Segmented Asset Markets and Optimal Exchange Rates

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Abstract

The paper studies the issue of optimal exchange rate regimes for developing countries in a flexible price environment. The key innovation is that we analyze the implications of segmented asset markets — only a fraction of agents participate in asset market transactions — for the optimal exchange rate regime. We show that even though all shocks in the model are monetary, asset market segmentation implies an inherent welfare bias towards flexible exchange rates. This finding is in stark contrast to the standard Mundellian prescription that if shocks are mostly monetary then fixed exchange rates are optimal. The paper also shows that the optimal policy regime hinges crucially on whether traders or non-traders face most of the monetary shocks.

Keywords: Optimal exchange rates, asset market segmentation

JEL Classification: F1, F2
1 Introduction

Almost 50 years after Milton Friedman’s (1953) celebrated case for flexible exchange rates, the debate on the optimal choice of exchange rate regimes rages on as fiercely as ever. Friedman argued that, in the presence of sticky prices, floating rates would provide better insulation from foreign shocks by allowing relative prices to adjust faster. In a world of capital mobility, Mundell’s (1963) work implies that the optimal choice of exchange rate regime should depend on the type of shocks hitting an economy: real shocks would call for a floating exchange rate, whereas monetary shocks would call for a fixed exchange rate. Ultimately, however, an explicit cost/benefit comparison of exchange rate regimes requires a utility-maximizing framework, as argued by Helpman (1981) and Helpman and Razin (1979). In such a framework, Engel and Devereux (1998) reexamine this question in a sticky prices model and show how results are sensitive to whether prices are denominated in the producer’s or consumer’s currency. On the other hand, Cespedes, Chang, and Velasco (2000) incorporate liability dollarization and balance sheets effects and conclude that the standard prescription in favor of flexible exchange rates in response to real shocks is not essentially affected.

An implicit assumption in most, if not all, of the literature is that economic agents have unrestricted and permanent access to asset markets. This, of course, implies that in the absence of nominal rigidities, the choice of fixed versus flexible exchange rates is irrelevant. In practice, however, access to asset markets is limited to some fraction of the population (due to, for example, fixed costs of entry). This is likely to be particularly true in emerging markets where it is still the case that asset markets are much smaller in size than in industrial countries.\(^1\) Since asset markets are at the heart of the adjustment process to different shocks in an open economy, it would seem natural to analyze how “asset market segmentation” affects the choice of exchange rate regime.\(^2\)

\(^1\) Even for the case of the United States, evidence cited by Mulligan and Sala-i-Martin (1996) – based on a 1989 survey of consumer finances – reveals that 59 percent of U.S. households do not hold any interest bearing assets (defined as money market accounts, certificates of deposit, bonds, mutual funds, and equities).

\(^2\) In closed economy macroeconomics, asset market segmentation has received widespread attention ever since the
To this end, this paper abstracts from any nominal rigidity and focus on a standard monetary model of an economy subject to stochastic velocity shocks in which the only friction is that an exogenously-given fraction of the population can access asset markets. The analysis makes clear that asset market segmentation introduces a fundamental asymmetry in the choice of fixed versus flexible exchange rates. To see this, consider first the effects of a positive velocity shock in a standard one-good open economy model in the absence of asset market segmentation. Under flexible exchange rates, the velocity shock gets reflected in an excess demand for goods, which leads to an increase in the price level (i.e., the exchange rate). Under fixed exchange rates, the adjustment must take place through an asset market operation whereby agents exchange their excess money balances for foreign bonds at the central bank. In either case, the adjustment takes place instantaneously with no real effects. How does asset market segmentation affect this adjustment? Under flexible rates, the same adjustment takes place. Under fixed exchange rates, however, only those agents who have access to asset markets (called “traders”) may get rid of their excess money balances. Non-traders – who are shut off asset markets – cannot do this. Non-traders are therefore forced to buy foreign goods (which implies lower consumption in the future), which is costly from a welfare point of view. Hence, under asset market segmentation, flexible exchange rates are superior than fixed exchange rates.

A key assumption behind the logic just described is the assumption of symmetric velocity shocks for all agents (i.e., traders and non-traders). In practice, however, there is no reason to believe that all economic agents face the same velocity shocks. Accounting for asymmetric shocks can alter the above results. As an example, consider the extreme case in which only traders are hit by velocity shocks. Clearly, under fixed exchange rates, traders can easily adjust to this shock through an asset pioneering work of Grossman and Weiss (1983) and Rotemberg (1984) (see also Chatterjee and Corbae (1992) and Alvarez, Lucas, and Weber (2001)). The key implication of these models is that open market operations reduce the nominal interest rate and thereby generate the so-called “liquidity effect”. In an open economy context, Alvarez and Atkeson (1997) and Alvarez, Atkeson, and Kehoe (2002) have argued that asset market segmentation models may help in resolving outstanding puzzles in international finance such as volatile and persistent real exchange rate movements as well as volatility of nominal exchange rates which exceeds the corresponding volatility of fundamentals.
market transaction with the central bank. Since non-traders have not been affected, the economy adjusts instantaneously with no real effects. Under flexible exchange rates, however, traders’ excess money balances will result in the price level increasing by the average velocity shock. This leads to real effects since non-traders’ real money balances fall and traders’ increase (since the rise in the price level will not fully offset their velocity shock). In this case, therefore, fixed exchange rates are welfare superior.

It follows that whether flexible rates dominate fixed rates or not will depend on the relative shocks to traders and non-traders. In general, flexible exchange rates will be superior except for cases where shocks to traders (which proxy shocks to assets/capital markets) are large relative to shocks to non-traders. Since one would expect that the more financially open an economy is, the more important are shocks to assets markets, our model would imply that economies with very open financial systems would be better off with fixed exchange rates but, short of that, flexible exchange rates would be preferable.

For a given composition of shocks between traders and non-traders, how does a higher variance of velocity shocks affect the choice between fixed and flexible rates? We find that while a higher variance does not alter the range of parameters for which flexible rates dominate, it makes for a much more “steeper” trade-off. In other words, deviations from the optimal exchange rate regime become more costly as the variance of shocks increases. This would imply that the choice of exchange rate regime is more important for, say, Argentina than for the United States.

In sum, the paper shows that asset market segmentation may be a critical friction in determining the optimal choice of exchange rate regime. Asset market segmentation introduces a fundamental bias in favor of flexible rates even when though shocks are exclusively monetary. This result runs counter to the Mundellian prescription that, if monetary shocks dominate, fixed rates are preferable. This discrepancy reflects the difference in the underlying friction. In the Mundell-Fleming world, sticky prices presumably reflect some imperfection in goods markets, whereas in our model asset market segmentation captures some imperfection in asset markets (for example, fixed cost of entry). Of course, which friction dominates in practice is ultimately an empirical issue.
The paper proceeds as follows. Section 2 presents the model and the equilibrium conditions while Section 3 describes the allocations under alternative exchange rate regimes. Section 4 contains the analytical results regarding optimal exchange rate for some interesting special cases. In Section 5 we present numerical simulations of the general version of the model while the last section concludes. Algebraically tedious proofs are consigned to an appendix at the end.

