Why do Banks Fail before BOP Crises? An East Asian Perspective

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Abstract

Recent emerging market experience highlights that, in general, banking crises begin before currency crises. While empirical research is yet to establish the direction of causality, BOP crises have been interpreted and modeled only as consequence of financial sector meltdowns in the recent crisis literature. The evidence points out that equity and real estate prices decline well in advance of BOP crisis and stabilize only after the worst of the currency crisis is over. Drawing our motivation from the observed timing of stock market declines and subsequent bank failures before BOP crises, we develop a framework where we establish a reverse causation between the ‘twin crises’. We show that, if firms heavily rely on debt finance and banks are exposed to firms’ asset valuations, a sudden prospective BOP crisis can cause a banking crisis in the empirically observed chronological order. Next, we explore the existence of self-fulfilling crisis equilibria under the assumption of government bailout guarantees and a predetermined level of net government transfer to the private sector - if banks fail, the government assumes additional public liabilities, which it is subsequently expected to finance through seignorage. It is shown that, if in addition, a decline in firms’ valuations constrain foreign credit, a BOP crisis can not only have severe adverse output effects; it can also be self-fulfilling.

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1 Introduction

One of the main stylized facts that has been established in the wake of recent emerging market crises is that banking and currency crises tend to occur together. This has shifted the focus of the crisis literature from balance-of-payments (BOP) to the ‘twin crises’ - a term coined by Kaminsky and Reinhart (1999) in their study of 26 banking and 76 currency crises episodes. The study also highlights that although banking crises often precede BOP crises, “they are not necessarily the immediate cause of currency crises”. Yet in the recent literature, BOP crises have been interpreted and modeled only as consequence of financial sector crises.

The post-East Asian crisis literature has grown into three directions. One class of models is based on hidden subsidies to investment - via implicit government bailout guarantees to banks. These distortions lead to moral hazards in financial markets, which induce overborrowing and overinvestment and generate a latent fiscal deficit for the government. These deficits being unsustainable, a ‘first-generation’ BOP crisis\(^1\) is inevitable. Some of the prominent works in this category are Krugman (1998), Corsetti, Pesenti, and Roubini (1998 a, b, c), and McKinnon & Pill (1998). On the other hand, Radelet and Sachs (1998) propose that these economies essentially fell victim to investor pessimism, which caused financial panic and self-fulfilling runs. Chang and Velasco (1998a) further add that with fixed exchange rates bank runs lead to BOP crisis if the central bank attempts to act as a lender of last resort. Thus, both of these views essentially hold that banking sector problems cause BOP crisis.

Finally, another class of ‘third generation’ models hold agency problems in financial markets as responsible for generating multiple equilibria. In all these non-monetary expositions, as in Krugman (1999), Aghion, Bacchetta, and Banerjee (1999), Caballero and Krishnamurthy (1999), and Schneider and Tornell (1999), capital inflows depend on the net worth of the domestic sector. If the price of the non-tradable good collapses, the tradable good investment falls, which validates the former and, thus, generates a self-fulfilling equilibrium. In these models, while financial and BOP crises are coterminous, the latter is, again, an implicit outcome of the former.

In this paper, we establish a reverse causality between the ‘twin crises’. Specifically, we ask whether a sudden anticipation of a BOP crisis can cause a banking crisis while generating the empirically observed timing of events. Employing a ‘first-generation’ BOP crisis model, we develop a framework where we find that, if firms heavily rely on debt finance and banks are exposed to firms’ asset valuations, the answer is “yes”. We then go a step further to ask whether a suddenly

\(^1\)See, for example, Krugman (1979) and Flood and Garber (1984). In these models, the BOP crisis results from an obvious inconsistency in the monetary and fiscal policy. The central bank expands domestic credit while keeping a fixed exchange rate. This leads to a secular loss of reserves, eventually forcing the central bank to abandon the peg when reserves reach a lower bound.
anticipated BOP crisis can validate itself through a banking crisis. In particular, we explore the existence of self-fulfilling crisis equilibria under the assumption of government bailout guarantees and a predetermined level of net transfers to the private sector - if banks fail, the government assumes additional public liabilities, which it is subsequently expected to finance through seignorage. We show that if, in addition, a decline in the firms’ valuations restrict their access to foreign credit, a BOP crisis can not only have severe adverse output effects; it can also be self-fulfilling.

Our motivation stems from the following stylized facts, which characterized the recent experience of emerging markets, particularly the East Asian crisis countries\textsuperscript{2}. First, equity prices began declining in advance of BOP crises. Second, the East Asian corporate debt/equity ratios were exceptionally high. Third, banks were exposed to real estate and equity loans, which made them vulnerable to asset prices. Fourth, during the crisis, major part of capital outflow was due to foreign banks. Fifth, firms were credit-constrained during the crisis. Finally, depositors and foreign creditors were often guaranteed by the governments, at least implicitly.

BOP crises are almost invariably associated with a rise in interest rates due to inflationary expectations, increase in default premium, or the government’s defense of the peg. As high-leverage corporates confront a higher prospective debt service, their future balance sheets are expected to deteriorate. Since equity prices are forward-looking, their decline in advance is consistent with an incipient BOP crisis\textsuperscript{3}. Very often banks hold substantial ownership stakes either as equity investments or as real estate loans - the sectors that are highly interest rate sensitive. Hence, a simultaneous decline in equity and real estate prices subsequently causes a deterioration in corporate balance sheets. Moreover, with large-scale loan defaults, banks also end up with collaterals whose market valuations decline. If the asset price impact is large enough, banks’ net worth may become negative - this is what we consider as a banking crisis. Under this scenario, banks fail and declare bankruptcy unless the government comes to their rescue and injects public funds to recapitalize them.

Lacking deep equity and debt markets, firms in emerging markets very often rely on bank-intermediated foreign currency debt. Precisely when domestic banks and firms are in trouble with their deteriorating balance sheets, foreign banks refuse to roll over the existing debt. Even otherwise viable firms face a liquidity crunch and credit-constraints that further aggravate their balance sheets. Thus, we can see the outline of a positive feedback process - an incipient BOP crisis leads to a decline in asset valuations and consequent credit-constraints that restrict firms’ production plans and further depress equity prices. Lower out-

\textsuperscript{2}Namely, Korea, Indonesia, Malaysia, Philippines, and Thailand.
\textsuperscript{3}The decline in equity prices can also partly be due to deteriorating growth prospects or portfolio investment outflows. Here we abstract from these issues.
put and consumption reduce money demand, hence the anticipated BOP crisis is brought forward. Moreover, with the presence of credit-constraints banks’ capital losses are relatively more severe. Hence, in the presence of government bailout guarantees a mere change in agents’ beliefs can be sufficient to generate a self-fulfilling banking and BOP crisis, in an empirically observed order. We feel that this mechanism reasonably explains the severity of the East Asian crises that led many observers to comment that asset price declines and currency devaluations were “well beyond what was justified by any reasonable reassessment of economic fundamentals”.

To address these issues, we use a standard perfect foresight small open economy model that closely follows Lahiri and Végh (2000). In our version of the model, firms need bank credit to hire productive capital and to import inputs. While capital rents are paid in domestic currency, foreign currency credit is required to pay for imports. An increase in domestic interest rates lowers the equilibrium return on capital and, thus, its price. With a sudden anticipation of a BOP crisis, although interest rates are expected to rise in the future when the peg is eventually abandoned, a decline in prospective returns on capital immediately affects asset prices. However, as the productive capital is fixed in the aggregate, output is not directly affected. This is our benchmark model, where although a future BOP crisis generates a banking crisis, it does not have any real effects on firms’ output. In this set up, we find that self-fulfilling equilibria are ruled out if the government runs a budget surplus in the pre-crisis equilibrium.

For asset prices to have real effects and generate self-fulfilling equilibria, we assume the existence of agency problem in foreign credit markets, which takes the form of collateral requirement by foreign creditors. A fall in asset prices leads to binding collateral constraints and consequent adverse real effects. Even if the government runs a budget surplus in the pre-crisis equilibrium, self-fulfilling crisis

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4IMF World Economic Outlook, May 1998, p4
5They introduce bank credit channel in an optimizing, small open economy model, which is anticipating a ‘first-generation’ BOP crisis. They show how policymakers can delay a crisis by raising interest rates. However, it leads to an output loss and may not be optimal from a welfare point of view.
6In our model, a post-BOP crisis nominal interest rate rise leads to an increase in the real lending rate. The channel we utilize for this purpose is the central bank’s requirement of cash reserves, which remains fixed at a predetermined level. We abstract from increase in default premium, or the government’s policy response to fight speculative attacks. Although we assume that the required reserve ratio remains fixed, it is not uncommon for central banks to increase this requirement during a crisis, especially if it is accompanied by bank failures. These considerations will unnecessarily complicate our analysis without adding any further insights. However, it is easy to see that including these factor will only enhance our results.
7Tornell and Schneider (1999b) provide a rationale for this assumption. Typically, in the first stage domestic banks borrow from foreign banks and in the second stage lend it to domestic firms. While the first stage requires no collaterals as foreign creditors enjoy government bailout guarantees, loans to domestic firms are collateralized. For modeling convenience, we merge the two stages.
equilibria exist. We find their existence to be contingent on the level of foreign reserves - above a threshold, a self-fulfilling crisis equilibrium can be ruled out.

Although our model shares the agency problem element with the ‘third-generation’ models as discussed before, in our setting the essential role is played by the sudden anticipation of a BOP crisis. The agency problem along with an incipient ‘first-generation’ BOP crisis allows us to produce much richer dynamics that replicate the stylized facts; banking crises precede currency crises, asset prices decline in advance of currency crises, and capital outflows begin with the banking crises. Further, the agency problem they employ is along the lines of Bernanke and Gertler (1989)\(^8\), while ours is based on the model developed by Kiyotaki and Moore (1995), which emphasizes the dual role that durable assets play as factors of production and as collateral for loans\(^9\). Our motivation for using the latter alternative stems from the fact that productive capital prices can represent equity prices in our representative agent model.

‘First-generation’ crisis model has been used by Burnside, Eichenbaum, and Rebelo (1998) for explaining the East Asian BOP crises. They hold the crises as the outcome of prospective deficits, which the governments had to assume in order to bail out the failing banks. In Burnside, Eichenbaum, and Rebelo (1999), implicit bailout guarantees are the underlying distortion that cause lending booms and encourage banks to take excessive risks. Ultimately a devaluation leads to the bankruptcies that create the anticipated deficits. Further, in Burnside, Eichenbaum, and Rebelo (2000), the presence of bailout guarantees makes a speculative attack inevitable; but its timing depends on self-fulfilling beliefs. Although we share the common features of bailout guarantees and the use of a ‘first-generation’ crisis model, in our framework a banking crisis occurs in anticipation of a BOP crisis, while in their setup - contrary to the empirical evidence - it is a post-devaluation event. In their approach, a BOP crisis is accompanied by a discrete devaluation that forces unhedged banks to default. In our model, banks incur capital losses in anticipation, and the bailout may have to be implemented before the BOP crisis; if the bailout is large enough to validate the anticipation, it generates a self-fulfilling crisis.

In section 2, which follows the introduction, we provide empirical facts that motivate our analysis. In section 3 we set up and analyze the pre-crisis equilibrium of our optimizing, small open economy model. Next, in section 4 we evaluate the

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\(^8\)In their framework, project performance depends on the net worth positions of borrowers. The smaller the net worth of a borrower relative to the size of her project, the more her interests will diverge from those of the financial entity. Asymmetric information about borrower types, and the fact that borrowers end up being insufficiently selective in undertaking investments, result in higher agency costs.

\(^9\)A rationale for collateral constraints is provided in Hart and Moore (1994). If managers have idiosyncratic skills for production, then creditors cannot obtain output from a liquidated firm without the manager. As a result, creditors will never let the size of debt exceed the value of collateral.
real and nominal effects of a ‘first-generation’ BOP crisis where the central bank inconsistently fixes the nominal exchange rate and at the same time expands domestic credit at a constant rate. In this set up, although banks suffer capital losses, there are no real effects. Collateral constraints are introduced in section 5 where we analyze the real effects of the unsustainable exchange rate policy. In section 6, we analyze the existence of self-fulfilling equilibria in the presence of government bailout guarantees and a predetermined level of net transfers to the private sector. Section 7 concludes, and is followed by an appendix.

