) have no incentive to move as their trading opportunities abroad are worse than they are domestically and thus \( \gamma_{2A} = \gamma_{1B} = 0 \).

4.2 International Equilibrium

We now consider the characteristics of the trading equilibrium in this symmetric environment. From the above discussion, the market in Country A will be
comprised of three types of agents, 1A, 2A, and 2B, comprising proportions 
\((1 - \gamma)/2, \gamma/2,\) and 1/2 respectively of the trading population in A. Furthermore, 
these three types will form two distinct categories of appropriate matches: 1A/2A and 
1A/2B. From (11), we can express the matching efficiencies as of the agents 
in each of the matches as:

\[
\delta_1 = \frac{\beta}{2r + \beta(1 + \gamma)}; \quad \delta_1^* = \frac{\beta\gamma}{2r + \beta(1 + \gamma)}; \quad \delta_2 = \frac{\beta(1 - \gamma)}{2r + \beta(1 - \gamma)}; \quad \delta_2^* = \frac{\beta^*(1 - \gamma)}{2r + \beta^*(1 - \gamma)}
\]

where to conserve of notation, any variable associated with type 2B will be 
differentiated with a * superscript.. Note that the matching efficiencies will be 
determined endogenously by the equilibrium value of \(\gamma\). The associated asset 
value function of agents in A are then given by:

\[
V_1 = \delta_1 \overline{u}_1(q_1, q_2) + \delta_1^* \overline{u}_1(q_1^*, q_2^*); \quad V_2 = \delta_2 \overline{u}_2(q_1, q_2); \quad V_2^* = \delta_2^* \overline{u}_2(q_1^*, q_2^*) \tag{15}
\]

Note that the symmetry assumption implies that the analysis for country B in 
the international equilibrium is identical and only requires the interchange of the 
“1” and “2” subscripts and thus we will focus only on country A in the following 
analysis. In the international equilibrium, three conditions must be satisfied. 
First, for each type of match, the quantities exchanged between the agents must 
be consistent with the solution to the Nash bargaining problem described by (10). 
Additionally, for an interior equilibrium with \(\gamma \epsilon (0, 1)\), \(\gamma [V_2^* - V_1] = 0\) requires 
that \(V_2^* = V_1\).

Consider the Nash bargaining problem between type 1A and 2B agents. Using 
the assumptions described in the preceding section, (10) provides the necessary 
conditions as:

\[
2cq_2^* = \frac{1}{2cq_1^*} = \frac{\overline{u}_2(q_1^*, q_2^*) - \overline{V}_2^*}{\overline{u}_1(q_1^*, q_2^*) - \overline{V}_1^*} \tag{16}
\]

The first equality provides that \(q_2^* = \phi (q_1^*)^{-1}\) where \(\phi^* \equiv (1/4c_2^2).\) Using this 
expression in the second equality and the definition of equilibrium asset value 
funtions provides:

\[
\Omega^*(q, q^*, \gamma; \beta^*) \equiv (1 - \delta_1)\overline{u}_1(q) - \delta_1^* \overline{u}_1(q^*) - (1 - \delta_2^*) [2cq^* \overline{u}_2(q^*)] = 0 \tag{17}
\]

where \(q^*\) and \(q\) denote the quantity of good 1 exchanged in a 1A/2B or 1A/2A 
match respectively and note that \(q_2 = \phi q_1^{-1}\) for \(\phi \equiv (1/4\beta^2).\) Similarly, the 
solution to the Nash Bargaining problem for the 1A/2A match is characterized by:\(10\)

\[
\Omega(q, q^*, \gamma) \equiv (1 - \delta_1)\overline{u}_1(q) - \delta_1^* \overline{u}_1(q^*) - (1 - \delta_2) [2cq \overline{u}_2(q)] = 0 \tag{18}
\]

\[\text{10} \text{Note that evaluating (17) and (16) at } \gamma = 0 \text{ determines } V_{1AUT}^1 \text{ and } (V_2^2)_{AUT}. \text{ By symmetry recall that } V_{1AUT}^1 = V_{2AUT}^2. \text{ A sufficient condition for agents to move internationally is then}

\[V_{2AUT} < (V_2^2)_{AUT}\]
Finally, the equilibrium condition that $\nabla_1 = \nabla_2$ is given by:

$$\Upsilon(q, q^*, \gamma; \beta^*) \equiv \delta_1 \tilde{u}_1(q) + \delta_1^* \tilde{u}_1(q^*) - \delta_2^* \tilde{u}_2(q^*) = 0$$  \hspace{1cm} (19)$$

(17), (18), and (19) provide a system of three equations which completely characterize the endogenous parameters $q$, $q^*$, and $\gamma$. Although the model does not allow for closed form solutions for all of these parameters, consider the Nash bargaining solution for the $1A/2B$ match given by (16).