2 Model

The basic model is an open economy variant of the model outlined in Alvarez, Lucas and Weber (2001). Consider a small open economy with perfect mobility in the goods market. There is a unit measure of households who consume an internationally traded good. The world currency price of the consumption good is fixed at one. Each household receives a fixed endowment $y$ of the consumption good in each period. The households’ derive utility from consumption as

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \right\}$$

where $\beta$ is the households’ time discount factor, $c_s$ is consumption in period $s$, while $E_t$ denotes the expectation conditional on information available at time $t$.

The households face a cash-in-advance constraint. As is standard in these models, the households are prohibited from consuming their own endowment. We assume that a household consists of a seller-shopper pair. While the seller sells the household’s own endowment, the shopper goes out with money to purchase consumption good from other households. Following Alvarez et al, we assume that the shopper can also access a proportion $v_t$ of current period $(t)$ sales receipts, in addition to the cash carried over from the last period ($M_t$), to purchase consumption.\footnote{Following Alvarez, Lucas, and Weber (2001) one can ‘think of the shopper as visiting the seller’s store at some time during the trading day, emptying the cash register, and returning to shop some more.’} We assume that $v_t$ is an independently and identically distributed random variable with mean $\bar{v} \in [0,1]$ and variance $\sigma^2_v$.

Only a fraction $\lambda$ of the population, called traders, have access to the asset markets, where $0 \leq \lambda \leq 1$. The rest, $1 - \lambda$, called non-traders, can only hold domestic money as an asset. We
allow for the possibility that velocity shocks may be specific to each group. In particular, we assume that:

\[ v_{t}^T = \theta v_{t}, \]
\[ v_{t}^{NT} = (1 - \theta) v_{t}. \]

where \( 0 \leq \theta \leq 1 \).

The timing runs as follows. First, the velocity shock is realized at the beginning of every period. Then the household splits. Before the opening of the goods market, shoppers of trading households visit the asset market where they trade in government and foreign bonds. Shoppers of the non-trading households, on the other hand, are excluded from the asset market and only transact on the goods market where they buy consumption goods. Goods markets open after asset markets close.

### 2.1 Households’ Problem

#### 2.1.1 Non-traders

The non-trader’s cash-in-advance is given by:

\[ M_{t}^{NT} + (1 - \theta)v_{t}S_{t}y = S_{t}c_{t}^{NT}, \] (2)

where \( M_{t}^{NT} \) is the beginning of period \( t \) nominal money balances while \( S_{t} \) is the period \( t \) exchange rate (the domestic currency price of foreign currency). Equation (2) shows that for consumption purposes, the non-traders can augment the beginning of period cash balances by withdrawals from current period sales receipts \((1 - \theta)v_{t}\) (the velocity shocks). Money balances at the beginning of period \( t + 1 \) are given by sales receipts net of withdrawals for period \( t \) consumption:

\[ M_{t+1}^{NT} = S_{t}y \left[ 1 - (1 - \theta)v_{t} \right], \] (3)

where \( S_{t} \) denotes the domestic currency price of consumption goods at time \( t \).

The usual flow constraint follows from combining (2) and (3):

\[ M_{t+1}^{NT} = M_{t}^{NT} + S_{t}y - S_{t}c_{t}^{NT}. \] (4)
Given the cash-in-advance (2), it follows that:
\[
c_t^{NT} = \frac{M_t^{NT} + (1 - \theta)v_tS_ty}{S_t}.
\] (5)

2.1.2 Traders

The traders begin any period with assets in the form of money balances and bond holdings carried over from the previous period. Armed with these assets the shopper of the trader household visits the asset market where she rebalances the households asset position and also receives the lump sum asset market transfers from the government. Thus, for any period \(t\), the accounting identity for the asset market transactions of a trader household gives
\[
\hat{M}_t^T = M_t^T + (1 + i_t) \frac{B_t}{\lambda} - \frac{B_{t+1}}{\lambda} + S_t(1 + r)f_t - S_tf_{t+1} + \frac{T_t}{\lambda}
\] (6)

where \(\hat{M}_t^T\) denotes the money balances with which the trader leaves the asset market and \(M_t^T\) denotes the money balances with which the trader entered the asset market. Also, \(B\) denotes aggregate nominal government bonds, \(f\) are foreign bonds (denominated in terms of the good), \(r\) is the exogenous and constant world real interest rate, and \(T\) are aggregate (nominal) lump-sum transfers (i.e., negative taxes) from the government.\(^4\) After asset markets close the shopper proceeds to the goods market with \(\hat{M}_t^T\) in nominal money balances to purchase consumption goods. Like non-traders, traders can also augment these starting money balances with random withdrawals from current sales receipts to carry out goods purchases. Thus, the cash-in-advance constraint for

\[\text{We assume that these transfers are made in the asset markets, where only the traders are present. Note that since } B \text{ and } T \text{ denote aggregate bonds and aggregate transfers, their corresponding per trader values are } B/\lambda \text{ and } T/\lambda \text{ since traders comprise a fraction } \lambda \text{ of the population.}\]

\[\text{The assumption of endogenous lump-sum transfers will ensure that any monetary policy may be consistent with the intertemporal fiscal constraint. This becomes particularly important in this stochastic environment where these endogenous transfers will have to adjust to ensure intertemporal solvency for any history of shocks. To make our life easier, these transfers are assumed to go only to traders. If these transfers also went to non-traders, then (5) would be affected.}\]
a trader is given by
\[ S_t c_t^T = \hat{M}_t^T + \theta v_t S_t y \] (7)

Combining equations (6) and (7) gives
\[ M_t^T + T \frac{\lambda}{\alpha} + \theta v_t S_t y = M_t^T + \frac{B_t + 1}{\lambda} - (1 + it) \frac{B_t}{\lambda} + S_t f_{t+1} - S_t (1 + r) f_t, \] (8)

In this set-up the only reason that traders hold money overnight is the sequential nature of market openings. In particular, if both asset and goods market were open simultaneously, then at the end of the day the trading household would use all their remaining sales receipts from the period to buy interest bearing bonds. However, since asset markets close before the opening of the goods market, traders are forced to hold money overnight. Thus, period-\( t \) sales receipts net of withdrawals become beginning of next period’s money balances
\[ M_{t+1}^T = S_t y [1 - \theta v_t]. \] (9)

Note that since \( v, S, \) and \( y \) are all exogenous, the traders’ money holdings evolve exogenously over time.

A trader chooses \( c_t, B_{t+1} \) and \( f_{t+1} \) to maximize (1) subject to the flow constraint (8). Combining first-order conditions, we obtain:
\[ u'(c_t^T) = \beta (1 + r) E_t \left\{ u'(c_{t+1}^T) \right\} \] (10)
\[ \frac{u'(c_t^T)}{S_t} = \beta E_t \left[ \frac{(1 + it_{t+1}) u'(c_{t+1}^T)}{S_{t+1}} \right]. \] (11)

Equation (10) is the standard Euler equation for the trader which relates the expected marginal rate of consumption substitution between today and tomorrow to the return on savings (given by \( 1 + r \)) discounted to today. Equation (11), on the other hand, determines the optimal holdings of nominal government bonds. Equations (10) and (11) jointly determine the modified interest parity

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6Throughout the analysis we shall restrict attention to ranges in which the cash-in-advance constraint binds for both traders and non-traders. In general, this would entail checking the individual optimality conditions to deduce the parameter restriction for which the cash-in-advance constraints bind.
condition for this economy which reflects the standard portfolio choice decision between safe and risky assets.