2 Empirical Motivation

As mentioned in the introduction, the following stylized facts have characterized the recent emerging market crisis, particularly the East Asian crisis: (1) equity prices began declining in advance of BOP crises; (2) the East Asian corporate debt/equity ratios were exceptionally high; (3) banks very often made loans to related parties and were heavily exposed to real estate and equity loans; (4) during the crisis, the major part of capital outflows was due to the foreign banks; (5) firms were credit-constrained; and (6) the depositors and the foreign bank creditors were often guaranteed by the governments. We present some evidence on each of the above in the following paragraphs:

(1) In their study of 76 emerging markets currency crises episodes, Kaminsky and Reinhart (1999) report that “during the 18 months prior to a balance-of-payments crisis, the equity market steadily underperforms (relative to tranquil times).” They further find that the equity prices are past their cyclical peak before the onset of banking crises. Figure 1 shows the evolution of exchange rates and stock indexes in Thailand, Korea, Malaysia, and Philippines - before, during, and after the crises. Clearly, equity prices in all these countries had been declining before the onset of currency crises, and they began to recover only after the culmination of currency crises. Since equity prices are forward-looking, these pictures vividly establish the role of equity prices as leading indicators of an incipient currency crisis.

Declining asset prices in Thailand and Korea provide one the earliest signs of the trouble in the region. During 1996 itself, stock prices (in domestic currency terms) fell by more than 20 percent in Korea and by almost one third in Thailand. It’s worth noting that Korea and Thailand suffered with the earliest major bank failures in the region. For these two countries, Table 1 presents the date-

\footnote{In comparison, no major bank failures/closures were reported in Malaysia and Philippines. However in Malaysia, with the onset of the regional crisis, banks and finance companies experienced a significant decline in profitability and asset quality deteriorated sharply. The level of non-performing loans increased from 3.6 percent in June 1997 to 5.7 percent at end-1997. In Philippines, the deterioration in asset quality led to difficulties in some small banks, but most of the large commercial banks were relatively better capitalized and were able to withstand the}
sequence of the beginning of stock market declines, bank failures/closures, and the eventual end of the exchange rate pegs.

Figure 1: Korea: KOSPI, Thailand: Bangkok S.E.T, Malaysia: Kuala Lumpur Composite, Philippines: Philippines SE composite (Source: Datastream). Exchange rates for Thailand, Philippines, and Malaysia have been scaled up by a factor of 40, 100 and 300 respectively. Exchange rates are shown by the thick lines.

<table>
<thead>
<tr>
<th>Country</th>
<th>Stock market peak</th>
<th>Major bank failures</th>
<th>Currency float</th>
<th>% decline in stock index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thailand</td>
<td>Jan 4, 94</td>
<td>Jan-Aug, 97</td>
<td>Jul 2, 97</td>
<td>67.6</td>
</tr>
<tr>
<td>Korea</td>
<td>Nov 8, 94</td>
<td>Jan-Sep, 97</td>
<td>Nov 17, 97</td>
<td>56.3</td>
</tr>
</tbody>
</table>

increase in bankruptcies and debt restructuring.
We identify major banking sector problems as depositor runs, suspensions, closures or large financial assistance by the government\(^\text{11}\). In Korea, the banking system exhibited increasing signs of stress during the first half of 1997 as a number of major conglomerates went bankrupt\(^\text{12}\). In the first quarter of 1997, two rating agencies downgraded the three major creditor banks of two bankrupt conglomerates and subsequently placed them under review for further downgrades during the summer. By the first week of September, six highly leveraged chaebols had failed or been placed under bankruptcy protection. The increasing weaknesses in Thailand’s financial system were already apparent in the first half of 1997, as demonstrated by the solvency problem of several finance companies. Many commercial banks reported a significant increase in non-performing loans in late 1996 and there were run on the deposits of the Bangkok Bank of Commerce in May 1997. On June 27, just before the baht was allowed to float, the Bank of Thailand suspended 16 finance companies; another 42 were suspended on August 5\(^\text{13}\).

While equity and property prices had been declining in advance of the crises leading to the banks’ deteriorating balance sheets, the crises did not come through without advance triggers. It is worth stressing that the Thai baht came under attack already in November and December 1996. The Korean won was also under pressure in 1996, and had been allowed to depreciate in 1996 and early 1997. These speculative attacks show that the markets were anticipating a future currency crisis.

Hence, the advance decline of equity prices, the timing of major bank failures, the continuing decline in equity prices until the culmination of currency crises, and the recurrent speculative attacks on the Thai baht and the pressure on the Korean won can be interpreted as the evidence that the equity prices had been declining in anticipation of a currency crisis. The declines in stock prices and property prices led to a self-perpetuating process of bankruptcies and bank failures\(^\text{14}\).

(2) It has been argued that one of the main factors that left the East Asian countries vulnerable to a shift in market sentiment was exceptionally high leverage, which was a symptom of excessive risk taking\(^\text{15}\). High leverage, in turn, implies high interest payments on debt. In Table 1, we provide a cross-country comparison of corporate debt/equity ratio and EBITDA/Interest ratio\(^\text{16}\).

\(^{11}\)Although the financial problems usually begin well before a bank is finally closed, we date the crisis when these banks were publicly known to be insolvent.

\(^{12}\)See Coresetti, Pesenti, and Roubini (1998b), pp 4-8 for a detailed description of these events.

\(^{13}\)See IMF (1998a) pp 159-60.


\(^{15}\)See, for example, IMF (1998a), pp34-35, and pp153-156, and Lane et al. (1999), p19. Pomerleano (1998) and Classens et al. (1998) have analyzed the East Asian corporate finance issues in detail and find their leverage to be amongst the highest in the world.

\(^{16}\)Earnings before interest, taxes, and depreciation is an indicator of the adequacy of cash
<table>
<thead>
<tr>
<th>Country</th>
<th>Leverage(^{17})</th>
<th>EBITDA/Interest(^{18})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hong Kong</td>
<td>1.6</td>
<td>11.07</td>
</tr>
<tr>
<td>Indonesia</td>
<td>1.9</td>
<td>2.44</td>
</tr>
<tr>
<td>Korea</td>
<td>3.5</td>
<td>1.07</td>
</tr>
<tr>
<td>Malaysia</td>
<td>1.2</td>
<td>6.74</td>
</tr>
<tr>
<td>Philippines</td>
<td>1.3</td>
<td>3.68</td>
</tr>
<tr>
<td>Singapore</td>
<td>1.0</td>
<td>8.05</td>
</tr>
<tr>
<td>Thailand</td>
<td>2.4</td>
<td>1.92</td>
</tr>
<tr>
<td>Germany</td>
<td>1.5</td>
<td>7.09</td>
</tr>
<tr>
<td>USA</td>
<td>1.1</td>
<td>7.62</td>
</tr>
</tbody>
</table>

These ratios indicate that in the event of a hike in interest rates, countries such as Korea, Thailand, and Indonesia would find their cash flows to be inadequate for interest payments. This clearly explains the reason for the acute cash flow problems, which the corporate sector in these countries faced during the crises. Consequently, their equity prices were most susceptible to an anticipation of a balance-of-payments crisis, which again explains the substantial decline in equity prices in Korea and Thailand.

(3) In the context of the East Asian crisis countries, it is widely held that banks were often part of big conglomerates (e.g. Korean Chaebols) and typically made loans to the firms in the same group, thus, indirectly acquiring ownership stakes in the related firms. Next, banks were heavily exposed to the real estate sector. When the real estate prices fell the banks were very often left with their collateral value. Finally, banks held substantial equity investments both directly or by making loans for equity purchases. Subsequently when equity prices fell, banks incurred direct capital losses or were left burdened with non-performing loans.

Although no quantitative data are available to show the extent and the magnitude of the problem, government directed lending to chaebols in Korea, lending to related parties within large financial (and non-financial) conglomerates in Korea and Thailand, and ownership of weakly regulated banks by non-banks in Indonesia have been extensively documented in past studies\(^{19}\). Moreover, banks also acquired downside risks by lending heavily for investments in property sector flows to pay back the interest on outstanding debt.

\(^{17}\)Source: Classens et al (1998). These numbers are reported to be as at the end of 1996. Also see Lane et al (1999), p19 who report average leverage ratios of 395 and 450 for Korea and Thailand, respectively.

\(^{18}\)Source: Pomerleau (1998). These numbers are as at the end of 1996.

and stock markets. Again, these elements have already been extensively documented in the literature\textsuperscript{20}

(4) Figure 2 shows the net capital flows (in billion dollars) to the five worst-hit East Asian economies. Contrary to the popular notion, it shows that the main culprits in creating a financial panic were the foreign banks and not the portfolio investors. Corporates heavily dependent on foreign bank credit were the worst-hit.

![Figure 2. Capital flows (in billion dollars) to Indonesia, Korea, Malaysia, Philippines, and Thailand. The thick line represents banking sector flows while the thin line represents net portfolio and direct investment.](image)

(5) There appears to be substantial evidence that the firms in the East Asian crisis countries were credit constrained. Ding, Domac and Ferri (1998) and Domac and Ferri (1998) found widespread credit crunch in the crises countries, and they conclude that it particularly affected the small-sized banks and enterprises. In another study, Classens, Djankov, and Ferri (1999) find that “63 % of firms in the five most affected countries are currently illiquid, and risk insolvency unless their liquidity constraints are relieved”.

(6) Finally, the bailout guarantees have been extensively discussed and documented in the literature\textsuperscript{21}. These studies also provide the data related with the extent to which the implicit guarantees became explicit once the crises occurred.

3 The Model

We consider a small open economy perfectly integrated with the rest of the world in both goods and capital markets. The economy is inhabited by a unit measure


each of identical households, firms, and banks. An infinitely lived representative household consumes a perishable good whose world price is fixed at unity. Free goods movement across borders implies that the law of one price holds. A representative firm rents capital from domestic banks and imports an input good from abroad to produce the consumption good. The relative world price of the imported input is fixed at $p$. The firm needs both domestic currency and foreign currency bank credit to make rental payments and to import inputs, respectively. A representative domestic bank intermediates domestic currency denominated demand deposits held by consumers as loans to firms, and also intermediates foreign bank credit, which firms need for imports. The representative bank also owns productive capital, which it leases to firms. The productive capital is fixed in the aggregate\(^{22}\). Finally, firms and banks are owned by households, who receives their profits as dividends. All markets are perfectly competitive.

The assumption of the banks’ ownership of the productive capital has been made to replicate the lending patterns observed in emerging markets, especially the East Asian countries, as has been discussed in section 2. This assumption not only incorporate these features, but also facilitates an analytically tractable transmission of the firms’ cash flow problems to the banks’ balance sheets from the modeling point of view\(^{23}\).

The productive capital is valued by its discounted future earnings and, therefore, can be thought of as the value of the firm if it owned its capital. Since the value of the firm in turn reflects equity prices, we will term the price of productive capital as the asset price in our future references.

### 3.1 Households

The representative household’s lifetime utility is given by

\[
\int_0^\infty e^{-\beta t} (\ln c_t + \nu \ln d_t) \, dt
\]

where $c$ denotes consumption and $d$ denotes the liquidity service offered by domestic banks as domestic currency demand deposits\(^{24}\), which earns a nominal interest at the rate of $i^d$. In addition to demand deposits, households can also hold an internationally traded bond denoted by $b$, with a rate of return $r$. Perfect capital mobility implies that the nominal interest rate is given by $i = r + \epsilon$; where

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\(^{22}\)For example, land, buildings, plant and machinery, which we can assume to remain fixed in the short run without any loss of generality.

\(^{23}\)Our results would be the same if the firms owned their productive capital and the banks held equity ownership in these firms. This requires an exogenous debt finance constraint for acquiring capital, so as to make firms’ capital choice sensitive to domestic lending interest rate. Incorporating these features will unnecessarily complicate our model without adding any further insights.

\(^{24}\)Here $d$ has been expressed in real terms, i.e. in terms of the consumption good.
\[ \dot{a}_t = ra_t + \tau_t + \Omega^f_t + \Omega^b_t - I^d_t d_t - c_t \]  

where \( a = d + b \) is real financial wealth, \( \tau \) denotes real lump sum transfers from the government, and \( \Omega^f \) and \( \Omega^b \) are dividends received from firms and banks, respectively. The household’s opportunity cost of holding demand deposits at banks is obtained as \( I^d_t \equiv i - i^d \). Integrating (2) after pre-multiplication by \( e^{-rt} \), and imposing transversality condition, the household’s intertemporal budget constraint is

\[ a_0 + \int_0^\infty (\tau_t + \Omega^f_t + \Omega^b_t) e^{-rt} \, dt = \int_0^\infty (c_t + I^d_t d_t) e^{-rt} \, dt \tag{3} \]

The household maximizes (1) subject to (3), taking \( \tau, I^d, r, \Omega^f, \Omega^b, \) and \( a_0 \) as given. Assuming \( \beta = r \), the optimal allocations are characterized by

\[ \frac{1}{c_t} = \lambda \tag{4} \]

\[ \frac{\nu c_t}{d_t} = I^d_t \tag{5} \]

where \( \lambda \) is the Lagrangian multiplier associated with (3). The first order conditions have standard interpretation. Equation (4) implies that the marginal utility of consumption is constant over time along a perfect foresight equilibrium path. Further, equation (5) indicates that the marginal utility of holding demand deposits is equal to its opportunity cost in terms of lost marginal utility from consumption. Thus from (5) we have a demand schedule for real demand deposits which positively depends on the household’s consumption and negatively on its opportunity cost given by \( I^d \).