The two agents have identical cost and preference parameters by assumption and in equilibrium the threat points of the two agents will be identical as $\nabla_1 = \nabla_2$. Thus neither agent in such a match has a relative advantage in matching or costs, and as result in equilibrium, both agents will receive the same surplus from the match and $\tilde{u}_2(q^*_1, q^*_2) = \tilde{u}_1(q^*_1, q^*_2)$. Given the assumptions on the cost and utility parameters, this provides in equilibrium,

$$q^*_1 = q^*_2 = (1/2c) \equiv \bar{q} \text{ and } \tilde{u}_1(q^*_1, q^*_2) = \tilde{u}_2(q^*_1, q^*_2) = (1/4c) \equiv \bar{u}^*$$

### 4.2.1 Participation Externalities

(18) and (19) now describe the remaining endogenous variables $q$ and $\gamma$ which can be completely characterized as a functions of the parameters of the system such that \(^{11}\)

$$q'(\beta^*) < 0 \text{ and } \gamma'(\beta^*) > 0$$

The initial effect of an increase in $\beta^*$ is an increase in the matching efficiency of the $2B$ agent, $\delta_2^*$ from (??), and a consequent increase in the asset value function $V_2^*$. This increase now provides an incentive for additional type $2B$ agents enter the market in $A$ as (19) no longer holds with equality and $\gamma$ increases, $\gamma'(\beta^*) > 0$. The entering agents take $V_2^*$ given, however their entrance generates a number of externalities which impact on the parameters of the marketplace. The increase in $\gamma$ will increase the relative abundance of appropriate partners for the type $1A$ agent (Increasing $V_1$) and decreasing that of both $2A$ and $2B$ generating decreases in both $\nabla_2$ and $\nabla_2$. The increase in $\nabla_1$ will return (19) to equality, while increasing the threat point of the $1A$ agents in their matches with domestic partners and

Straightforward manipulation of these equations provides this condition as:

$$\left(\frac{\tau}{c}\right)^{2/3} > \frac{2r + \beta^*}{\beta^*} \cdot \frac{\beta}{2r + \beta}$$

Note that when international exchange is more difficult, i.e. $\beta^*$ is low, the potential gains from trade, captured by the magnitude of the cost differences between agents, must be large in order to induce agents to move internationally.

\(^{11}\)The details of the comparative statics are provided in the appendix. Also note that $q$ and $\gamma$ are additionally determined by $\beta$ in addition to the cost parameters. However, in the following analysis, we wish to focus on the difficulties in international exchange, $\beta^*$. 

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improving the terms of trade in such matches - decreasing the quantity of good 1 exchanged in such a match \( q'(\beta^*) < 0 \).

Figures 1, 2, and 3 illustrate the impact of changes in \( \beta^* \) in addition to the impact on relative price of good 1 in matches between domestic agents in both countries A and B. for a numerical case in which \( r = 0.05, \beta = 0.3, \mu = \frac{1}{2}, c_1 = 0.5, \) and \( c_2 = 1.12 \). Note in the figures that there exists a range of values for \( \beta^* \) such that changes such that changes in \( \beta^* \) have no impact on endogenous variables. For these values, although the autarkic relative prices will differ across the two countries (illustrated in Figure 2), the costs associated with the difficulties of international search dominate the positive incentives to trade. In the next section, we consider the net effects of changes in \( \beta^* \) on overall welfare.

### 4.3 Welfare Analysis

In this section, we seek to understand the connections between international exchange and the degree of market frictions in terms of their impact on the aggregate welfare of agents. The aggregate welfare of agents in country A’s market can be expressed as:

\[
W(\beta^*) = (1 - \gamma)V_1 + \gamma V_2^* + V_2 = V_1(\beta^*) + V_2(\beta^*)
\]  

(20)

where the latter equality results from the imposition of the equilibrium condition that \( V_2^* = V_1 \). Consider how an increase in \( \beta^* \) affects equilibrium welfare in country A.

\[
\frac{\partial W}{\partial \beta^*} = \frac{\partial V_1}{\partial \beta^*} + \frac{\partial V_2}{\partial \beta^*}
\]

As argued in the preceding section, the effects of the increase on the individual types of agents are unambiguous:

\[
\frac{\partial V_1}{\partial \beta^*} > 0 > \frac{\partial V_2}{\partial \beta^*}
\]