### 2.2 Government

The government in this economy holds foreign bonds (reserves) which earn the world rate of interest $r$. The government can sell nominal domestic bonds, issue domestic money, and can make lump sum transfers to the traders. Thus, the government’s budget constraint is given by

$$S_t h_{t+1} - (1 + r)S_t h_t + (1 + i_t)B_t - B_{t+1} + T_t = M_{t+1} - M_t$$

where $B$ denotes the amount of nominal government bonds held by the private sector, $h$ are foreign bonds held by the government, $M$ is the aggregate money supply, and $T$ is government transfers to the traders. Equation (12) makes clear that money supply can be altered either through open market operations or through interventions in the foreign exchange market. Importantly, both methods impact only the traders since they are the only agents present in the asset market.

### 2.3 Equilibrium conditions

Equilibrium in the money market requires that

$$M_t = \lambda M^T_t + (1 - \lambda) M^{NT}_t.$$  

(13)

The flow constraint for the economy as a whole (i.e., the current account) follows from combining the flow constraint for non-traders (4), traders (7 and 9), government ((12) and money market equilibrium (13):

$$\lambda c^T_t + (1 - \lambda) c^{NT}_t = y + (1 + r)k_t - k_{t+1},$$  

where $k \equiv h + \lambda f$ denotes per-capita foreign bonds for the economy as a whole.

To obtain the quantity theory, we combine (3), (9) and (13) to get:

$$\frac{M_{t+1}}{1 - \phi v_t} = S_t y,$$

(15)

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7 In principle, the government could also inject money through lump-sum transfers. However, to fix ideas, we prefer to think of the government as injecting money only through open market operations or foreign exchange market intervention.
where $\phi$ is the average velocity shock:

$$\phi \equiv \lambda \theta + (1 - \lambda)(1 - \theta).$$  

(16)

Notice that the stock of money relevant for the quantity theory is the end of period $t$ money balances (i.e., $M_{t+1}$). This reflects the fact that, unlike standard CIA models (in which the good market is open before the asset market and shoppers cannot withdraw current sales receipts for consumption), in this model (i) asset markets open and close before goods market open (which allows traders to change this period’s money balances for consumption purposes); and (ii) both traders and non-traders can access current sales receipts.

Combining (5) and (3) gives the consumption of non-traders:

$$c_{NT}^t = \left(\frac{S_{t-1} + (1 - \theta)(v_tS_t - v_{t-1}S_{t-1})}{S_t}\right) y.$$  

(17)

To derive the consumption of traders, we use equation (9) to substitute out for $M_t^T$ in equation (8). Then, subtracting $S_{ty}$ from both sides allows us to rewrite (8) as

$$(1 + r)f_t - f_{t+1} + \frac{(1 + i_t)B_t + B_{t+1}}{S_t} + B_{t+1} + \frac{T_{t+1}}{S_t} + y - c_t^T + (\theta - \phi)\left(\frac{v_tS_t - v_{t-1}S_{t-1}}{S_t}\right) y = \frac{M_{t+1} - M_t}{S_t}$$

where we have used equation (15) to get $M_{t+1} - M_t = [(S_t - S_{t-1}) - \phi(v_tS_t - v_{t-1}S_{t-1})] y$. Using equation (12) in the equation above gives

$$\frac{k_{t+1}}{\lambda} - (1 + r)\frac{k_t}{\lambda} = y - c_t^T + \left(\frac{1 - \lambda}{\lambda}\right)\left(\frac{M_{t+1} - M_t}{S_t}\right) + (\theta - \phi)\left(\frac{v_tS_t - v_{t-1}S_{t-1}}{S_t}\right) y$$  

(18)

where $k_0$ is given exogenously. Equation (18) gives the trader’s flow constraint in equilibrium. The left hand side gives the net acquisition of foreign assets by the economy per trader while the right hand side gives periodic trader income net of consumption. Given the precise monetary regime, we can iterate forward equation (18) and impose the trader household’s first order condition for optimal consumption (equation (10)) to derive the trader’s policy function for consumption along a rational expectations equilibrium path.

It is worth noting that the last two terms on the right hand side of (18) capture the two sources of redistribution in this economy. First, any changes of money supply occur through central bank
operations in the asset market where only traders are present. Hence, the traders receive the entire incremental money injection while their own increase in money balances is only a fraction $\lambda$ of the total. This leads to redistribution of $\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{M_{t+1}-M_t}{S_t}\right)$ from non-traders to traders. Note that as $\lambda \to 1$ this term goes to zero. Second, the traders own share of the velocity shock is given by the parameter $\theta$ while the average velocity shock is given by $\phi$, the term $(\theta - \phi)\left(\frac{v_t S_t - v_{t-1} S_{t-1}}{S_t}\right) y$ captures the second source of redistribution which arises due to asymmetric velocity shocks. In the special case where shocks are symmetric so that $\theta = \phi = 1/2$, this last term disappears from equation (18). It is important to note that the first channel exists solely due to asset market segmentation while the second is due to asymmetric velocity shocks between traders and non-traders.

3 Alternative Exchange Rate Regimes

Having described the model and the equilibrium conditions above, we now turn to allocations under specific exchange rate regimes. We will look at two pure cases: flexible exchange rates and fixed exchange rates. The end goal, of course, is to evaluate the welfare implications under the two regimes. In all the policy experiments below, we shall assume that the initial distribution of nominal money balances across the two types of agents is invariant. In particular, we assume that

$$M_0^T = \bar{M} \left(\frac{1 - \theta \bar{v}}{1 - \phi \bar{v}}\right)$$

$$M_0^{NT} = \bar{M} \left(\frac{1 - (1 - \theta) \bar{v}}{1 - \phi \bar{v}}\right)$$

This assumption about the initial distribution across agents ensures that at $t = 0$ the velocity shocks do not create any expected redistribution.

In order to make the analytics of the welfare comparisons tractable, we shall also assume from hereon that the periodic utility function of both agents is quadratic:

$$u(c) = c - \zeta c^2.$$
### 3.1 Flexible exchange rates

We assume that under flexible exchange rates, the monetary authority sets a constant path of the money supply:

\[ M_t = \bar{M}. \]

Further, the government does not intervene in foreign exchange markets and, for simplicity, we assume that initial foreign reserves are zero. Then, the government’s flow constraint reduces to:

\[ (1 + \bar{i})B_t - B_{t+1} + T_t = 0. \]  
\[(22)\]

The quantity theory equation (15) determines the exchange rate:

\[ S_t = \frac{\bar{M}}{(1 - \phi\bar{v})y}. \]
\[(23)\]

The exchange rate will thus follow the velocity shock and be high (low) when the shock \( v \) is high (low).