### 3.2 Firms

The representative firm produces the consumption good using the following technology

\[ y_t = n_t^\alpha k_t^{1-\alpha} \tag{6} \]

where \( k \) denotes rented capital and \( n \) denotes perishable imported input. The productive capital is owned by banks and is fixed at \( \bar{k} \) in the aggregate. This particular ownership structure ensures that firms operate with borrowed capital, which makes their capital hiring decisions sensitive to interest rate changes.
the capital is fixed in the aggregate, interest rates affect capital rentals, which in turn affects its price\textsuperscript{25}.

Firms need bank credit both for importing inputs and rental payments. While rental payments require domestic currency loan advances, imports have to be paid through foreign currency credit. These restrictions induce demand for both domestic and foreign bank credit, which are essential elements of our model\textsuperscript{26}. Formally, these ‘credit-in-advance’ constraints are given by

\[ z_t = \phi \rho_t k_t, \quad 0 < \phi \leq 1 \] (7)

and

\[ b^f_t = \psi p n_t, \quad 0 < \psi \leq 1 \] (8)

where \( z \) and \( b^f \) denote the firm’s real domestic and foreign credit respectively, both in terms of the consumption good. \( \rho \) is the rental rate on capital, payable in terms of the consumption good. The lending interest rate on domestic currency loans is \( i^l \), implying an opportunity cost of loans given by the lending spread \( I^l = i^l - i \). The interest rate on foreign currency loans is \( r^b \). Firms choose a path for \( \{k_t, n_t\} \) given the lending and rental rates \( \{i^l, r^b, \rho\} \). A representative firm has the following flow constraint

\[ \dot{a}^f_t = r a^f_t + y_t - p n_t (1 + \psi (r^b_t - r)) - \rho_t k_t (1 + \phi I^l_t) - \Omega^f \] (9)

where \( a^f = -b^f - z \). Firms are owned by households who use their discount factor to maximize the discounted sum of infinite stream of dividends, given by (9) and subject to (6). The firm’s choice for renting capital implies a rental rate given by

\[ \rho_t = \left( \frac{1 - \alpha}{1 + \phi I^l_t} \right) \left( \frac{n_t}{k_t} \right)^\alpha \] (10)

Thus, the capital rental is negatively related to the domestic lending spread. The first order condition with respect to imported input is

\[ n_t = \left( \frac{\alpha}{p (1 + \psi (r^b_t - r))} \right)^{\frac{1}{1 - \alpha}} k_t \] (11)

Equation (11) implies that the demand for the imports is inversely related with its price and the interest rate on foreign currency loans.

\textsuperscript{25}Banks’ ownership of fixed capital serves two purposes; first, they can use it as collateral for intermediating foreign credit to firms, and second, their balance sheets would be sensitive to interest rate changes through its effect on asset valuations.

\textsuperscript{26}The reason behind having the two credit channels is to develop a framework where domestic credit channel affects only the asset price, while the foreign credit channel generates the real effects. Although domestic interest rate affects the capital valuation, that does not, by itself, have an effect on the aggregate output since the capital is fixed in the aggregate. However, if due to agency problems in the credit markets foreign creditors lend to domestic firms only against collateral, a fall in its price may reduce the volume of imports, which in turn will reduce the output.
3.3 Banks

The representative bank takes domestic currency demand deposits from households \( (d) \) and advances domestic currency loans \( (z) \) to firms. It faces a cash reserve constraint imposed by the central bank, which requires it to keep a fraction \( \delta \) of domestic currency deposits as cash reserves \( m \). Banks also intermediate foreign credit to firms, which they obtain from foreign banks at the world interest rate \( r \). Here they play a pure intermediary role without any reserve restrictions imposed by the central bank. In addition, banks can also borrow and lend in the foreign bond markets at the world interest rate \( r \).

For producing domestic currency loans banks use a linear technology, which costlessly converts a unit of demand deposits into domestic currency loans. However, converting foreign borrowing into domestic currency loans entails a per unit cost of \( \eta \). It is straightforward to show that if \( \eta > I^l \), it’s not optimal to convert foreign borrowings into domestic currency loans. We assume that \( \eta \) is high enough, so that this condition always holds in any equilibrium. These assumptions on the banks’ production technology imply

\[
z_t = (1 - \delta) d_t \tag{12}
\]

Banks earn rentals on the capital they own. At every instant they adjust their capital holdings, whose relative price in terms of consumption good is denoted by \( q \). Taking (12) into account, the bank’s flow constraint is given by

\[
-b_t^b = -r b_t^b + I_t^l (1 - \delta) d_t + I_t^d d_t + (r^b - r) b_t^f - i_t \delta d_t + \rho_t k_t - q_t k_t - \Omega_t^b \tag{13}
\]

where \( b_t^b \) is the bank’s real foreign borrowing in terms of the consumption good. Banks are owned by households as shareholders, who maximize the discounted stream of banks’ profits using their own discount rates. Profit maximization with respect to demand deposits yields

\[
I_t^l (1 - \delta) = \delta i_t - I_t^d \tag{14}
\]

Further, zero profit condition for intermediating firms’ foreign credit implies

\[
r^b = r \tag{15}
\]

The first order condition with respect to capital holdings yield the following asset pricing equation

\[
\dot{q}_t = rq_t - \rho_t \tag{16}
\]

---

\textsuperscript{27}Banks may have to incur these costs due to hedging. It is straightforward to show that if \( \eta > I^l \), it’s not optimal to convert foreign borrowings into domestic currency loans. We assume that \( \eta \) is high enough, so that this condition always holds in any equilibrium.
which simply states that the asset price at any point of time is given by the discounted sum of stream of rental payments. To see this, premultiply (16) by $e^{-rt}$, and integrate. After using transversality condition $\lim_{t \to \infty} q_t e^{-rt} = 0$, we get

$$q_t = \int_t^\infty \rho_s e^{-r(s-t)} ds$$

(17)

It follows from (17) that future changes in rental will be immediately reflected in current asset price.

### 3.3.1 Bank Balance Sheet

The representative bank’s balance sheet at any instant of time $t$ is shown below. The bank’s net worth is given by $q \bar{k} - b^b$, which can be assigned to owners’ equity. It is clear that a fall in the value of the bank’s assets will reduce its net worth. If the initial owners’ equity is low, an asset price collapse may, thus, lead to the bank’s net worth becoming negative. In this case, we assume that the bank declares bankruptcy. This is what we consider as a banking crisis.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_t \bar{k}$</td>
<td>$b^b_t$</td>
</tr>
<tr>
<td>$z_t$</td>
<td>$d_t$</td>
</tr>
<tr>
<td>$m_t$</td>
<td>$b^f_t$</td>
</tr>
</tbody>
</table>

To avoid a complete breakdown of the corporate sector in which firms are dependent on bank credit, the government may intervene by bailing out the bank. The recapitalization cost would depend on the magnitude of capital loss suffered by the bank\(^{28}\) at any instant of time, and the pre-crisis level of owners’ equity. The discrete capital loss is given by $\bar{k} \Delta q_t$, where $\Delta q_t$ denotes a discrete fall in asset prices at any instant of time $t$.

### 3.4 Government

The government includes the central bank. Their combined flow constraint is given by

$$-\dot{m}_t + \dot{h}_t = rh_t + \epsilon_t m_t - \tau_t$$

(18)

where $h$ denotes interest paying foreign currency reserves held at the central bank. We assume that the government chooses a domestic credit policy given by

\(^{28}\)In reality, this would be reflected as an increase in non-performing loans.
\[
\frac{\dot{B}^g_t}{B^g_t} = \mu_t \tag{19}
\]

where \( B^g \) is the amount of nominal government bonds held by the central bank. Using central bank’s balance sheet identity \( m = h + b^g \), (18) and (19) imply a specific transfer policy given by

\[
\tau_t = r h_t + \epsilon_t m_t + (\mu_t - \epsilon_t)b^g_t \tag{20}
\]

where \( b^g = \frac{B^g}{E} \), i.e. domestic credit expressed in terms of consumption good.

### 3.5 Resource Constraint

Combining the flow constraints of households, firms, banks, and the government as given by (2),(9),(13), and (18), respectively, the economy’s flow constraint is given by

\[
\dot{f}_t = r f_t + n_t^a k^{1-\alpha} - p m_t - c_t \tag{21}
\]

where \( f = b - b^k - b^f + h \). Notice we have assumed that only households hold positive amount of foreign bonds; banks and firms are net borrowers.

### 3.6 Pre-Crisis Stationary Equilibrium

In this section, we analyze the stationary equilibrium of the economy under a sustainable fixed exchange rate policy. We assume that the government fixes the nominal exchange rate at \( \bar{E} \), and sets the domestic credit growth rate \( \mu \) equal to zero for all times. In this stationary equilibrium, the level of foreign reserves remains constant at \( h_0 \) and the net lump sum transfer is given by \( \tau = rh_0 \) as implied by (20).

Since the aggregate capital is fixed at \( \bar{k} \), the representative firm’s optimal input choice as given by equation (11) implies that the demand for imported inputs is given by (using the fact that \( r^b = r \), from equation (15))

\[
n_t = \bar{n} = \left( \frac{\alpha}{\beta} \right)^{\frac{1}{1-\alpha}} \bar{k} \tag{22}
\]

which implies that the unconstrained demand for imports\(^{29}\) is constant over time. From the consumer’s optimal consumption choice given by equation (4), the consumption is also constant over time; we denote it as \( \bar{c} \). Then, the economy’s

\(^{29}\)If firms have unconstrained access to foreign credit, they obtain a constant level of foreign currency credit equal to \( \psi \bar{p} \bar{n} \) for import payments, where \( \bar{n} \) is given by (22).
resource constraint (21) implies that constant level of consumption will be given by
\[ \bar{c} = rf_0 + (1 - \alpha) \left( \frac{\alpha}{p} \right) \bar{r} \bar{k} \] (23)
where \( f_0 \) denotes the stationary level of net foreign assets of the economy. Thus, the representative household’s permanent consumption is equal to the sum of the interest earned from foreign bonds and the economy’s constant level of output.

The firms’ first order condition (10) for renting capital, with equations (5) and (14) yields a rental rate given by
\[ \rho_t = \frac{(1 - \alpha)(\frac{\sigma}{k})^\alpha + \frac{\nu c}{k}}{1 + \phi_{1-\delta}^{\delta \mu_t}} \] (24)
which implies that the rate of return on capital varies positively with the input use and negatively with the economy’s interest rate. A higher use of input leads to the capital’s higher marginal productivity, thus, earning a higher rent. A higher interest rate leads to higher lending rate, which depresses its equilibrium demand and, therefore, the rent. Its positive dependence on the consumption is explained by the fact that the higher the consumption, the higher will be the demand deposits, which lead to a lower lending rate.

If firms have unconstrained access to foreign credit, the rental rate will be only affected by the interest rate \( i_t \) since both the input \( \bar{n} \) and the consumption \( \bar{c} \) are constant as given by (22) and (23). Further, since the domestic inflation rate is equal to zero, \( i_t \equiv r \). From (16), the stationary level of the asset price will be given by
\[ \bar{q} = \frac{\bar{p}}{r} = \frac{1}{r} \frac{(1 - \alpha)(\frac{n}{k})^\alpha + \frac{\nu c}{k}}{1 + \phi_{1-\delta}^{\delta \mu_t}} \] (25)

4 BOP Crisis and Asset Prices: The Benchmark Model

After having analyzed the economy’s stationary equilibrium with a sustainable exchange rate policy, we now proceed to examine the impact of a sudden anticipation of a future BOP crisis. Specifically, we assume that the economy is originally in an unconstrained stationary equilibrium as described in the previous section. At some time \( t = 0 \), the government adopts a positive domestic credit growth rate given by (19), while keeping the exchange rate pegged at \( \bar{E} \) as before. In the previous steady state the domestic credit growth rate \( (\mu_t) \) was identically set to zero, but now it is fixed at a positive value \( \mu \) (perhaps as a result of a suddenly realized additional expenditure, which can only be financed through inflation). As is standard in Krugman type ‘first-generation’ balance-of-payments crisis models,
we assume that the nominal exchange rate remains fixed at $\bar{E}$ until the central bank’s foreign reserves hit a lower bound $h = 0$. This policy implies that if the rate of money demand growth is lower than the domestic credit growth $\mu$, reserves would be exhausted in a finite time which would force the policy makers to switch to floating exchange rates.