Figure 5 illustrates these effects for the specific numerical example. This result does not seem unique - in any standard model of trade, a movement from trade to autarky will increase the welfare of exporters and harm import competing sectors. However, unlike standard models of trade the net effect on welfare needn’t be positive in this setting. The increase in \( \beta^* \) will have two distinct effects on equilibrium welfare: a terms of trade effect, \( q'(\beta^*) \), as well as an effect on the

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12 Recall that the relative price of good 1 is given by \( p = q_2/q_1 \) and given that \( q_2 = \phi q_1^{-1} \), \( p = \phi q^{-2} \). By the symmetry assumption the relative price of good 1 in country B in the trading equilibrium is simply \( p^{-1} \). Note that relative prices in the two countries will never completely converge due to the differences in search frictions across the two marketplaces and the degree to which they converge is dependent on the magnitude of these frictions.
relative matching efficiencies of agents, \( \gamma'(\beta^*) \), deriving from \( q'(\beta^*) \) and \( \gamma'(\beta^*) \):

\[
\frac{\partial W}{\partial \beta^*} = q'(\beta^*) \left[ \frac{\partial V_1}{\partial q} + \frac{\partial V_2}{\partial q} \right] + \gamma'(\beta^*) \left[ \frac{\partial V_1}{\partial \gamma} + \frac{\partial V_2}{\partial \gamma} \right] \leq 0
\]

Figure 6 illustrates the net effect on welfare for the specific numerical case. As illustrated in the Figure 5, for initially low \( \beta^* \) and correspondingly small \( \gamma \), the welfare is initially improving as the economy benefits from the inclusion of the efficient type 2B producers and the any negative market participation externalities are small. However, as \( \beta^* \) increases, the negative market participation externalities dominate any efficiency gains from trade and as illustrated in Figure 6 welfare may be lower under international trade than autarky.

As noted in the preceding section, in addition to the impact of \( \beta^* \), welfare in equilibrium will vary with domestic market frictions, \( \beta \) as well as the cost parameters and similar analyses could be performed for variations in these parameters. However, \( \beta^* \) is of particular interest in terms of its trade policy interpretations. As noted in the introduction, \( \beta^* \) may be interpreted in many fashions. Interpreted as language and/or cultural differences, \( \beta^* \) is arbitrary and variations in \( \beta^* \) are not of particular interest for policy discussions. However, many difficulties encountered by agents in international markets can be interpreted as endogenous policy decisions i.e. non-tariff barriers to trades of a variety of forms. Interpreting increases in \( \beta^* \) as movements towards freer trade, the above analysis suggests that in this setting global free trade may be welfare reducing. In the example illustrated in Figure 6, a global social planner in this symmetric setting would select \( \beta^* < \beta \) as trade barriers may actually enhance welfare when the market participation and the resulting externalities are more severe.

## 5 Conclusions

Within this framework, there are a host of issues which could be examined. In the simple case here, we motivated international exchange through differences in the agents’ technologies, a Ricardian style framework. However, we can equally consider differences in the relative abundance of agents in the two countries in autarky - a Heckscher-Ohlin style framework and derive similar effects. We intend to pursue a number of extensions of this work in future research.

\(^{13}\)It is possible to demonstrate that when the nature of comparative advantage is more striking (cost differences are higher), the ability to exchange goods across countries may actually lower welfare below the autarky level. This occurs because of market participation externalities arising due to the decentralized nature of exchange. Whereas, increases in \( \beta^* \) lower the costs associated with international exchange encouraging more extensive international market participation, greater cost differences increase the benefit from such participation.
Our first extension involves taking a closer look at the effects of trade policy such as the effects of tariffs and production subsidies in our model. A second extension involves incorporating the idea that individuals must exert effort in attempting to find trading partners as in Hosios (1990). Introducing search intensity provides another dimension in which international trade may affect relative prices, total real production, and welfare. If individuals can raise their levels of search effort in response to the market participation externalities which arise with international trade, it may be possible that the welfare effects of international trade may differ from those presented here. In addition, we believe it would be interesting to examine the interactions between trade policy, search intensity, welfare, and relative prices under this setting.

We also believe there are fundamental issues in international economics which will be interesting to examine in our random-matching context. An interesting example involves studying the interactions between capital accumulation and international trade. Under domestic matching, the productivity of a match may be more certain than when meeting foreign agents. Because of these information problems associated with international exchange, individuals may not have as much incentive to trade with foreigners which, in turn, may affect the initial human capital investments individuals select. Additional issues we intend to pursue involve exploring the interactions between international trade and specialization and studying the links between international trade and the effects of money as a medium of exchange as in Matsuyama et al (1993) and Trejos and Wright (1995).