Using (23), consumption of non-traders (given by equation(17)) under flexible exchange rates can be written as:

\[ c_{t,\text{NT,flex}} = y \left( 1 + \frac{(1 - \theta - \phi)(v_t - v_{t-1})}{1 - \phi \bar{v}_{t-1}} \right), \]  
\[(24)\]

\[ c_{0,\text{NT,flex}} = y \left( 1 + \frac{(1 - \theta - \phi)(v_0 - \bar{v})}{1 - \phi \bar{v}} \right). \]  
\[(25)\]

Equations (24) and (25) show that consumption of non-traders will fluctuate with the velocity shock as long as shocks are not symmetric between traders and non-traders. Note that in the special case where \( \theta = 1/2 \), we have \( \theta = \phi \) and hence, \( c_{t,\text{NT}} = y \) for all \( t \). Thus, under a floating exchange rate consumption of non-traders fluctuates solely due to asymmetric shocks. Intuitively, under floating exchange rates, prices change by the average of the velocity shocks. If all agents get the average of the velocity shock then there is no redistribution of purchasing power between agents. However, when shocks are asymmetric then a redistribution does occur and hence, causes consumption of non-traders to fluctuate with real balances which fluctuate with the velocity shock.
To determine consumption of traders under the floating exchange rate regime, we can iterate forward equation (18) under the condition $M_t = \bar{M}$ to get
\[
c_{0,\text{flex}} = r \frac{k_0}{\lambda} + y \left[ 1 + (\theta - \phi) \left( \frac{v_0 - \bar{v}}{1 - \psi} + E_0 \left( \sum_{t=1}^{\infty} \left( \frac{1}{1 + r} \right)^t \frac{v_t - v_{t-1}}{1 - \psi v_{t-1}} \right) \right) \right] \tag{26}
\]
where we have used the fact that under the quadratic utility specification adopted above, equation (10) — which describes the optimal consumption plans for traders — reduces to $c_0 = E_0(c_t)$ for all $t > 0$.\(^8\) Note that under asymmetric shocks ($\theta \neq \phi$), traders are also subject to real balance fluctuations due to velocity shocks. However, access to world capital markets allows them to smooth out their consumption through borrowing and lending at the fixed interest rate $r$. Note that this is the standard prediction of the permanent income theory of consumption.

### 3.2 Fixed exchange rates

Under fixed exchange rates, the monetary authority sets a constant path of the exchange rate equal to $\bar{S}$. In particular, we assume that the nominal exchange rate is fixed at
\[
\bar{S} = \frac{\bar{M}}{(1 - \psi \bar{v})y}. \tag{27}
\]
In effect, we are assuming that at time $t = 0$ the monetary authority pegs the exchange rate at the deterministic equilibrium level.

Under this specification, it is easy to see from equation (17) that consumption of non-traders under a fixed exchange rate is given by
\[
c_{t,\text{NT,peg}} = y \left[ 1 + (1 - \theta)(v_t - v_{t-1}) \right] \tag{28}
\]
\[
c_{0,\text{NT,peg}} = y \left[ 1 + (1 - \theta)(v_0 - \bar{v}) \right] \tag{29}
\]
Equation (28) shows that under an exchange rate peg, consumption of non-traders will fluctuate by the full amount of the velocity shock. Intuitively, velocity shocks change the nominal balances

\(^8\)More generally, consumption of traders under floating exchange rates at any point in time $t > 1$ is given by
\[
c_{t,\text{flex}} = r \frac{k_t}{\lambda} + y \left[ 1 + (\theta - \phi)E_t \left( \sum_{s=t}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} \frac{v_s - v_{s-1}}{1 - \psi v_{s-1}} \right) \right]
\]
with non-traders that are available for consumption. Since the price level is now fixed, any change in nominal balances also implies a one-for-one change in real balances and, hence, affects the consumption of non-traders.

To determine the consumption of traders we again iterate forward on equation (18) by using the Euler equation \( c_0 = E_0(c_t) \) and after imposing the condition \( S_t = \bar{S} \) for all \( t \), get

\[
\begin{align*}
\frac{k_0}{\lambda} + y \left[ 1 + \left( \theta - \frac{\phi}{\lambda} \right) \left( v_0 - \bar{v} \right) + E_0 \left( \sum_{t=1}^{\infty} \left( \frac{1}{1 + r} \right)^t (v_t - v_{t-1}) \right) \left( v_t - v_{t-1} \right) \right] = \frac{k_0}{\lambda} + y \left[ 1 + \left( \theta - \frac{\phi}{\lambda} \right) \left( v_0 - \bar{v} \right) + E_0 \left( \sum_{t=1}^{\infty} \left( \frac{1}{1 + r} \right)^t (v_t - v_{t-1}) \right) \left( v_t - v_{t-1} \right) \right].
\end{align*}
\]

In deriving (30) we have used the fact under pegged exchange rates, equation (15) implies that \( M_{t+1} - M_t = -\phi_0 (v_t - v_{t-1}) \bar{S} y \).

To understand the consumption function of traders, note that under fixed exchange rates, the nominal value of GDP remains unchanged, i.e., \( S_t y = \bar{S} y \) for all \( t \). The quantity theory relationship demands that aggregate nominal money balances plus the aggregate withdrawal from current period sales must be sufficient to purchase current nominal output. To keep nominal output unchanged over time, any change in cash withdrawals from current receipts, i.e., \( \phi v_t \neq \phi v_{t-1} \), must be met by the monetary authority with an offsetting change in aggregate nominal money balances. This intervention must happen through transactions in the asset market where only traders are present. On a per trader basis then, the change in nominal money balances needed for keeping the exchange rate fixed is \(-\frac{\phi}{\lambda} (v_t - v_{t-1})\). Thus, under fixed exchange rates, a velocity shock of \( \Delta v \) changes real balances of traders by \( \theta \) but also changes her real balances by \(-\frac{\phi}{\lambda} \) due to central bank interventions. The net effect is \( \theta - \frac{\phi}{\lambda} \) which is the term that shows up in the coefficient on the velocity shocks in equation (30).

\[\text{More generally, consumption of traders under fixed exchange rates at any point in time } t > 1 \text{ is given by}
\[
\begin{align*}
ct_{t, peg} = r \frac{k_t}{\lambda} + y \left[ 1 + \left( \theta - \frac{\phi}{\lambda} \right) E_t \left( \sum_{s=1}^{\infty} \left( \frac{1}{1 + r} \right)^{s-t} (v_s - v_{s-1}) \right) \right].
\end{align*}
\]
Having described allocations under the alternative exchange rate arrangements, we now turn to the key focus of the paper: determination of the optimal exchange rate regime. We shall conduct our analysis by comparing the unconditional expectation of lifetime welfare at time $t = 0$ (i.e., before the revelation of any information at time 0). In terms of preliminaries, it is useful to define the following:

$$W^{i,j} = E\left\{ \sum \beta^t \left[ c^i_t - \zeta \left( \bar{c}^i_t \right)^2 \right] \right\}, \quad i = T, NT, \quad j = \text{flex, peg}$$

(31)

$$W^j = \lambda W^{T,j} + (1 - \lambda)W^{NT,j}, \quad j = \text{flex, peg}$$

(32)

Equation (31) gives the welfare for each agent under a specific exchange rate regime where the relevant consumption for each type of agent is given by the consumption functions derived above for each regime. Equation (32) is the aggregate welfare for the economy under each regime which is the sum of the regime specific individual welfares weighted by their population shares. Note that the quadratic utility specification implies that the expected value of periodic utility can be written as

$$E\left( c - \zeta c^2 \right) = E(c) - \zeta \left[ E(c) \right]^2 - \zeta \text{var}(c).$$

(33)

where $\text{var}(c)$ denotes the variance of consumption.

Since, the general version of the model is too complicated to facilitate analytical results on optimal exchange rate regimes, in the next section we proceed by conducting numerical simulations. In this section we make some analytical progress by studying some interesting special cases.