Notice that if firms have unconstrained access to foreign credit, the level of imports and hence the aggregate output will remain constant as in the previous section. Hence, the consumption will also be constant as given by (23). As the rental rate given by (25) remains constant as long as the exchange rate peg prevails (i.e. $i = r$), the level of domestic currency advances to the firms will also remain constant from (7). This further implies that the level of demand deposits will be constant from (12). Therefore, the central bank’s balance sheet identity implies that foreign reserves evolve according to

$$\dot{h}_t = -\dot{b}_t = -\mu b_t = -\mu b_0 e^{\mu t}.$$ 

Hence, it is obvious that the reserves will be exhausted in finite time and the central bank will be forced to abandon the peg at some time $T$ in future. The expected devaluation rate will be given by

$$\epsilon_t = \begin{cases} 0, & 0 \leq t < T, \\ \mu, & t \geq T. \end{cases}$$ (26)

which implies that the path of nominal interest rate will be given by

$$i_t = \begin{cases} r, & 0 \leq t < T, \\ r + \mu, & t \geq T. \end{cases}$$ (27)

At time $T$, the nominal interest rate jumps, and the equilibrium rental rate falls. Accordingly, the level of loan advances given by (7) and the level of demand deposits given by (12) also fall. The quantity of demand deposits $d_T$ will be given by (from equations (7), (12), (22)-(24), and (27))

$$d_T = \phi \bar{k} \frac{(1 + \nu)(1 - \alpha) (\frac{\alpha}{\rho})^{1-\alpha} + \nu r f_0}{1 - \delta + \phi \delta (r + \mu)}.$$

We know that at time $T$, the central bank reserves are exhausted, i.e. $h_T = 0$. Then, the central bank’s balance sheet identity implies that $m_T = \delta d_T = b_T^g = b_0^g e^{\mu T}$. This pins down the time of run on reserves, $T$, which is given by

$$T = \frac{1}{\mu} \log \left( \frac{\delta E d_T}{B_0^g} \right).$$ (29)

Finally, the loss of foreign reserves at time $T$ is given by a fall in money demand, which occurs precisely at time $T$

$$-\Delta h_T = \delta (d_T - d_T) = \delta^2 \phi^2 \mu \bar{k} \left( \frac{(1 + \nu)(1 - \alpha) (\frac{\alpha}{\rho})^{1-\alpha} + \nu r f_0}{(1 - \delta + \phi \delta r)(1 - \delta + \phi \delta (r + \mu))} \right).$$ (30)
Thus, we get the standard result that is obtained from the ‘first-generation’ currency crisis models - an unsustainable exchange rate policy leads to the abandonment of the peg at some point of time in the future. This is perfectly anticipated by the agents who know the exact time of the float and react with a run on the reserves, precisely at the time $T$, which results in a sudden loss of reserves.

Although the sudden change in the policy has nominal implications; it has no real effects. As the capital is fixed in the aggregate and the domestic interest rate does not affect the amount of imported inputs, the output and the consumption remain at their previous stationary level. Nevertheless, it is interesting to note that this policy has an adverse implication for the asset price. Using the pricing equation (17), with the equation for rental rate (24) and the interest rate time path (27), the asset price time path can be derived as

$$q_t = \begin{cases} 
\frac{(1+\nu)(1-\alpha)\left(\frac{\nu}{p}\right)^{\frac{1-\alpha}{1-\phi}} + \frac{\nu r f_0}{k}}{r(1+\phi)^{-\frac{1-\alpha}{1-\phi}}} 
\left[ 1 - \left( 1 - \frac{1-\delta+\phi \delta r}{1-\delta+\phi \delta(r+\mu)} \right) e^{r(t-T)} \right], & 0 \leq t < T, \\
\frac{(1+\nu)(1-\alpha)\left(\frac{\nu}{p}\right)^{\frac{1-\alpha}{1-\phi}} + \frac{\nu r f_0}{k}}{r(1+\phi)^{-\frac{1-\alpha}{1-\phi}}} 
\left[ 1 - \left( 1 - \frac{1-\delta+\phi \delta r}{1-\delta+\phi \delta(r+\mu)} \right) e^{r(t-T)} \right], & t \geq T 
\end{cases}$$

We simulate our model for parameter values of $\nu = 0.05, \alpha = 0.8, \delta = 0.7, \mu = 0.15, p = \bar{k} = 1, f_0 = 2, \phi = 1$, and $\theta = 0.2$. The time of run on reserves is found to be $T = 17.85$. The asset price time path is shown below in figure 3.

**Asset price Dynamics**

![Figure 3](attachment:image_url)

Although there is no real effect on the output and consumption, the price of asset collapses on the knowledge of the new credit growth policy. In other words, the anticipation of an approaching balance of payments crisis has an adverse impact on the asset price which deteriorates banks balance sheets. If the banks have low net worth in the stationary equilibrium, a sudden collapse in the asset price can
cause bankruptcies unless the banks are recapitalized and additional liabilities are assumed by the government. An increase in the government liability can force the government to seek inflationary finance in the absence alternative sources. In section 5 we analyze if this can be the cause a self-fulfilling crisis, which begins with a banking crisis and government bailouts, and then eventually leads to a BOP crisis whose anticipation generated the whole sequence.

5 BOP Crisis with Credit Constraints: Real Effects

In the last section we found that although a sudden anticipation of a BOP crisis affected the time path of asset prices and interest rates, it did not have any real effects. In this section, we introduce financial market imperfections in the foreign credit market. Specifically, due to agency problems, foreign lending institutions require domestic firms to provide collateral against their borrowings. Since firms don’t have the direct asset ownership, collateral on their behalf is provided by domestic banks\(^{30}\). Formally, the credit constraint takes the following form

\[
 b^f_t = \psi p_n t \leq \omega q_t k_t, \quad 0 \leq \omega \leq 1, \quad \gamma \geq 0
\]

which means that foreign banks lend only a fraction \(\omega\) of the current value of the collateral. As long as the asset price is large enough, such that the unconstrained need for credit is less than the value of the collateral, i.e. \(\psi p_n < \omega q_t k\), firms obtain the desired foreign credit given by \(\psi p_n\). Otherwise, if the asset price is low, the constraint will bind. This in turn would constrain the firms’ imports to

\[
 n_t = \frac{\theta q_t k}{p}, \quad \text{where} \quad \theta = \frac{\omega}{\psi}
\]

Suppose that the economy is initially in a stationary equilibrium as described in section 2. Implicitly, we have assumed that the stationary level of the asset price and the credit market parameter values are such that the foreign credit constraint (33) does not bind and the input choice made by firms is unconstrained as given by (11). This will hold if and only if

\[
 (1 + \nu)(1 - \alpha) + \frac{\nu r f_0}{k(\frac{a}{2})^{\frac{\alpha}{1-\alpha}}} \geq \frac{r \alpha}{\theta}
\]

\(^{30}\)One can think of a Korean ‘Chaebol’ with its own bank, which would provide collateral for one of its manufacturing units.

\(^{31}\)Fraction \(\omega\) is indicative of the level of capital market development. Higher the agency problem, the lower would be the fraction of collateral value, which foreign lenders are willing to lend.
Mainly, if the imported input is not too important for production, i.e. its technology share parameter is not too high, and the credit constraint parameter $\theta$ is not too small; equation (34) will hold and firms will not be constrained in the original stationary equilibrium where the domestic interest rate is supposed to remain fixed at the world rate of interest $r$. Graphically, the initial price path corresponds to the time period between -5 to 0 of the figure shown above. Now suppose that at date 0 the government resorts to the exchange rate and the domestic credit policy described in the last section. As discussed before, agents know that this policy is unsustainable and fixed exchange peg will be abandoned at a future date $T$, which is determined endogenously. If firms remain unconstrained, the time path of asset price will follow equation (31). However, it is now possible that even while (34) holds, the asset price becomes low enough between $T \geq t \geq 0$ (as shown in the fourth quadrant of above figure I), such that the collateral value falls below the unconstrained amount of credit. In this case, firms will be constrained for foreign credit, and hence imports, and the pricing equation will no longer be given by (31). There are following three possibilities:

**Case I** The time path is such that the constraint (32) does not bind at $t = 0$; it binds at some point of time in future, $T_u$. We begin by assuming that once the constraint binds at $T_u$, it will keep on binding from $T_u$ onwards. Later, we will show that this is the only possible equilibrium path. Using equations (22), (24), and (33) in the asset pricing equation given by (16), the price time path can be derived as

\[ \dot{q}_t = r q_t - \frac{(1 - \alpha) \left( \frac{\bar{c}}{p} \right)^{\frac{\alpha}{1-\delta}} + \frac{\nu c}{k}}{1 + \phi \frac{\delta i}{1-\delta}}, \quad 0 \leq t < T_u \quad (35) \]

\[ \dot{q}_t = r q_t - \frac{(1 - \alpha) \left( \frac{\bar{c}}{p} \right)^{\frac{\alpha}{1-\delta}} - \frac{\nu c}{k}}{1 + \phi \frac{\delta i}{1-\delta}}, \quad t \geq T_u \quad (36) \]

where the path of the nominal interest rate $i_t$ is given by (27). Notice that we have again assumed a constant level of consumption $\bar{c}$, which is obvious from equation (4). However, when firms are constrained its level will be different from $\bar{c}$ in (23). In what follows we will assume a time-invariant consumption level $\bar{c}$, to be later evaluated as part of the equilibrium. Essentially, we aim to find a fixed point for the consumption as part of the constrained equilibrium.

Crucial to understanding the price time-path is that with perfect foresight it can not jump at any time in the future. The path described by (35) corresponds to the time period while firms remain unconstrained. This time path is unstable around the steady state $\frac{\partial \bar{q}}{\partial q} = r >$; hence, it diverges downwards from its steady state\(^3\). Diverging above its steady state can not be an equilibrium because then

\(^3\text{This steady state lies below the stationary level since } \bar{c} < \bar{c}, \text{ as will be shown later.}\)
it will never be constrained contradicting the initial assumption. Assuming that $T_u$ is known, the initial price level $q_0$ of this path is determined from the fact that at $T_u$ the constraint (32) just binds, implying $q_{T_u} = \frac{\mu}{\delta k}$. For $t \geq T_u$, the time path is characterized by (36), with the initial price level given by $q_{T_u}$. Given $\tilde{c}$, it has two unique positive steady states corresponding to the two different nominal interest rates as given in (27). Around these steady states

$$\frac{\partial \hat{p}}{\partial q} \bigg|_{ss} = r - \frac{(1 - \alpha) \left( \frac{\theta q_{ss}}{p} \right) \alpha - 1 \cdot \frac{\alpha \theta}{p}}{1 + \phi \frac{\delta q_{ss}}{1 - \delta}} = \frac{\nu \bar{c}}{k} + (1 - \alpha) \frac{2 \left( \frac{\theta q_{ss}}{p} \right)^\alpha}{q_{ss} \left( 1 + \phi \frac{\delta q_{ss}}{1 - \delta} \right)} > 0 \quad (37)$$

where the second equality follows from the asset price equation (36) directly. Thus, price path is unstable around the steady state - away from the steady state it would diverge. Using implicit function theorem (36) yields,

$$\frac{\partial q_{ss}}{\partial i} = -\frac{q_{ss} \phi \frac{\delta}{1 - \delta} \left( (1 - \alpha) \left( \frac{\theta q_{ss}}{p} \right) \alpha + \frac{\nu \bar{c}}{k} \right)}{\left( 1 + \phi \frac{\delta q_{ss}}{1 - \delta} \right) \left( \frac{\nu \bar{c}}{k} + \left( 1 - \alpha \right) \frac{2 \left( \frac{\theta q_{ss}}{p} \right)^\alpha}{q_{ss} \left( 1 + \phi \frac{\delta q_{ss}}{1 - \delta} \right)} \right)} < 0 \quad (38)$$

Hence, the steady state of (36) corresponding to $t \geq T$ is lower in value than for $t < T$. In equilibrium, it follows from equation (37) that

$$\dot{q}_t = 0 = r q_{T} - \frac{(1 - \alpha) \left( \frac{\theta q_{T}}{p} \right)^\alpha}{1 + \phi \frac{\delta (r + \mu)}{1 - \delta}} - \frac{\nu \bar{c}}{k} \left( 1 + \phi \frac{\delta (r + \mu)}{1 - \delta} \right), \quad t \geq T \quad (39)$$

Equation (39) is implied from the fact there is no exogenous change in the system beyond $T$, and since there are no inherent dynamics in the model, all variables must be stationary after $T$. The asset price must also reach its steady state value at $T$ in an equilibrium; otherwise it will diverge indefinitely. Above analysis of price time path implies that between time 0 to $T_u$ it diverges downwards from steady state of equation (35), before joining the time path described by equation (36), whence it again diverges downwards from its higher steady corresponding to the nominal interest rate $i = r$, eventually reaching its steady state value corresponding to the nominal interest rate $i = r + \mu$, where it finally settles down from $T$ onwards. It should be obvious that the price level at $T_u$ must be below the first steady state of (36) for this path to be an equilibrium; if it’s above - it will diverge indefinitely upwards and the constraint will never bind, again contradicting our initial assumption. But we still have to pin down $T_u$ and $T$, which we shall do now.