Given our framework, we can ask if money promotes international trade and welfare. We can also investigate the effects of monetary integration on production specialization, the volume of international trade, and world welfare.

6 Appendix

0.1 Comparative Statics: Autarky Quantities

Defining \( \bar{\delta} \equiv (1 - \delta_1)/(1 - \delta_2) \), Total differentiation of (7) provides:

\[
q_1(\delta) = \frac{\partial q_1}{\partial \bar{\delta}} = \frac{-c_1'\bar{u}_2}{|H|} [c'_1c''_2 - u'_1u'_2] < 0
\]

\[
q_2(\delta) = \frac{\partial q_2}{\partial \bar{\delta}} = \frac{c'_1\bar{u}_2}{|H|} [(c''_1c'_2 - u'_1u''_2)] > 0
\]

where

\[
|H| \equiv \begin{vmatrix}
(c''_1c'_2 - u'_1u''_2) & (c'_1c''_2 - u'_1u''_2) \\
(u''_2\bar{u}_1 - c''_1\bar{u}_2\bar{\delta} - u'_2c'_1(1 + \bar{\delta}) & u'_1u''_2(1 + \bar{\delta})
\end{vmatrix}
\]
Further defining \( p = q_2/q_1 \), the relative price of good 1, as the equilibrium terms of trade, the above provides:

\[
p'(\delta) = \frac{q_1(\delta)q'_2(\delta) - q_2(\delta)q'_1(\delta)}{q_1(\delta)^2} > 0
\]

### 0.2 Proportions of Agents

As noted in the text, in the international equilibrium, \( \gamma_{ik} \) defines the proportion of type \( ik \) agents who are seeking trading opportunities abroad in country \( l \neq k \). Normalize the autarky populations of both countries to 1 and Consider the population of country \( l \) in the international equilibrium. \( \gamma_{il} \) domestic \( i \) agents will have sought international opportunities, leaving \( (1 - \gamma_{il})\mu_{il} \) such agents in country \( l \)'s market for \( i = \{1, 2\} \). Similarly, \( \gamma_{ik}\mu_{il} \) foreign agents (from \( k \)) will arrive in \( l \)'s market to search for potential partners. Thus, the total population of agents in the domestic economy will be given by:

\[
P^T \equiv \sum_{i=1,2} [(1 - \gamma_{il})\mu_{il} + \gamma_{ik}\mu_{il}]
\]

Now the proportions of each type of agent in country \( l \)'s market are easily derived as:

\[
\tilde{\mu}_{il} = \frac{(1 - \gamma_{il})\mu_{il}}{P^T} ; \tilde{\mu}^*_{i} = \frac{(1 - \gamma_{ik})\mu_{ik}}{P^T}
\]

### 0.3 Comparative Statics: The Simple Case:

To derive the equilibrium comparative statics for \( q \) and \( \gamma \), we first completely differentiate (19) and (11) with respect to \( q \), \( \gamma \), \( \beta^* \) which provides the following system:

\[
\Omega_q = \left[ (1 - \delta_1) \frac{\partial \tilde{u}_1(q)}{\partial q} - (1 - \delta_2) \frac{\partial [2cq\tilde{u}_2(q)]}{\partial q} \right] < 0 \tag{21}
\]

\[
\Omega_\gamma = - \left[ \delta_1 [\bar{u}^* - V_1] + \frac{(1 - \delta_2)\delta_2}{(1 - \gamma)} \left[ 2cq\tilde{u}_2(q) \right] \right] < 0 \tag{22}
\]

\[
\Omega_{\beta^*} = 0 \tag{23}
\]

\[
\Upsilon_q = \delta_1 \frac{\partial \tilde{u}_1(q)}{\partial q} < 0
\]

\[
\Upsilon_\gamma = \delta_1 (\bar{u}^* - V_1) + \frac{\delta^*_2(1 - \delta^*_2)}{(1 - \gamma)} \bar{u}^* > 0 \tag{24}
\]

\[
\Upsilon_{\beta^*} = -\frac{(1 - \delta^*_2)}{\beta^*} V_2^* < 0 \tag{25}
\]
and the above expressions can be employed to find:

\[
q'(\beta^*) = -\left(\Omega_\gamma \Omega_\gamma - \Omega_\gamma \Omega_\gamma\right)^{-1} \Omega_\gamma \Omega_\gamma < 0
\]

\[
\gamma'(\beta^*) = \left(\Omega_\gamma \Omega_\gamma - \Omega_\gamma \Omega_\gamma\right)^{-1} \Omega_\gamma \Omega_\gamma > 0
\]
References


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