### 4.1 Perfect capital mobility: $\lambda = 1$

The first case of interest is one where all agents are traders, i.e., $\lambda = 1$. This case corresponds to a representative agent, small open economy with perfect capital mobility. In this event $W^{T,j} = W^j$. Hence, we can focus exclusively on the welfare comparison for traders between the two exchange rate regimes.
**Proposition 1** When all agents in the economy are traders, i.e., $\lambda = 1$, the fixed and flexible exchange rate regimes are welfare equivalent.

**Proof.** Noting that for $\lambda = 1$ we must have $\phi = \theta = \phi / \lambda$, it follows directly from equations (26) and (30) that

$$c^T_{T, \text{flex}} = c^T_{T, \text{peg}} = r \frac{k_0}{\lambda} + y$$

Hence, for $\lambda = 1$, consumption for traders is identical under both regimes. Moreover, since no stochastic terms enter the consumption function, welfare of traders (and hence aggregate welfare as well) must be identical under both regimes. Thus, welfare is independent of the exchange rate regime. ■

This result is similar to the well known result of Helpman and Razin (1979) who showed the welfare equivalence between fixed and flexible exchange rates for representative agent economies with perfect capital mobility where agents are subject to cash-in-advance constraints. Intuitively, when all agents are traders, the trader’s own velocity shock is also the average shock for the economy. Hence, under flexible exchange rates the price level adjusts exactly in proportion to the trader’s velocity shock which leaves her real balances unchanged and thereby insulates her completely from any wealth effects due to real balance fluctuations.\textsuperscript{10} Symmetrically, when exchange rates are fixed, the monetary authority pegs the exchange rate by exactly offsetting the aggregate velocity shock through a corresponding intervention in asset markets. When all agents are in the asset market, the intervention amount in asset markets corresponds exactly to the size of the trader’s velocity shock which leaves their real balances unchanged. As in the flexible exchange rate case, this intervention effectively insulates traders from any wealth effects due to their velocity shocks. Hence, the two regimes are identical from a welfare standpoint.

\textsuperscript{10}Note that though traders do have access to asset markets, these markets are incomplete. More specifically, traders do not have access to asset markets where they can trade in state contingent assets spanning all states. Hence, velocity shocks induce wealth effects and consumption volatility for traders as well despite their access to competitive world capital markets.
4.2 No capital mobility: $\lambda = 0$

The polar opposite case to the one analyzed above is when there are no traders in the economy at all. Hence, this is an economy which corresponds to a representative agent, small open economy which is subject to complete capital controls. Put differently, this is an economy where there is current account convertibility but no capital account convertibility. Since all agents in the economy are non-traders we must have $W_{NT,j} = W_j$. Also, for $\lambda = 0$ we must have $\phi = 1 - \theta$.

It is easy to check that in this case equations (24), (25), (28) and (29) reduce to

\[
\begin{align*}
\bar{c}^{NT,\text{flex}}_t &= c_0^{NT,\text{flex}} = y, \\
\bar{c}^{NT,\text{peg}}_t &= y \left[ 1 + (1 - \theta)(v_t - v_{t-1}) \right] \\
\bar{c}^{NT,\text{peg}}_0 &= y \left[ 1 + (1 - \theta)(v_0 - \bar{v}) \right]
\end{align*}
\]

**Proposition 2** When all agents in the economy are non-traders, i.e., $\lambda = 0$, the flexible exchange rate regime welfare dominates the fixed exchange rate regime.

**Proof.** Since $v_t$ is identically and independently distributed with $E(v_t) = \bar{v}$ and $\text{var}(v_t) = \sigma_v^2 > 0$, it is clear from the above that $E(\bar{c}^{NT,\text{flex}}_t) = E(\bar{c}^{NT,\text{peg}}_t) = y$ for all $t$ where $E$ is the unconditional expectation operator. It is easy to check that for $\lambda = 0$, \( \text{var}(\bar{c}^{NT,\text{peg}}_t) = [y(1-\theta)]^2 2\sigma_v^2 \), \( \text{var}(\bar{c}^{NT,\text{peg}}_0) = [y(1-\theta)]^2 \sigma_v^2 \), while \( \text{var}(\bar{c}^{NT,\text{flex}}_t) = 0 \). Hence, for $\theta < 1$, expected periodic utility (see equation (33)) for non-traders under flexible exchange rates is strictly greater than the corresponding value under fixed exchange rates. It then follows directly that $W^{\text{flex}} > W^{\text{peg}}$ for all $\theta < 1$. \( \square \)

Intuitively, when all agents are non-traders, the aggregate velocity shock is also the shock faced by the non-traders. Hence, under flexible exchange rates the price level adjusts by the same proportion as the velocity shock. Thus, real balances of non-traders are completely insulated from the shocks under flexible exchange rates which leaves their consumption invariant to the shocks. Under fixed rates on the other hand, the monetary authority allows the full amount of the shock to pass through to consumption by buying and selling reserves at an unchanged exchange rate.\(^{11}\)

\(^{11}\)Recall that the monetary authority permits current account transactions for all agents. Only capital account
This causes consumption to be volatile under pegged exchange rates which extracts a welfare cost even though the i.i.d. nature of the shocks imply that expected consumption is identical across the two exchange rate regimes.

4.3 The symmetric shock case: $\theta = 1/2$

Recall that, in general, this model has two sources of redistribution: one coming from the fact that shocks may be asymmetric and the second coming from asset market segmentation. In the two special cases analyzed above ($\lambda = 1$ and $\lambda = 0$) we have looked at the implications of no segmentation and hence, no heterogeneity across agents. Hence, those two cases focused exclusively on the role played by the shocks to velocity. The next special case that is of interest is when both traders and non-traders face symmetric shocks. In particular, this corresponds to the case where $\theta = 1/2$. This case is interesting since under symmetric shocks the only source of redistribution across agents comes from the fact that asset market are segmented.

From equation (16) it follows that if $\theta = 1/2$ then $\phi = 1/2$. Under flexible exchange rates, it is easy to check from equations (24), (25), (28) and (29) that consumption of non-traders in this case is given by

$$c_{NT,flex}^t = y \quad \text{for all } t$$

$$c_{NT,peg}^t = y [1 + (1 - \theta)(v_t - v_{t-1})]$$

$$c_{NT,peg}^0 = y [1 + (1 - \theta)(v_0 - \bar{v})]$$

Moreover, from equations (26) and (30), it follows that consumption of traders in the symmetric case is given by

$$c_{T,flex}^0 = r \frac{k_0}{\lambda} + y$$

$$c_{T,peg}^0 = r \frac{k_0}{\lambda} + y \left[ 1 + \left( \frac{\theta}{\lambda} - \frac{\phi}{\lambda} \right) \left( v_0 - \bar{v} \right) + E_0 \left\{ \sum_{t=1}^{\infty} \left( \frac{1}{1 + \lambda r} \right)^t \left( v_t - v_{t-1} \right) \right\} \right]$$

transactions are restricted to traders. Thus, excess money balances with non-traders would induce them to buy goods abroad which would imply a corresponding reduction in foreign reserves with the central bank.
Proposition 3 When shocks are symmetric across the two types of agents, i.e., $\theta = 1/2$, the flexible exchange rate regime welfare dominates the fixed exchange rate regime for both agents and hence, is the optimal exchange rate regime for the economy.