From equation (39), the post-BOP crisis asset price $q_T$ is uniquely determined as a function $q_T(\tilde{c}, \mu, \theta)$. Understanding its properties will be crucial in determining the fixed point of $\tilde{c}$. First, equation (38) directly implies that $\frac{\partial q_{ss}}{\partial \mu} < 0$. Next, we apply the implicit function theorem to equation (39) to get...
\[ \frac{\partial q_T}{\partial \theta} = \frac{1 - \alpha}{\nu c} \left( \frac{\theta q_T}{p} \right)^{\alpha - 1} \frac{\alpha}{p} > 0 \]  \hspace{1cm} (40) \]

Equation (40) implies that the less restrictive the collateral constraint, as measured by the credit market parameter \( \theta \), the higher would be the steady state asset price under the constrained economy. Further, \[ \frac{\partial q_T}{\partial \tilde{c}} = \frac{\nu c}{\kappa} \left( \frac{\alpha}{\nu q_T} \right)^{\alpha} > 0 \]  \hspace{1cm} (41) \]

Equation (41) implies that a higher level of consumption will lead to a higher steady state asset price. This is due to the fact that higher consumption leads to a higher money demand, which increases the supply of domestic currency funds, thus, lowering the lending rate.

Once \( q_T \) is known from equation (39), the demand deposit at \( T \) is obtained using equations (16), (7), and (12) as \[ d_T = \frac{\phi q_T}{1 - \delta} = \frac{\phi q_T}{1 - \delta}. \]

The time \( T \) - when peg is abandoned - can be pinned down using equation (29) as before. We find that \( T \) is a function of steady state price \( q_T \), and can be represented as an implicit function \( T(q_T) \). However, it remains to be verified whether a run on reserves takes place at the time \( T \), i.e. whether level of demand deposits falls down at \( T \). Substituting the constrained input level from (33) in the rental rate equation (24), it’s easy to verify that there is a sudden loss of reserves at time \( T \), which is given by

\[ -\Delta h_T = \delta (d_{T_0} - d_T) = \delta^2 \phi^2 \mu \ k \left( \frac{1 - \alpha}{\nu q_T} \right)^{\alpha} + \frac{\nu c}{\kappa} \left( 1 - \delta + \phi \delta r \left( 1 - \delta + \phi \delta (r + \mu) \right) \right) \]  \hspace{1cm} (42) \]

which is clearly positive; hence, it stands verified that a run on reserves occurs at time \( T \). To complete the analysis, we still need to pin down \( T_u \). Having already found \( q_T \) and \( T \), we have to go backwards to see that equation (36) implies a unique value of \( T_u \) for \( q_T = \frac{\nu c}{\kappa} \). Thus, we have seen that given parameters \( \theta, \mu \), and an assumed value of \( \tilde{c} \), \( q_T(\tilde{c}, \mu, \theta) \) is obtained from equation (39) and the time path of the asset price is completely characterized. Our next concern is to find a solution for \( \tilde{c} \). We shall look into this after we have described the other two possible time paths.

**Case II** This corresponds to the special case of I in which constraint (32) binds from the beginning, i.e. \( T_u = 0 \). The asset price path is described by (36) and (39). Following the same argument as in the last subsection, it is easy to see that the price path diverges downward from its first higher steady state to eventually reach its lower steady state level at time \( T \). As before, given \( \tilde{c} \), \( q_T \) and \( T \) can be
found which implies a unique value of $q_0$. Thus, the equilibrium price path is completely characterized.

**Case III** Notice that $T = 0$ is another possibility when economy reaches its new stationary equilibrium on impact. Given assumed value of $\tilde{c}$, $q_0(= q_T)$ can be obtained from (39). Pinning down the size of the run is straight-forward.

To visualize the difference between the unconstrained and the constrained cases, for the most general case I, the model was simulated for parameter values (same as in section 3), $\nu = 0.05, \alpha = 0.8, \delta = 0.7, \mu = 0.15, p = \bar{k} = 1, f_o = 2, \phi = 1$, and $\theta = 0.2$. The time of speculative attack for unconstrained case was found to be $T = 17.85$. For the model with collateral constraints the time of attack is found to be $T = 16.98$, and $T_u = 1.23$. The price time path is shown below in figure 4.

![Asset Price Dynamics](image)

**Figure 4**

Notice that although the constraint binds only at $T_u = 1.23$, no kink is observed in the time path. This is due to the fact that at $T_u$ the credit constraint just binds so that $\bar{n} = \frac{\theta q_{Tu}}{p}$. Hence, the value of $\dot{q}_t$ is same from both equations (35) and (36), which implies that $\dot{q}_t$ is continuous at $T_u$.

### 5.0.1 Constraint Conditions

We have seen that the economy is unconstrained under a sustainable exchange rate policy if (34) holds. It would remain unconstrained even under an unsustainable policy if the policy and financial market parameters are such that $\frac{\theta q_{Tu}}{p} \geq \bar{n}$, where $q_T$ is given by (31), i.e. even the post-BOP crisis steady state asset price provides a sufficient collateral value to ensure an unconstrained supply of imported inputs. This condition can be written as
\[
\frac{(1 + \nu)(1 - \alpha) + \frac{\nu r f_0}{k(p)} \frac{\alpha}{1 - \alpha}}{(1 + \phi \frac{\delta (r + \mu)}{1 - \delta})} \geq \frac{r - \alpha}{\theta}
\]  
(43)

It is easy to see that if \(\frac{\theta q k}{p} < \bar{n}\), i.e. if (43) does not hold, the economy will be constrained either from the date of announcement of a new policy or from a later date (before \(T\)) depending on the initial level of the asset price described by the time path in (35) and (36). However, it is obvious that once the constraint (32) binds, it will keep binding because the price path can only go downwards. Equation (34) and (43) can be combined to provide a range of values for the credit constraint parameter \(\theta\) as

\[
\frac{r \alpha \left(1 + \phi \frac{\delta (r + \mu)}{1 - \delta}\right)}{(1 + \nu)(1 - \alpha) + \frac{\nu r f_0}{k(p)} \frac{\alpha}{1 - \alpha}} > \theta > \frac{r \alpha \left(1 + \phi \frac{\delta r}{1 - \delta}\right)}{(1 + \nu)(1 - \alpha) + \frac{\nu r f_0}{k(p)} \frac{\alpha}{1 - \alpha}}
\]  
(44)

Notice that even with a high value of \(\theta\), the credit constraint may bind in the interval \([0, T]\), if the post-BOP crisis inflation rate \(\mu\) is large enough to ensure that the first inequality holds. We have already assumed that the second inequality holds, so that the economy is initially in the unconstrained stationary state with the domestic credit growth rate \(\mu\) fixed at 0.

5.1 The Equilibrium Consumption

In all of the above analyses, we have taken a constant value of consumption as given. But it’s obvious that the consumption itself depends on the output stream. In the constrained economy it will, in turn, be determined by the asset price time path. Using the economy’s resource constraint (21), the constant level of consumption for the most general case described under case I, is given by

\[
\tilde{c} = r f_0 + r \int_0^\infty \left(n_t^\alpha k^{1 - \alpha} - pn_t\right) e^{-rt} dt
\]  
(45)

where

\[
\begin{align*}
n_t &= \begin{cases} 
\bar{n}, & 0 \leq t < T_u \\
\frac{\theta q k}{p}, & T > t \geq T_u \\
\frac{\theta q k}{p}, & t \geq T
\end{cases}
\end{align*}
\]  
(46)

\(^{33}\)This condition implies that only for an intermediate range of values for \(\theta\), the economy can switch from an unconstrained pre-crisis stationary equilibrium to a constrained equilibrium, which leads to a higher volatility for asset prices. For values above the range, firms will never be constrained. On the other hand, for very low values of \(\theta\), even the pre-crisis equilibrium will be constrained. Our results are similar to that obtained by Aghion, Bacchetta, and Banerjee (1999) and Caballero and Krishnamurthy (1999) who find that the volatility of capital markets is highest for the intermediate level of capital market development.
Hence, from (45) and (46) the consumption is given by

\[ \tilde{c} = r f_0 + (1 - \alpha) \left( \frac{\alpha}{p} \right)^{\frac{\alpha}{\alpha - 1}} \bar{k} \left[ 1 - e^{-r T_u} \right] \]

\[ + r \bar{k} \int_{T_u}^{T} \left( \left( \frac{\theta q_t}{p} \right)^\alpha - \theta q_t \right) e^{-r t} dt + \bar{k} \left( \left( \frac{\theta q_T}{p} \right)^\alpha - \theta q_T \right) e^{-r T} \]  

(47)

Thus, equations (35), (36) and (47) describe the equilibrium. We have seen that the asset price path is completely characterized for a given value of \( \tilde{c} \). First from equation (39) \( q_T \) is characterized as a function \( q_T(\tilde{c}, \mu, \theta) \). In appendix 8.1, we show that the time path of \( q_t \) in (35) and (36) can be described as a function \( q(t, T(q_T), q_T) \). We can write (47) as a fixed point problem given by \( \tilde{c} = \Gamma^q(q_T(\tilde{c}, \mu, \theta)) \). For the given values of \( \mu \) and \( \theta \), we can represent (47) as

\[ \tilde{c} = \Gamma(\tilde{c}, .) \]  

(48)

We look for a fixed point of (48) to solve for \( \tilde{c} \). In appendix 8.2 we show that \( \Gamma(.) \) exhibits the following properties

\[ \Gamma(., .) > 0, \quad \Gamma_{\tilde{c}} \geq 0, \quad \text{and} \quad \Gamma_{\tilde{c}\tilde{c}} < 0 \]  

(49)

Hence, a fixed point exists. Having established the existence of an equilibrium we proceed further to analyze its properties. In particular, we analyze the dependence of the equilibrium on the credit growth rate \( \mu \), and the agency problem parameter \( \theta \). In our model, a rise in \( \mu \) is an indication of higher future lending rates and thus should have a negative impact on asset prices, output, and consumption. Similarly, a very restrictive foreign credit market as measured by a lower \( \theta \), would also be expected to have adverse real effects.

5.1.1 Effect of Domestic Credit Growth Rate \( \mu \)

Again, we use the properties of \( q_T(\tilde{c}, \mu, \theta) \) from (39) into (47) to characterize \( \tilde{c} \). In appendix 8.2 (79) we show that \( \Gamma_{\tilde{c}} = \Gamma_{q_T} \frac{\partial q_T}{\partial \tilde{c}} > 0 \), which also implies that \( \Gamma_{q_T} > 0 \). We differentiate the fixed point equation (48) with respect to \( \mu \) to obtain

\[ \frac{d\tilde{c}}{d\mu} = \frac{\Gamma_{q_T}}{(1 - \Gamma_{\tilde{c}})} \frac{\partial q_T}{\partial \mu} < 0 \]  

(50)

since from (38) \( \frac{\partial q_T}{\partial \mu} < 0 \). We have utilized the fact that at the fixed point of \( \tilde{c} \), \( \Gamma_{\tilde{c}} < 1 \) since the graph of \( \Gamma \) cuts the 45° line from the above. Equation (50) has an obvious interpretation. With higher credit growth rate, lending rates are expected to be higher. Therefore, in a constrained economy with lower level of asset prices, the output is lower that leads to a lower level of consumption. To contrast this with the model without the credit constraint, remember that the output and the consumption remained unaffected by a change in the domestic credit growth rate.
5.1.2 Effect of Credit Constraint Parameter $\theta$

Similarly, using (40) and the fact that \( \left( \frac{\alpha}{p} \left( \frac{q_t}{p} \right)^{\alpha-1} - 1 \right) > 0 \), the differentiation of the fixed point equation (48) with respect to $\theta$ yields

\[
\frac{d\tilde{c}}{d\theta} = \frac{1}{(1 - \Gamma \tilde{c})} \left( \Gamma \frac{\partial q_T}{\partial \theta} + r \tilde{k} \int_{T_u(q_T)}^{T(q_T(\tilde{c}))} q_t \left( \frac{\alpha}{p} \left( \frac{\theta q_t}{p} \right)^{\alpha-1} - 1 \right) e^{-rt} dt \right)
\]

\[
+ \frac{\tilde{k} q_T}{(1 - \Gamma \tilde{c})} \left( \frac{\alpha}{p} \left( \frac{\theta q_T}{p} \right)^{\alpha-1} - 1 \right) e^{-rT} > 0 \tag{51}
\]

Again equation (51) has an obvious interpretation. The more restricted the foreign credit, the lower will be the consumption, essentially because of the output effect of constrained inputs. Having established these properties for $\tilde{c}$, we can go back to equation (39) and verify that at the equilibrium solution $\tilde{c}$, the properties established partially also hold totally, i.e. $\frac{dq_T}{dT} < 0$, and $\frac{dq_T}{d\theta} > 0$. Since $\frac{dT}{dq_T} > 0$, these results imply that $\frac{dT}{d\mu} < 0$, and $\frac{dT}{d\theta} > 0$.

5.2 The Model With Credit Constraints Vs The Benchmark Model

It is easy to show that the model with the credit constraint brings forward the BOP crisis and results in a lower consumption level relative to the benchmark model. To see the effect on consumption we compare equation (23) with (47). Since the net output in the unconstrained economy is higher, clearly the consumption given by (23) in the benchmark model is higher. Next, to examine the impact on the time of run $T$, we compare the post-crisis stationary level of asset price $q_T$ obtained from equation (31) with that from (39). Clearly $q_T$ is lower in (39), which from (29) also implies a lower time of run $T$. If the credit constraint binds, the dependence of all endogenous variables on the credit growth rate $\mu$ and the credit market parameter $\theta$ has been already described in the previous subsections.

5.2.1 Simulations

We simulate the benchmark model and the model with credit constraints for studying the impact of the credit constraint on the consumption and the time of collapse $T$. In particular, we plot the consumption and the time of collapse for the model with credit constraints, as a fraction of the benchmark model values. The results are presented in figure 5. The top two panels exhibit the effect of credit growth parameter $\mu$ for a fixed value of the credit market parameter $\theta$. 
while in the bottom two panels $\theta$ is variable and $\mu$ is kept fixed.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{\( \nu = 0.05, \alpha = 0.8, \delta = 0.7, \mu = 0.4 \) (with variable $\theta$), $p = \bar{k} = 1$, $f_o = 2$, $\phi = 1$, and $\theta = 0.2$ (with variable $\mu$).}
\end{figure}

5.3 Credit Constraints, Asset Prices, and Banks’ Balance Sheets

Both analytically and through numerical simulations, it has been shown that the presence of collateral constraints brings the BOP crisis forward and has severe adverse real effects. The output is adversely affected, which also affects the level of consumption and welfare. It is shown that the collapse of the asset price is much more severe relative to the BOP crisis without collateral constraints. Again, as in the previous section a sudden anticipation of a BOP crisis will collapse the banks’ balance sheets. If their own initial net worth is low, a collapse in the asset price may force the banks to declare bankruptcy as their net worth becomes negative. Moreover, in comparison to the benchmark model, with the credit constraints banks suffer relatively higher capital losses. Thus, even if they have sufficiently high initial net worth to survive a sudden anticipation of a BOP crisis, they are likely to fail and declare bankruptcy in the presence of the imperfections in the foreign credit market. In the next section we will attempt to formalize the capital losses incurred by banks, which may have to be assumed by the government to avoid bankruptcies. Taking over these additional public liabilities can be a source of a self-fulfilling crisis.

6 Exogenous Government Transfer, Bailout, and Self-Fulfilling BOP Crisis

In previous sections, we have shown that an inconsistent monetary and fiscal policy not only leads to a BOP crisis, but it also generates a banking crisis at
the instant of the sudden policy change, through its impact on the asset prices. Moreover, the crisis is more severe and more likely\footnote{In our perfect foresight model, likelihood relates to the level of initial net worth of banks. Even with a sufficiently high initial net worth, banks may face bankruptcy in the presence of credit constraints.} if firms are constrained for foreign credit. Now we want to proceed a step further to examine whether the capital loss incurred by banks can, by itself, cause the anticipated crisis if the government takes some or all of the suddenly created liabilities.

With the sudden anticipation of a BOP crisis, banks suffer a net worth loss due to the asset price collapse, which is given by

\[ -\Delta Nw^b = (\bar{q} - q_0)\bar{k} \tag{52} \]

where \(\bar{q}\) is the pre-crisis stationary level of the asset price given by (25), and \(q_0\) is the post-shock price at \(t = 0\), when agents suddenly anticipate a BOP crisis to occur at some future date \(T\). We assume that foreign investors would only invest in domestic banks to the point where the bank’s net worth is non-negative and if banks net worth becomes negative they refuse to roll over the existing level of debt. If the banks’ net worth becomes negative, banks go bankrupt unless the government bails them out by assuming their losses as public liabilities. We can think of the government taking over the failed banks and handing it over to new managements after recapitalizing them and making them viable again. Then the additional public liability taken over by the government at time \(t = 0\) is given by \(\Delta Nw^b\).\footnote{We are essentially assuming here that the banks begin with a zero initial net worth. This assumption is only for the analytical convenience. Our results will hold even if the banks have an initial positive net worth.} Next, we proceed to examine if this sudden increase in government liability can validate the anticipated BOP crisis in a self-fulfilling manner.

6.1 Transfer Policy, Post-Crisis Devaluation Rate and The Size of Run

For simplicity, we assume that the government does not issue public debt. It holds foreign reserves and makes a constant level of net lump sum transfer \(\tau\) to consumers. Hence, \(\tau\) is in a sense primary deficit of the government. However, before time \(t = 0\), the government is running net surplus so that at time \(t = 0_\), \(rh_{0_\slo} > \tau\). We further assume that due to political economy considerations, the lump sum transfers can not be reduced in the event of any additional burden of fiscal liabilities to be undertaken by the government. As before, we assume that the government abandons the exchange rate peg once its foreign reserves are exhausted. If a BOP crisis is expected to occur on a future date \(T\), the government’s flow constraint (18) implies that

\[ \mu = \epsilon = \frac{\tau}{\delta d_T} \tag{53} \]
where $\mu$ and $\epsilon$ denote the constant level of the base money growth rate and the devaluation rate, respectively, and $d_T$ denotes the constant domestic currency demand deposit from time $T$ onwards. It is easy to see that these variables will be constant from time $T$ onwards, as no exogenous changes occur after this date. Further, under this policy the expected devaluation rate from time $0$ onwards will be again given by (26). Using the capital rental rate equation (24) with (7), (12), (26), and (53) the rental rate from time $T$ onwards is given by

$$\rho_T = \frac{(1 - \alpha)\left(\frac{n_T}{k}\right)^\alpha + \frac{\nu e}{k} - \frac{\tau}{k}}{1 + \phi \frac{\delta r}{1 - \delta}}$$

(54)

while at time $T_-$, it is given by

$$\rho_{T_-} = \frac{(1 - \alpha)\left(\frac{n_T}{k}\right)^\alpha + \frac{\nu e}{k}}{1 + \phi \frac{\delta r}{1 - \delta}}$$

(55)

Where $n_T = \bar{n}$ or $= \frac{\theta q_T k}{p}$ depending on whether firms are unconstrained or constrained for foreign credit. From equations (54) and (55), and using (7) and (12), the size of the run on reserves is given by

$$- \triangle h_T = \delta (d_T - d_T) = \frac{\tau \phi}{1 - \delta} \frac{1 - \phi \frac{\delta r}{1 - \delta}}{1 + \phi \frac{\delta r}{1 - \delta}}$$

(56)

Equation (56) shows that the fall in money demand at $T$ is proportional to the government level of net transfers, which is now completely financed by the seignorage.

6.2 The Benchmark Model: Can a Self-Fulfilling BOP Crisis Occur?

In section 3 we showed that if firms are not credit constrained, there are no real effects. However, the asset price collapses at the instant of the announcement of an inconsistent fiscal and monetary policy, and follow a downward trend thereafter, before reaching its minimum at the instant of the exchange rate collapse. We begin by assuming that the government assumes total capital losses of banks as an additional public liability. This implies that at the time of the sudden anticipation of the BOP crisis, i.e. at time $t = 0$, the government’s foreign reserves fall discretely by an amount given by $(\bar{q} - q_0)k$, where $\bar{q}$ is the stationary asset price level before the anticipation of BOP crisis. The government’s flow constraint (18) for the interval $T > t \geq 0$, can be simplified as

36 We can think that in order to salvage the failed banks, the government pays off part of their outstanding foreign debt from its foreign reserves.
\[
\dot{h}_t = rh_t - \tau
\]  
(57)

since the money demand is constant and the rate of devaluation is equal to 0.

Integrating equation (57) over the interval \([0, T)\), after pre-multiplication with \(e^{-rt}\), we obtain

\[
h_0 - h_{T-} e^{-rT} = \frac{\tau}{r}(1 - e^{-rT})
\]  
(58)

where the post-shock level of the government’s foreign reserves \(h_0 = h_{0-} - \bar{q}\bar{k} + q_0\bar{k}\), where \(h_{0-}\) denotes the pre-shock government foreign reserves. Further, the pre-collapse level or reserves is same as the size of run; therefore, \(h_{T-} = - \Delta h_T\) as given in equation (56). From the asset pricing equation (16), the asset price at time \(t = 0\) is given by

\[
q_0 = \bar{q}(1 - e^{-rT}) + q_T e^{-rT}
\]  
(59)

where \(\bar{q}\) is given by (25). Using equation (16) and (54), the post-collapse price \(q_T\) is given by

\[
q_T = \frac{1}{r} \left[ (1 - \alpha) \left( \frac{\bar{a}}{\bar{k}} \right)^{\alpha} + \frac{\nu e}{\bar{k}} - \frac{\tau}{r} \right]
\]  
(60)

Using equations (25), (59), and (60) equation (58) can be rewritten as

\[
h_{0-} - \frac{\tau}{r} = \frac{\tau}{r} \frac{e^{-rT}}{1 + \phi \frac{\delta r}{1 - \delta}} e^{-rT} + h_{T-} e^{-rT} - \frac{\tau}{r} e^{-rT} = 0
\]  
(61)

The last equality follows from since \(h_{T-}\) is given by (56). Hence, equation (61) can hold if and only if \(h_{0-} = \frac{\tau}{r}\), in which case any value of \(T\) is consistent with the crisis equilibrium. If \(h_{0-} > \frac{\tau}{r}\), as we initially assumed, a self-fulfilling crisis equilibrium is ruled out. Again, if \(h_{0-} < \frac{\tau}{r}\), in a world of perfect foresight crisis would have already occurred, hence this can not be an \emph{anticipated} self-fulfilling crisis equilibrium.

The knife-edge characteristic of crisis for the balanced budget case \((h_{0-} = \frac{\tau}{r})\) with an indeterminate \(T\) is very interesting and intuitive. As discussed before, a BOP crisis in our benchmark model does not have any real effects since there are no external constraints. Corporate sector is vulnerable to the capital loss only because their operating costs are higher due to the anticipated higher lending spread in the wake of the BOP crisis. In a completely circular manner, these extra costs are incurred due to the government’s need of extra revenues to bear the capital loss suffered by the corporate sector. Thus, if there is any foreign reserve surplus with the government, BOP crisis can never occur. If the budget is in exact balance initially, the time of run becomes indeterminate. Although an initial condition such as \(h_{0-} = \frac{\tau}{r}\), has interesting implications for crisis timing, we should be skeptical about its practical relevance. An interesting case to analyze
would be when the government has a comfortable reserve surplus position, i.e. $h_0 > \frac{\tau}{r}$. If this is so, self-fulfilling crisis is ruled out in the benchmark model.

### 6.3 Credit Constraints and Self-Fulfilling Crisis Equilibrium

Next, we examine the possibility of a self-fulfilling BOP crisis in our model with credit constraints. For notational consistency we will keep the same notations as in section 4.

With a fixed domestic credit growth policy such as in section 4, we know that if the policy announced at time 0 is inconsistent, a BOP crisis is bound to occur at a future date $T$, which is endogenously determined. In the present case we start with the assumption that there is no policy inconsistency to begin with, and economy can indefinitely remain at its unconstrained steady state; agents never expect a BOP crisis to occur. Then, we want to specifically analyze that if at any time $t = 0$, agents suddenly change their beliefs and expect a BOP crisis to occur at some time $T$ in future, what are the conditions on the initial level of reserves and the level of government net lump-sum transfers which can make this belief as self-fulfilling. Note that in our benchmark model, if the government has been running a fiscal surplus, i.e. if $h_0 > \frac{\tau}{r}$, a self-fulfilling crisis is ruled out.

We start by assuming that at time $t = 0$, agents expect a BOP crisis to occur at time $T$. This implies that even if firms are not credit constrained, the path of asset prices follows a downward trend, reaching its minimum level $q_T$ at time $T$. We assume that this level is such that $\theta q_T \bar{k} < p \bar{n}$, so that firms become credit constrained at some point of time in the interval $[0, T)$. Using the post-crisis pricing equation (60) with (23), this condition implies that

$$
\tau > \left(1 + \nu \right) \left(1 - \alpha \right) - \frac{r \alpha}{\theta} \left(1 + \phi \frac{\delta r}{1 - \phi}\right) \left(\frac{\alpha}{p}\right) \frac{1}{\theta} \bar{k} + \nu f_0 .
$$

We assume that net government transfer $\tau$ is large enough to satisfy equation (62), so that the crisis equilibrium is credit constrained.