Proof. It is easy to see that $E(c_t^{NT, flex}) = E(c_t^{NT, peg}) = y$ while $E(c_0^{T, flex}) = E(c_0^{T, peg}) = r_k + y$. Hence, for both types of agents, expected consumption under the two regimes is identical. However, $\text{var}(c_t^{T, peg}) > \text{var}(c_t^{T, flex}) = 0$ and $\text{var}(c_t^{NT, peg}) > \text{var}(c_t^{NT, flex}) = 0$ for all $t$. From the expression for expected periodic utility given by (33), it then follows directly that

$$W^{i, flex} > W^{i, peg} \quad i = T, NT$$

Hence, when shocks are symmetric, welfare under flexible exchange rates is greater than welfare under fixed exchange rates for both agents. Thus, aggregate welfare under flexible exchange rates is unambiguously greater than under fixed rates. ■

Intuitively, under flexible exchange rates the adjustment of the price level is proportional to the velocity shock of both agents when the shocks are symmetric for both agents. Hence, flexible exchange rates completely insulate the real balances of both agents which allows them to smooth consumption completely. Under fixed exchange rate on the other hand, a wealth redistribution occurs across agents due to velocity shocks. Specifically, in order to keep the exchange rate unchanged, the monetary authority intervenes in the asset market to accommodate the average effect of the velocity shock. This affects transfers to traders which induces redistributions. Hence, consumption of non-traders fluctuates over time while consumption of traders is affected by a wealth effect coming from asset market transfers.

4.4 Shocks only to traders: $\theta = 1$

We now analyze the case where non-traders do not face any velocity shocks at all and the shocks are restricted to traders only. In particular, this is the case in which $\theta = 1$. Thus, $v_i^{NT} = 0$ while $v_i^T = v_t$. We will show that in this case it is optimal for the monetary authority to follow a fixed exchange rate regime.
From equation (16) it is easy to verify that in this case, $\phi = \lambda$. Hence, equations (24), (25), (28) and (29) imply that consumption of non-traders is given by

$$c_{i}^{NT, flex} = y \left[ 1 - \lambda \left( \frac{v_{t} - v_{t-1}}{1 - \lambda v_{t-1}} \right) \right]$$

$$c_{0}^{NT, flex} = y \left[ 1 - \lambda \left( \frac{v_{0} - \bar{v}}{1 - \lambda \bar{v}} \right) \right]$$

$$c_{i}^{NT, peg} = y \quad \text{for all } t$$

Moreover, from equations (26) and (30), it follows that for $\theta = 1$ consumption of traders is given by

$$c_{0}^{T, flex} = \frac{k_{0}}{\lambda} + y \left[ 1 + (1 - \lambda) \left( \frac{v_{0} - \bar{v}}{1 - \lambda \bar{v}} + E_{0} \left\{ \sum_{t=1}^{\infty} \left( \frac{1}{1 + r} \right)^{t} \left( \frac{v_{t} - v_{t-1}}{1 - \lambda v_{t-1}} \right) \right\} \right) \right]$$

$$c_{0}^{T, peg} = c_{i}^{T, peg} = r \frac{k_{0}}{\lambda} + y$$

It is useful to start by noting that consumption of both types is constant over time under fixed exchange rates. Intuitively, since non-traders do not face any velocity shocks, any fluctuation of their consumption can only occur through fluctuations in the price level. Since, the monetary authority keeps the exchange rate fixed, consumption of non-traders is constant over time. For the traders, a shock to their velocity induces an incipient change in the exchange rate which is exactly proportional to the shock. The intervention by the monetary authority in the asset market to keep exchange rates unchanged compensates traders exactly by the proportion of their velocity shock thereby keeping their real balances unchanged. Hence, consumption of traders is also constant over time. Constant consumption for both agents also implies that their consumption variances are zero.

The expressions for $c_{i}^{T, flex}$ and $c_{i}^{NT, flex}$ above make clear that under flexible exchange rates, consumption of both types will fluctuate over time. Hence $\text{var}(c_{i}^{flex}) > \text{var}(c_{i}^{peg}) = 0$ for $i = T, NT$ and for all $t$. In the appendix we show that $E \left( \frac{v_{t} - v_{t-1}}{1 - \lambda v_{t-1}} \right) = \frac{-\lambda \sigma_{v}^{2}}{(1 - \lambda \bar{v})^{2}} < 0$. Moreover, $E \left( \frac{v_{0} - \bar{v}}{1 - \lambda \bar{v}} \right) = 0$. Hence,

$$E \left( c_{i}^{T, flex} \right) = r \frac{k_{0}}{\lambda} + y \left[ 1 - \frac{\lambda (1 - \lambda) \sigma_{v}^{2}}{r (1 - \lambda \bar{v})^{2}} \right], \quad \text{for all } t.$$
Thus, $E\left(c^T_{t,\text{flex}}\right) < E\left(c^T_{t,\text{peg}}\right)$ for all $t$ while $\var(c^T_{t,\text{flex}}) > \var(c^T_{t,\text{peg}}) = 0$. Recalling from equation (33) that $E(c - \zeta c^2) = E(c) - \zeta [E(c)]^2 - \zeta \var(c)$, it is clear that $W^T_{\text{peg}} > W^T_{\text{flex}}$. Hence, traders unambiguously prefer fixed exchange rates to floating exchange rates.

The comparison for non-traders is more complicated. First, $E\left(v_{t-1} - v_t - \lambda v_{t-1} \right) = -\lambda \sigma^2 v_t (1 - \lambda \bar{v})^2$ implies that

$$E\left(c^\text{NT,flex}_{t}\right) = y \left[ 1 + \frac{\lambda^2 \sigma^2 v_t}{(1 - \lambda \bar{v})^2} \right] > c^\text{NT,peg}_{t} = y \quad \text{for all } t > 0$$

$$E\left(c^\text{NT,flex}_{0}\right) = y = c^\text{NT,peg}_{0}$$

Hence, expected consumption of non-traders is always weakly greater under flexible exchange rates than under fixed exchange rates. However, the appendix also shows that $\var\left(v_{t-1} - v_t - \lambda v_{t-1} \right) = \frac{\sigma^2}{(1 - \lambda \bar{v})^2}$. Hence,

$$\var(c^\text{NT,flex}_{t}) = 2 \var(c^\text{NT,flex}_{0}) = \left( \frac{y \lambda}{1 - \lambda \bar{v}} \right)^2 2 \sigma^2$$

Thus, the variance of consumption for non-traders is also higher under flexible rates relative to fixed exchange rates.

**Condition 1** *Curvature condition:*

$$\frac{1}{(3 + \frac{1}{\beta}) + \frac{\sigma^2}{(1 - \lambda \bar{v})^2}} < \zeta y < \frac{1}{(2 + \frac{\sigma^2}{(1 - \lambda \bar{v})^2})}$$

**Proposition 4** *When velocity shocks are only on traders, i.e., $\theta = 1$, Condition 1 is a sufficient condition for fixed exchange rates to be the optimal exchange rate policy.*

**Proof.** See Appendix. ■

The intuition behind Proposition 4 is simple. Condition 1 provides two bounds on $\zeta y$. $\zeta y < \frac{1}{(2 + \frac{\sigma^2}{(1 - \lambda \bar{v})^2})}$ guarantees that marginal utility is always positive (which needs to be ensured in the quadratic utility case). This ensures that the higher expected consumption under flexible exchange rates by itself makes the expected utility of non-traders under flexible rates greater than that under fixed exchange rates. However, the second condition $\zeta y > \frac{1}{(3 + \frac{1}{\beta}) + \frac{\sigma^2}{(1 - \lambda \bar{v})^2}}$ ensures that the utility losses of non-traders from the variable consumption under flexible exchange rates are high enough to offset the higher expected consumption that the non-traders enjoys under flexible rates.
Hence, fixed exchange rates are optimal for non-traders. Since, traders unambiguously prefer fixed exchange rates, Condition 1 is sufficient for fixed exchange rates to be optimal for the economy when $\theta = 1$.