We first show that an equilibrium exists and analyze the equilibrium properties. Then we proceed to determine the initial level of foreign reserves that will be consistent with this equilibrium. In particular, we pin down the level of reserves which will be consistent with the timing of the BOP crisis. For the economy with credit constrained firms, the asset price dynamics will still be described by equations (27), (35) and (36) and post-BOP crisis inflation rate $\mu$ will be given by equation (53). Using the post-collapse rental rate equation (54), the post-collapse pricing equation (39) can be rewritten as

$$
\dot{q}_T = rz_T - \frac{(1 - \alpha) \left(\frac{\theta q_T}{p}\right)^{\alpha}}{1 + \phi \frac{\delta r}{1 - \phi}} - \frac{\nu c}{\bar{k}} - \frac{\tau}{\bar{k}} = 0, \quad t \geq T
$$

(63)
which implies that $q_T$ can be expressed as a function $q_T(\tilde{c}, \tau, \theta)$. It is straightforward to show that

$$\frac{\partial q_T}{\partial \tilde{c}} > 0, \quad \frac{\partial q_T}{\partial \theta} < 0, \quad \text{and} \quad \frac{\partial q_T}{\partial \tau} < 0 \quad (64)$$

The time path of asset prices will be as described in section 4. For pinning down the time of run $T$, we have to look at the government’s flow constraint, which is given by

$$\dot{h}_t = rh_t + \delta \dot{d}_t - \tau, \quad T > t \geq 0 \quad (65)$$

In appendix 8.3 we show that equation (65) yields

$$h_{0_-} - \bar{k}(\bar{q} - q_0) \left(1 + \frac{\delta r \phi}{1 - \delta}\right) = \frac{\tau}{r} (1 - e^{-rT}) \quad (66)$$

where $h_{0_-}$ denotes the initial pre-crisis level of foreign reserves. Equation (66) implies that $T$ can be expressed as a function $T(q_T, \tau, h_{0_-})$. From appendix 8.1 we know that the asset price $q_t$ can be expressed as a function $q(t, T(q_T; \cdot), q_T)$, and its derivative with respect to $q_T$, $q_T$ is given by (77). Using (77) and differentiating (66) with respect to $q_T$, we obtain

$$\frac{\partial T}{\partial q_T} = \left(1 + \frac{\delta r \phi}{1 - \delta}\right) \frac{\partial q_0}{\partial q_T} > 0 \quad (67)$$

since from equation (77) we have $\frac{\partial q_0}{\partial q_T} > 0$. Thus, as in section 4, the equilibrium of a credit constrained economy anticipating a BOP crisis at $T$ is completely determined by equations as in section 4. Following the arguments made there, we can write the equilibrium as a fixed point of $\tilde{c} = \Gamma(q_T(\tilde{c}, \tau, \theta, h_{0_-})) = \Gamma(\tilde{c}; \cdot)$. In appendix 8.4 we show that $\frac{\partial q_T}{\partial \tilde{c}} > 0$, and $\frac{\partial^2 q_T}{\partial \tau^2} < 0$. Using these properties with appendix 8.1, we can again show that

$$\Gamma(\cdot; \cdot) > 0, \quad \Gamma_{\tilde{c}} \geq 0, \quad \text{and} \quad \Gamma_{\tilde{c} \tilde{c}} < 0 \quad (68)$$

which ensures that a fixed point of equilibrium consumption exists. Once $\tilde{c}$ is determined, all other endogenous time paths are known. It is straightforward to extend the analysis in sections 4.2 and 4.3 to the present set up and show that

$$\frac{d\tilde{c}}{d\tau} < 0, \quad \text{and} \quad \frac{d\tilde{c}}{d\theta} > 0 \quad (69)$$

which implies that the higher the post-collapse level of net lump-sum transfers, the lower will be the consumption. As in section 4, the second condition implies that the less restrictive the credit constraint (higher $\theta$), the higher will be the level of consumption. Moreover, we can use these properties to find the total derivatives.
\[ \frac{dq_T}{d\theta} > 0, \quad \text{and} \quad \frac{dq_T}{dT} < 0 \quad (70) \]

The first condition implies that the less restrictive the credit constraint, the higher will be the post-collapse level of asset price. The second condition implies that the higher the post-collapse level of net lump-sum transfers, the lower will be the post-collapse level of asset price. Further, since \( \frac{dT}{dq_T} > 0 \), equation (70) implies \[ \frac{dT}{d\theta} > 0, \quad \text{and} \quad \frac{dT}{dT} < 0 \quad (71) \]

Again, the first condition implies that the less restrictive the credit constraint, the later will the exchange rate peg collapse occur. The second condition implies that the higher the post-collapse level of net lump-sum transfers, the earlier will the peg be abandoned. Note that we started with an assumption that a crisis is anticipated to occur at time \( T \). Having shown that an equilibrium exists, we need to confirm that \( T \) can be uniquely and endogenously determined.

### 6.4 Foreign Reserves and the Time of Run

We begin by repeating equation (66) which relates the initial level of foreign reserves with the time of collapse of the peg

\[ h_{0\_} - \bar{k}(\bar{q} - q_0) \left( 1 + \frac{\delta r \phi}{1 - \delta} \right) = \frac{\tau}{r} (1 - e^{-rT}) \quad (66) \]

Equation (66) has a very intuitive interpretation. The right hand side is the net lump sum transfers made by the government between time 0 and \( T \) and the left hand side is the amount of reserves which will be spent for this purpose. While \( h_{0\_} \) is the pre-crisis amount of reserves, \( \bar{k}(\bar{q} - q_0) \left( 1 + \frac{\delta r \phi}{1 - \delta} \right) \) is the sum of (a) amount of the reserve loss on impact at \( t = 0 \) (if the constraint binds from the beginning), (b) loss of reserves from time 0 to \( T_\_, \) and (c) finally at \( T \). The difference must be spent on the net level of transfers between 0 to \( T \). Notice that \( h_{0\_} - \bar{k}(\bar{q} - q_0) - \delta(d - d_0) \) is the level of foreign reserves left with the government after accounting for the initial fall in demand deposits and after paying for bailouts. Out of this \( h_{T\_} e^{-rT} \) needs to be kept for the run on reserves at time \( T \), and the rest \( \delta d_0 - q_0 \frac{\delta r}{\bar{k}} \bar{k} - h_{T\_} e^{-rT} \) is the discounted value of the reserves lost from time 0 to \( T \) due to the continuously falling level of demand deposits. Notice that the above analysis is valid even if the constraint does not bind initially but binds from a later date \( T_u \) as in section 4. We can determine the time of collapse \( T \) directly from equation (66) as

\[ T = -\frac{1}{r} \ln \left[ 1 - \left( \frac{h_{0\_} - \bar{k}(\bar{q} - q_0) \left( 1 + \frac{\delta r \phi}{1 - \delta} \right)}{\frac{\tau}{r}} \right) \right] \quad (72) \]
From equation (72), it is obvious that a non-negative unique value of $T$ can be obtained if and only if

$$\frac{\tau}{r} > h_0_\_ - \bar{k}(\bar{q} - q_0) \left( 1 + \frac{\delta r \phi}{1 - \delta} \right) \geq 0$$

(73)

The first inequality in (73) states that if the anticipated loss in reserves due to the bail out and drop in demand deposits between time 0 to $T$, is greater than the total discounted surplus of the government, a crisis can be self-fulfilling. If a crisis is foreseen - the government will be at some stage be forced to resort to inflationary revenue generation to meet the shortfall, thereby validating the anticipation. In other words, if the inequality is violated the initial discounted surplus will be greater than the reserve loss due to anticipated BOP crisis which can not be a crisis equilibrium because the government will never exhaust its reserves. On the other hand, if the second inequality is violated the level of reserves is too small to even meet the loss in reserves. In this case even if a crisis occurs at time $t = 0$, the government will not have enough reserves to bail out the banks in addition to bearing the private sector’s run on reserves. In this event crisis will occur instantaneously, also requiring a discrete devaluation at time $t = 0$. This would also entail a discrete capital loss for optimizing consumers, which is possible only if the consumers never believed that a crisis can occur.

If the credit constraint binds, for any value of $T$ the price level at time $t = 0$ will be lower than when the firms are unconstrained. Equation (66) then implies that a higher initial level of reserve $h_0_\_$ would be required compared to the unconstrained case where the time of crisis is indeterminate if $h_0_\_ = \frac{\tau}{r}$. In the present scenario, it will be possible to determine a unique value of $h_0_\_$ consistent with a future date of collapse $T$ that will be obtained from equation (72). Alternatively, we can say that for every value of initial level of reserves $h_0_\_$ there will be a unique value of $T$ for an anticipated BOP crisis that can be self fulfilling. To look into the relationship between $h_0_\_$ and the time of crisis $T$, we differentiate equation (66) with respect to $T$ to obtain

$$\frac{\partial h_0_\_}{\partial T} = \tau e^{-rT} - \bar{k} \left( 1 + \phi \frac{\delta r}{1 - \delta} \right) \frac{\partial q_0}{\partial T}$$

(74)

In appendix 8.5 we show that $\frac{\partial q_0}{\partial T} = \frac{\tau}{(1 + \phi \frac{\delta r}{1 - \delta})} e^{\int_0^T f_q(t,q(t,T,q)) dt}$. Then equation (74) yields

$$\frac{\partial h_0_\_}{\partial T} = \tau e^{-rT} \left( 1 - e^{\int_0^T (r - f_q(t,q(t,T,q_T))) dt} \right) < 0$$

(75)

The last inequality is obtained due to the fact that on an equilibrium price path $r - f_q(t,q(t,T,q_T)) > 0$. Hence, the integrand is positive, which makes the term within brackets negative. This result seems counter-intuitive; implying that a higher initial level of foreign reserves will be consistent with a time of BOP crisis.
that is lower compared to the case when the initial reserves were lower. But from another viewpoint, the result implies that a higher initial level of foreign reserves would be needed to compensate for the corporate sector loss and the fall in money demand if the crisis occurs earlier. This is due to the fact that given the post-collapse inflation revenue $\tau$, a BOP crisis which occurs earlier results in lower time path of asset prices. Since post-collapse money demand is linked with the steady state level of asset prices, a higher rate of inflation is required to generate sufficient revenues to cover the constant level of net transfers. Again, a higher level of post-crisis inflation is consistent with a lower level of asset prices given the time of collapse $T$.

6.4.1 Simulations

We simulate the model to visualize the relationship between the initial level of foreign reserves which will be consistent with a self-fulfilling BOP crisis at time $T$. With parameter values given as $\nu = 0.03$, $\alpha = 0.8$, $\delta = 0.7$, $\theta = 0.16$, $p = \bar{k} = 1$, $f_o = 5$, $\phi = 1$, and $\tau = 0.01$, the plot of level of initial reserves consistent with anticipated crisis at $T$ is shown below in figure 6.

![Figure 6](image)

**Figure 6.** With parameter values given as $\nu = 0.03$, $\alpha = 0.8$, $\delta = 0.7$, $\theta = 0.16$, $p = \bar{k} = 1$, $f_o = 5$, $\phi = 1$, and $\tau = 0.01$.

It is evident that the pre-crisis level of reserves at $t = 0_{-}$ is decreasing with the time of BOP crisis which occurs at $T$. The earlier the exchange rate peg is anticipated to collapse, the higher will be the amount of reserves consistent with a self-fulfilling equilibrium. It is obvious that an initial level of reserves, which is higher than the level required for the collapse to occur at time 0 will rule out a self-fulfilling BOP completely. However, a lower level is consistent with a BOP crisis occurring at a future date. What if the agents’ believe that crisis is going to occur immediately. The answer is that it will also be validated.
Unless the government alters its policies i.e. generates more revenues through other than inflationary means or adopts expenditure cuts to reduce its level of net transfers; a self-fulfilling crisis which occurs on impact of change in beliefs will also potentially entail debt defaults and a discrete devaluation on impact.

7 Conclusions

In this paper, we have established a causality from a future currency crisis to an incipient banking crisis, which replicates crisis chronology and asset market dynamics that have been observed in the emerging market crises. We find that two primary features, which underlie this result are high corporate leverage and asset market exposure of banks - the result of a low level of financial market deepening in emerging markets. Moreover, in the presence of government bailout guarantees and credit market imperfections in cross-border credit markets, we show that the above features can generate a self-fulfilling banking and currency crises in the sequence as has been observed empirically.

We conclude by discussing two shortcomings of our analysis. First, in our model the root cause of the asset price disturbance is the post-crisis lending spread - the opportunity cost of firms’ domestic currency borrowings - that depends on the central bank’s exogenously fixed reserve requirement. Clearly, this is just a modeling simplification and not a feature of reality. In general, central banks aggressively defend their exchange rates by raising short-term interest rates. Moreover, often when the central bank gives up the defense of the peg, the collapse is accompanied by large exchange rate devaluations. While the first observation works in the favor of the way we model the crisis, the latter needs a different framework altogether. However, it is easy to see that with an assumption of nominal rigidities in domestic goods markets, currency mismatch between corporate earnings and debt would lead to same results as we have obtained. Furthermore, a discrete devaluation can only be modeled by introducing uncertainty, which will complicate the analysis without adding any further insights on the issues that we address in this paper. On the other hand, our perfect foresight continuous time model generates an empirically relevant sequence of crises and a rich output and asset price dynamics, in a much simpler and more direct manner.

Second, in our analysis of the self-fulfilling equilibria we found that the initial level of foreign reserves is inversely related to the time of run on reserves. Although, the result is contrary to conventional professional wisdom on this issue; it needs to be interpreted with an asset market perspective. Clearly, if a crisis occurs earlier, the discounted stream of associated output and capital loss of the corporate sector is higher. Thus the government is required to spend a higher amount of its resources to cover for the private sector’s loss, which in turn brings the crisis forward. However, at the same time our results also show that above a threshold level of reserves a self-fulfilling equilibrium can be ruled
out. The lesson to be learned from this result is that under the presence of financial market distortions, the level of initial foreign reserves (if it is below a threshold value) is not the right indicator for the assessment of vulnerability to the crisis. Again, our assumption of a fixed fiscal stance and the government’s passive role is a modeling artifact. Typically, governments respond to a crisis by seeking foreign exchange assistance from the IMF and also adjust their fiscal obligations. Moreover, agents’ beliefs about the probability of a crisis and their prior on the government’s credibility depend on fundamentals such as the initial level of foreign reserves. We may need to consider a multi-player game-theoretic set up, where the government plays an active coordination role and the equilibrium outcome is a not only a function of fundamentals but also incorporates the mechanism through which agents form their beliefs. Studying these issues is left for future research.

8 Appendix

8.1 Asset Prices and Terminal Conditions: Properties

Since asset price \( q_t \) is continuous on interval \([0,T)\) and it passes through \( q_T \) at \( T \) \((q_T)\), it is further implied that \( \dot{q}_t \) is continuous over the interval \([0,T)\) (notice that at \( T_u \), \( \tilde{n} = \frac{\delta_q \kappa}{\rho} \), hence \( \dot{q}_t \) is continuous at \( T_u \)). These properties imply that (Theorem 7.2, Theory of Ordinary Differential Equations, Coddington and Levinson, McGraw-Hill)

\[
\dot{q}_{qt}(t, T(q_T), q_T) = f_q(t, q(t, T(q_T), q_T)) \quad q_{qt}
\]

(76)

where function \( f(.,) \) denotes the expression on the R.H.S of equations (35) and (36). Further, (76) has a solution

\[
q_{qt} = \exp \left( \int_{T(q_T)}^t f_q(s, q(s, T(q_T), q_T)) \, ds \right)
\]

(77)

Differentiating (77) with respect to \( q_T \) and applying Leibnitz rule we obtain

\[
\frac{\partial^2 q_t}{\partial q_T^2} = \left( \exp \left( \int_{T(q_T)}^t f_q(s, q(s, T(q_T), q_T)) \, ds \right) \right) \ast
\]

\[
\left( - \int_t^{T(q_T)} f_{qq}(s, q(s, T(q_T), q_T)) \left( \frac{\partial q_t}{\partial q_T} \right) \, ds - f_q(T, q_T) \left( \frac{\partial T(q_T)}{\partial q_T} \right) \right) < 0
\]

(78)

From (37) we know that \( f_q(T, q_T) \left( \frac{\partial q_t}{\partial q_T} \right) |_{T>0} > 0 \). Differentiating (35) and (36) twice w.r.t. \( q_t \), it is easily seen that \( f_{qq}(t, q(t, T(q_T), q_T)) \geq 0 \). Next, using the fact that \( d_T = \frac{\delta_q \kappa k}{1-b} \), and \( T \) is given by (29); it’s straight-forward to show that \( \frac{\partial T(q_T)}{\partial q_T} > 0 \). Hence, we obtain the last inequality in (78).
8.2 Fixed Point for Consumption

$\Gamma(.;.) > 0$  This condition is imposed. We assume a non-negative $f_0$ (alternatively, we can assume that the consumer also exogenously receives a constant stream of endowment). All other terms in (47) are non-negative.

$\Gamma_{\tilde{c}} \geq 0$  Differentiating (47) with respect to $\tilde{c}$, and applying Leibnitz rule (since $T_u$ and $T$ are also functions of $\tilde{c}$) we get

$$
\Gamma_{\tilde{c}}(\tilde{c}; .) = r \int_{T_u(q_T(\tilde{c})))}^{T(q_T(\tilde{c})))} \theta \left( \frac{\alpha}{p} \left( \frac{\theta q_T}{p} \right)^{\alpha-1} - 1 \right) \frac{\partial q_T}{\partial q_T} \frac{\partial q_T}{\partial \tilde{c}} e^{-rt} dt
$$

$$
+ \hat{k} \theta \left( \frac{\alpha}{p} \left( \frac{\theta q_T}{p} \right)^{\alpha-1} - 1 \right) \frac{\partial q_T}{\partial \tilde{c}} e^{-rT} > 0 \tag{79}
$$

The inequality follows from the fact that $\frac{\partial q_T}{\partial \tilde{c}}, \frac{\partial q_T}{\partial q_T} > 0$, and $\frac{\partial}{\partial q_T} \left( \left( \frac{\theta q_T}{p} \right)^{\alpha} - \theta q_T \right) = \theta \left( \frac{\alpha}{p} \left( \frac{\theta q_T}{p} \right)^{\alpha-1} - 1 \right) > 0$, since for the constrained economy $\frac{\theta q_T}{p} \leq \frac{\bar{n}}{k} = \left( \frac{\alpha}{p} \right)^{\frac{1}{1-\alpha}}$.

$\Gamma_{\tilde{c}} \leq 0$  First, we get

$$
\frac{\partial^2 q_T}{\partial \tilde{c}^2} = \frac{\alpha(\alpha - 1)q_T^{\alpha-1} \left( \frac{\theta q_T}{p} \right)^{\alpha-1}}{\frac{\theta q_T}{p} + (1 - \alpha)^2 \left( \frac{\theta q_T}{p} \right)^{\alpha}} \left( \frac{\partial q_T}{\partial \tilde{c}} \right)^2 < 0 \tag{80}
$$

since $\alpha < 1$. Differentiating (79) once again w.r.t $\tilde{c}$ and applying Leibnitz rule, we get

$$
\Gamma_{\tilde{c}\tilde{c}}(\tilde{c}; .) = r \int_{T_u(q_T(\tilde{c})))}^{T(q_T(\tilde{c})))} \theta \left( \frac{\alpha}{p} \left( \frac{\theta q_T}{p} \right)^{\alpha-1} - 1 \right)
$$

$$
\left[ \frac{\partial^2 q_T}{\partial q_T^2} \left( \frac{\partial q_T}{\partial \tilde{c}} \right)^2 + \frac{\partial q_T}{\partial q_T} \frac{\partial^2 q_T}{\partial \tilde{c}^2} \right] e^{-rt} dt
$$

$$
+ r \int_{T_u(q_T(\tilde{c})))}^{T(q_T(\tilde{c})))} \left( \theta \left( \frac{\alpha}{p} \left( \frac{\theta q_T}{p} \right)^{\alpha-1} - 1 \right) \right) \left( \frac{\partial q_T}{\partial q_T} \frac{\partial q_T}{\partial \tilde{c}} \right)^2 e^{-rT} dt
$$

$$
+ \hat{k} \theta \left( \frac{\alpha}{p} \left( \frac{\theta q_T}{p} \right)^{\alpha-1} - 1 \right) \frac{\partial^2 q_T}{\partial \tilde{c}^2} e^{-rT}
$$
\[ + \left( \frac{\theta}{p} \right)^2 \alpha (\alpha - 1) \left( \frac{\theta q_T}{p} \right)^{\alpha - 2} \left( \frac{\partial q_T}{\partial c} \right)^2 e^{-rT} \]

\[ - r \bar{k} \theta \left( \frac{\alpha}{p} \left( \frac{\theta q_{Tu}}{p} \right)^{\alpha - 1} - 1 \right) e^{-rT_u} \partial T_u \frac{\partial T_u}{\partial c} \]  

(81)

The last term in above expression is equal to zero as at \( T_u, \frac{\alpha}{p} \left( \frac{\theta q_{Tu}}{p} \right)^{\alpha - 1} = 1 \). All other terms are negative. Hence, \( \Gamma_c \langle \partial c \rangle < 0 \).

### 8.3 Initial Foreign Reserves and the Time of BOP Crisis

Integrating equation (65) on the interval \( t \in [0, T] \), after pre-multiplying with \( e^{-rt} \), we obtain

\[ h_{T_\ast} e^{-rT} - h_0 = \frac{\tau}{r} (e^{-rT} - 1) + \delta d_{T_\ast} e^{-rT} - \delta d_0 + \delta r \int_0^{T_\ast} d_t e^{-rt} dt \]  

(82)

We note here that the level of foreign reserves at time \( t = 0 \), \( h_0 \) is not only lower than \( h_{T_\ast} \) by the capital loss \( (\bar{q} - q_0) \bar{k} \), but it is further reduced by \( \delta (\bar{d} - d_0) \), due to fall in demand deposits at time 0 which does not happen in the unconstrained case. Using equations (7), (12), and (16) with the above equation (82) and integrating by parts we can show that

\[ h_{T_\ast} e^{-rT} - h_{T_\ast} + (\bar{q} - q_0) \bar{k} + \delta (\bar{d} - d_0) = \frac{\tau}{r} (e^{-rT} - 1) + \delta d_{T_\ast} e^{-rT} \]

\[ - \delta d_0 - \frac{\phi \bar{k}}{1 - \delta} \delta r \int_0^T \left( \dot{q} e^{-rt} - r e^{-rt} \right) dt \]

\[ = \frac{\tau}{r} (e^{-rT} - 1) + \delta d_{T_\ast} e^{-rT} - \delta d_0 - \frac{\phi \bar{k}}{1 - \delta} \delta r (q_T e^{-rT} - q_0) \]  

(83)

Using the fact that \( \frac{\phi \bar{k}}{1 - \delta} \delta r q_T = \delta d_T \), and \( \delta \bar{d} = \frac{\phi \bar{k}}{1 - \delta} \delta r \bar{q} \), equation (83) can be simplified to equation (66) in the main text.

\[ h_{T_\ast} - \bar{k} (\bar{q} - q_0) \left( 1 + \frac{\delta r \phi}{1 - \delta} \right) = \frac{\tau}{r} (1 - e^{-rT}) \]  

(84)
8.4 Steady State Asset Price and Consumption: Fixed Net Government Transfer

Using equation (60) it is straightforward to show that

\[ \frac{\partial q_T}{\partial \tilde{c}} = \frac{\nu_T}{k} + (1 - \alpha)^2 \left( \frac{\theta q_T}{p} \right)^\alpha > 0 \]  

\[ \frac{\partial^2 q_T}{\partial \tilde{c}^2} = \frac{\nu c - \nu T}{k} + (1 - \alpha)^2 \left( \frac{\theta q_T}{p} \right)^\alpha \left( \frac{\partial q_T}{\partial \tilde{c}} \right)^2 < 0 \]  

(85)  

(86)

8.5 Time Path of Asset Price and the Time of Run

Since land price \( q_t \) is continuous on interval \([0,T)\) and it passes through \( q_T \) at \( T \); \( q_t \) is a function \( q(t,T,q_T) \). Further note that \( \dot{q}_t \) is continuous over the interval \([0,T)\). These properties imply that (Theorem 7.2, Theory of Ordinary Differential Equations, Coddington and Levinson, McGraw-Hill)

\[ \frac{\partial \dot{q}_t}{\partial T}(t, T, q_T) = f_q(t,q(t,T,q_T)) \frac{\partial q_T}{\partial T} \]  

where function \( f(.) \) denotes the expression on the R.H.S of equations (35) and (36). Integrating over the time interval \([0,T)\), we obtain

\[ \log \left[ \frac{\partial q_0}{\partial T} \right] - \log \left[ \frac{\partial q_T}{\partial T} \right] = \int_T^0 f_q(t,q(t,T,q_T)) \, dt \]  

(88)

which simplifies to

\[ \frac{\partial q_0}{\partial T} = \frac{\partial q_T}{\partial T} \left( \exp \left( \int_T^0 f_q(t,q(t,T,q_T)) \, dt \right) \right) \]  

(89)

Further, note that

\[ \frac{\partial q_T}{\partial T} = -f(T,q_T) \]  

(90)

where \( f(.) \) is the R.H.S. of equation (36). From equation (36) and (60) it is easy to show that

\[ f(T,q_T) = -\frac{\nu}{k} \frac{1}{1 + \phi \frac{\delta}{1-\delta}} \]  

(91)

Hence,

\[ \frac{\partial q_0}{\partial T} = \frac{\nu T}{k} \frac{1}{1 + \phi \frac{\delta}{1-\delta}} \left( \exp \left( \int_T^0 f_q(t,q(t,T,q_T)) \, dt \right) \right) \]  

(92)
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