More generally, when all the shocks are concentrated on traders, keeping the exchange rate fixed insulates the real balances of the non-traders from the shocks being faced the traders. Moreover, since the monetary authority fixes the exchange rate by intervening in the asset market, it is able to exactly target the group that is affected by the shocks. Hence, both types are strictly better off under fixed exchange rates when $\theta = 1$.

5 Numerical Analysis

In Section 4 we compared the two exchange rate alternatives for some specific cases of interest. To characterize the optimal exchange rate policies more generally, for all plausible parameter values, we solve our model numerically. In particular, we evaluate welfare gains under flexible exchange rates relative to fixed exchange rates for (1) all plausible degrees of market segmentation i.e., for $0 < \lambda < 1$; (2) all plausible degrees of velocity shock asymmetry i.e., for $0 < \theta < 1$; and (3) various values for the variance of velocity shocks.

The welfare rankings of the two regimes are invariant to aggregate endowment and expected money supply. Henceforth, we set $y = \hat{M} = 1$. Further, to abstract from the initial wealth level effects, we set aggregate foreign assets $f_0 = k_0 = 0$. The discount factor $\beta$ is set equal to its standard value of 0.95. For the utility parameter $\zeta$ we choose a value of 0.4, which ensures that Condition 1 holds. We assume a binomial stochastic process for velocity shocks, where the two states, high and low, occur with equal probabilities $\left(= \frac{1}{2}\right)$. We set its expected value $\bar{v} = 0.3$ and its standard deviation $\sigma_v = 0.2$.\footnote{For symmetric shocks $\left(= \frac{1}{2}\right)$, an expected velocity shock of 0.3 implies that, on an average, each household can supplement its beginning of period nominal balances by an additional 15% of its current period sales receipts. $\sigma_v = 0.2$ amounts to a standard deviation of 5% of the expected money supply.}
5.1 Asset market segmentation

First, we compute a weighted sum of the expected lifetime utilities of the two types under both flexible and fixed exchange rates over a range of market segmentation $0.04 < \lambda \leq 1$ for a given level of the velocity shock. \footnote{Note that for $\lambda = 0$ traders’ welfare under fixed exchange rates is not defined since their consumption variance becomes infinite. Therefore, we limit our analysis to $0.04 < \lambda \leq 1$.} Let

$$\Delta W = W^{\text{flex}} - W^{\text{peg}}$$

Hence, $\Delta W$ gives the welfare gain of a flexible exchange rate regime relative to a fixed exchange rate regime. Further, let $W^o = \frac{1}{1-\beta} (y - \zeta y^2)$. Hence, $W^o$ is the lifetime utility obtained by an agent with a certain consumption of $y$ every period. Now define $\Delta c$ as the solution to

$$W^o + \Delta W = \frac{1}{1-\beta} \left[ y + \Delta c - \zeta (y + \Delta c)^2 \right]$$

Substituting $W^o$ into this expression and simplifying gives

$$\Delta W = \frac{1}{1-\beta} \left[ \Delta c - \zeta (\Delta c)^2 - 2\zeta y \Delta c \right] \quad (34)$$

Thus, $\Delta c$ is the amount of certain consumption above (below) a certain level $y$ that will raise (lower) lifetime utility above (below) $W^o$ by $\Delta W$. To make our results readily interpretable in steady state consumption terms, we will henceforth represent $\Delta W$ in terms of $\Delta c$ using equation (34).

Figure 1 plots $\Delta c$ (as a percentage of steady state income $y$) against the degree of market segmentation $\lambda$. Moreover, we depict this relationship for three different levels of velocity shocks asymmetry: $\theta = 0.2, 0.5$, and $0.8$. Note that flexible (fixed) exchange rates are optimal when the graph lies above (below) the horizontal axis. Thus, for $\lambda = 0.5$ and $\theta = 0.5$, the welfare gain under a flexible exchange rate regime relative to a fixed exchange rate regime is the equivalent of an additional constant consumption stream (over and above the steady state income level) of approximately 2.5% of steady state income. Contrarily, for $\lambda = 0.5$ and $\theta = 0.8$, the welfare loss
under a flexible exchange rate regime is the equivalent of a 1% reduction in constant consumption below the steady state income level.

Notice that for $\lambda = 1$ both exchange rate regimes are welfare equivalent, as discussed in Section 4.1. On the other hand, as we showed in Section 4.2, for $\lambda = 0$ a flexible exchange rate regime is welfare superior. For intermediate degrees of market segmentation however, the optimal policy depends on the degree of asymmetry of the velocity shock, i.e., it depends on $\theta$. For $\theta \leq 0.5$, Figure 1 shows that welfare gains under flexible exchange rates monotonically increase with the fraction of non-traders. The intuition behind this is simple. When non-traders get a higher fraction of velocity shocks (low $\theta$), fixed exchange rates are dominated by flexible exchange rates since the latter better insulates the real balances of both types. Fixed exchange rates, on the contrary, induce large wealth redistributions that increase consumption volatilities of both types. Hence, as the fraction of non-traders $(1 - \lambda)$ increases, the case for flexible exchange rates becomes stronger.

5.2 Asymmetry of velocity shocks

Figure 2 exhibits welfare gains under flexible exchange rates relative to fixed exchange rates over a range of velocity shock asymmetry parameter $0 \leq \theta \leq 1$, for two different levels of market
segmentation $\lambda = 0.2$ and 0.5. As before, we set $\sigma_v = 0.2$. Figure 2 shows that for each level of market segmentation, there is a threshold value of $\theta$ above which fixed exchange rates are optimal, while below this value flexible exchange rates are optimal. Intuitively, when shocks largely go to the traders (high $\theta$), flexible exchange rates induce relatively larger wealth redistributions and, hence, higher consumption volatility and lower welfare, whereas fixed exchange do the same when non-traders get relatively higher fraction of shocks (low $\theta$). Thus for each $\lambda$, there is a unique threshold value of $\theta$, where welfare under both regimes become equal.

Increasing the fraction of traders ($\lambda$) decreases the threshold value of the fraction of shocks received by traders ($\theta$) beyond which fixed exchange rates become optimal. To understand this, note that as the fraction of traders increases, traders receive more of the velocity shocks in the aggregate. Therefore, the case for fixed exchange rates becomes stronger as now it provides better insulation to the real balances of the two types. Hence, at any given $\theta$, a higher $\lambda$ shifts the welfare gains curves downwards which implies a smaller threshold value of $\theta$. 

![Figure 2: Optimal exchange rate regime and asymmetric shocks](image-url)
5.3 Variance of velocity shocks

Finally, we analyze the impact of the variance of velocity shocks on the characterization of optimal exchange rate policies for $\sigma_v = 0.1, 0.2, \text{ and } 0.3$. The share of traders is held fixed at $\lambda = 0.5$. Figure 3 presents our results. First, observe that the ordinal rankings of the two regimes is invariant to the variance of the velocity shock. This is because the variance of the shock is just a scale parameter in our model. Second, a higher value of $\sigma_v$ makes the optimal exchange rate regime even more desirable. The intuition behind both these results is simple. The welfare difference between the two regimes is monotonic and separable in $\sigma_v^2$. However, its sign is determined by other parameters of the model, i.e., by $\theta, \lambda$ and $\zeta$. Hence, while $\sigma_v^2$ affects the welfare gains, it does not affect the relative ranking of the two regimes.

![Figure 3: Optimal exchange rate regime and volatility of velocity](image)

Figure 3: Optimal exchange rate regime and volatility of velocity

Note that with $\theta = 0.5$ and $\sigma_v = 0.1$ the steady state consumption equivalent of the welfare gain under flexible rates is about 0.5% of steady state consumption. On the other hand, for $\theta = 0.5$ and $\sigma_v = 0.3$, the consumption equivalent of the welfare gain under flexible rates is approximately 5% of steady state income. Hence, a three-fold increase in the volatility of money demand induces an approximately ten-fold increase in the welfare advantage of the optimal exchange rate regime.
Thus, for economies with more volatile money demands (e.g., Argentina), the benefits of following the optimal exchange rate policy are large in consumption terms and an order of magnitude greater than the corresponding welfare gains for economies with more stable money demands (say the USA).

6 Conclusion

Determination of the optimal exchange rate regime for a country has been one of the oldest issues in international economics. The single most influential idea in this context has been the Mundellian prescription that if shocks facing the country are mostly monetary then fixed exchange rates are optimal whereas flexible rates are optimal if the shocks are mostly real. The key friction underlying Mundell’s results was the assumption of sticky prices in the goods market. In this paper we have investigated the implications of frictions in the asset market as opposed to the goods market. We have shown that when only a fraction of agents are in the asset market, i.e., asset markets are segmented, the Mundellian prescription gets turned on its head. Fixing exchange rates entails central bank interventions in the asset market where only a fraction of agents are present. Hence, monetary shocks (shocks to velocity in our context) under fixed exchange rate regimes cause redistributions across agents thereby generating consumption volatility. On the other hand, when exchange rates are flexible, monetary shocks cause changes in the price level which insulates the real balances of agents. Thus, asset market segmentation causes an inherent welfare bias towards flexible exchange rate regimes even when shocks are monetary.

Another well known result in this area is that under flexible prices, fixed and flexible exchange rate regimes are welfare equivalent. We have shown that with flexible prices but segmented asset markets, it is feasible to break this welfare equivalence. In particular, if traders (i.e., agents who have access to asset markets) face most of the velocity shocks then a fixed exchange rate regime is optimal. Intuitively, if most of the nominal shocks are faced by traders then central bank interventions in the asset market to fix the exchange rate impact precisely the agents who are mostly affected by the shock. Hence, fixed rates provide a better insulation from the shocks than
do flexible rates.

We have also found that the choice of an optimal exchange rate regime is likely to be much more important in welfare terms for more volatile economies like the emerging countries than for stable economies like the industrialized countries. In particular, the welfare advantage of the optimal regime rises non-linearly with the underlying volatility of the economy. Thus, one of our numerical simulations of the model economy shows that a three-fold increase in the volatility of velocity can imply a ten-fold increase in the consumption equivalent welfare gain under the optimal exchange rate regime. We interpret these results as being suggestive of the quantitative importance of choosing the correct exchange rate regime for developing countries.

In this paper we have ignored the issue of endogeneity of market segmentation. In particular, one would expect that agents endogenously choose to be traders or non-traders with the choice being sensitive to the cost of participating in asset markets as well as the prevailing exchange rate and/or monetary regime. We conjecture that a particular choice of exchange rate regime could have important implications for participation in asset markets. Hence, the degree of capital mobility of the economy could itself be endogenous to the exchange rate regime chosen by the economy as people elect in or elect out of asset markets. This is the subject of ongoing and future work.
7 Appendix

7.1 Proof of Proposition 4

Under $\theta = 1$, for a fixed exchange rate regime to welfare dominate a floating exchange rate regime we must have $W_{peg}^N > W_{flex}^N$. Hence,

$$y - \zeta y^2 > (1 - \beta) \left( E \left[ c_0^N \right] - \zeta (E \left[ c_0^N \right])^2 - \zeta \text{var} \left[ c_0^N \right] \right)$$

$$+ \beta \left( E \left[ c_t^N \right] - \zeta (E \left[ c_t^N \right])^2 - \zeta \text{var} \left[ c_t^N \right] \right)$$

(35)

First, we take a second order approximation of $\frac{v_t - v_{t-1}}{1 - \lambda v_{t-1}}$ around $E(v_t) = E(v_{t-1}) = \bar{v}$. This gives

$$\frac{v_t - v_{t-1}}{1 - \lambda v_{t-1}} \approx \frac{(v_t - v_{t-1})}{1 - \lambda \bar{v}} - \frac{\lambda(v_t - \bar{v})^2}{(1 - \lambda \bar{v})^2} + \frac{\lambda(v_t - \bar{v})(v_{t-1} - \bar{v})}{2(1 - \lambda \bar{v})^2}$$

(36)

Taking expectations, (36) yields

$$E \left[ \frac{v_t - v_{t-1}}{1 - \lambda v_{t-1}} \right] = \frac{-\lambda \sigma_v^2}{(1 - \lambda \bar{v})^2}$$

(37)

For evaluating the variance of $\frac{v_t - v_{t-1}}{1 - \lambda v_{t-1}}$, we use equation (36) (ignoring the third and fourth order moments) to get

$$\text{var} \left[ \frac{v_t - v_{t-1}}{1 - \lambda v_{t-1}} \right] = 2 \text{var} \left[ \frac{v_t - v}{1 - \lambda \bar{v}} \right] = \frac{2\sigma_v^2}{(1 - \lambda \bar{v})^2}$$

In order to ensure that the marginal utility of consumption is positive, a sufficient condition is

$$y - \zeta y^2 < y \left( 1 + \frac{\lambda^2 \sigma_v^2}{(1 - \lambda \bar{v})^2} \right) - \zeta y^2 \left( 1 + \frac{\lambda^2 \sigma_v^2}{(1 - \lambda \bar{v})^2} \right)^2$$

(38)

On the other hand, for (35) to hold the necessary and sufficient condition is

$$y - \zeta y^2 > y \left( 1 + \frac{\lambda^2 \sigma_v^2}{(1 - \lambda \bar{v})^2} \right) - \zeta y^2 \left( 1 + \frac{\phi^2 \sigma_v^2}{(1 - \lambda \bar{v})^2} \right)^2 - (1 + \beta) \zeta y^2 \frac{\phi^2 \sigma_v^2}{(1 - \lambda \bar{v})^2}$$

(39)

Equations (38) and (39) imply that if the curvature parameter $\zeta$ such that

$$\frac{1}{(3 + \frac{1}{\beta} + \frac{\sigma_v^2}{(1 - \lambda \bar{v})^2})} < \zeta y < \frac{1}{(2 + \frac{\sigma_v^2}{(1 - \lambda \bar{v})^2})}$$

(40)

then, (35) holds.
References


