Provision Point Mechanisms and Over Provision of Public Goods[*]

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Abstract

Charities often let the sum of contributions determine the quantity of services to provide. Some organizations, however, have the option of setting a minimum threshold necessary for provision of the public good, allowing donors to pledge donations contingent on the threshold being reached. Contributions are only collected when sufficient funds have been pledged. We show that contribution-maximizing fundraisers who have such a strategy available to them will choose to use it. In contrast to the traditional model of voluntary contributions, in this model inefficiency arises as a result of over provision of the public good.

Keywords: Provision Point Mechanism, Fundraising, Commitment to Refund, Over Provision of Public Goods.

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1 Introduction

Fundraisers frequently announce a contribution goal at the beginning of their fundraising campaign. While these goals often are non-binding, there are some cases where the fundraiser can choose a strategy that commits her to either reaching the goal or providing none of the good. For instance, Bagnoli and Mckee (1991) present a case from Manitoba, Canada, where the New Democratic Party in 1980 and 1985 sent letters to its larger contributors soliciting additional funds to mount an upcoming election campaign. The letters stipulated that a target had been set at $200,000 and that the New Democratic Party would refund all contributions if the target were not reached by a certain date. Both campaigns succeeded.[1]

In this paper we determine whether and how a fundraiser will employ a threshold strategy similar to that described in the example above.[2] That is, we examine a case where the public good is continuous, and determine whether the fundraiser will want to artificially truncate the production function. This truncation is secured by setting a threshold for total contributions, and by allowing donors to make contribution pledges contingent on the threshold being reached. Alternatively, the fundraiser may collect contributions and refund them if they fall short of the goal.[3] The primary results of the paper are that a contribution-maximizing fundraiser who can commit to such a strategy always chooses to set a threshold, and that the chosen threshold is "too high." Hence, the equilibrium of this game is one of over provision of the public good.

The contribution game is similar to that commonly examined in the literature on private provision of public goods: A finite number of potential donors simultaneously allocate an exogenous finite income between consumption of a private good and contributions to a public good, and an individual's utility depends only on the consumption levels of private and public goods.[4] In such an environment the standard free-riding result holds, and there is under

[1]Other examples are presented in Bagnoli and Mckee as well as in Marks and Croson (1998) and Marks, Schansberg and Croson (1999).

[2]A series of recent papers have examined the strategic role of the fundraiser. See for example Andreoni (1998), Bac and Bag (1999a,b), Bilodeau and Slivinski (1997), Romano and Yildirim (1998), Slivinski and Steinberg (1999), and Vesterlund (1999).

[3]Marks, Schansberg and Croson (1999) argue that these two mechanisms are isomorphic.

[4]See for example the seminal work of Bergstrom, Blume, and Varian (1989).

provision of the public good. Bagnoli and Lipman (1989) show that this inefficiency result need not hold when a discrete public good is being provided. In fact, a simultaneous move provision game with a simple refund rule secures efficient provision.

Applying the same equilibrium concept as that of Bagnoli and Lipman (1989), we show that their result may be extended to the case where the public good is continuous. In particular, when contributors have quasi-linear preferences, then a welfare-maximizing fundraiser can secure efficient provision by committing to a threshold and to refunding donations when the threshold is not reached.

In many cases it is unlikely that the fundraiser's objective is to maximize social welfare. While some contracts between the agency and the fundraiser do not directly provide incentives to maximize total contributions, there is no penalty for doing so.[5] Hence, if the fundraiser's promotion and future employment depends on her past success, then contribution maximization appears to be a reasonable assumption. With such an objective it is questionable whether efficiency will result when the fundraiser is able to use a provision point mechanism. For a more general class of contributor preferences we show that a contribution-maximizing fundraiser will choose to set the threshold at a level which exceeds efficient provision. In contrast to the standard simultaneous problem, inefficiency arises from over provision, rather than under provision. Furthermore, when contributors do not have quasi-linear preferences, then over provision may result even when the fundraiser is a potential donor who cares about the provision level.

The paper is organized as follows. We first present the model, describing the contribution game for every threshold set by the fundraiser. Next, the equilibrium of the game is examined and some of the model's comparative statics are determined. In contrast to the standard simultaneous provision model, it is shown that an increase in the population need not decrease the total provision level. We then discuss how sensitive the results are to the fundraiser's objective and commitment ability. Finally, we show how threshold strategies can be endogenous in competitive public good provision problems.

[5] See Slivinski and Steinberg (1999) for a discussion of contracts between the agency and the fundraiser.

2 The Model

The model is one of continuous provision of a public good, where a contribution-maximizing fundraiser artificially truncates the production function by setting a minimum threshold for total donations. If the threshold is not reached, all contributions are returned to the respective donors. The optimal threshold choice depends on whether the fundraiser can commit to refunding all contributions when the threshold is not met and whether the fundraiser can commit to not accepting donations that are made after the refund. We start by considering a full-commitment game, where the fundraiser commits to returning all contributions if they do not meet the threshold and will refuse to accept donations following the refund. Following this analysis we consider a more realistic scenario where the fundraiser, after having made the refund, accepts voluntary donations. We refer to this level of commitment as partial commitment.

2.1 The Full-Commitment Game

Let n be a finite number of potential donors, and let each be endowed with an exogenous finite and strictly positive endowment, w_i. Each donor allocates his endowment between a private good, x_i, and contributions to a public good, g_i. The game has an additional player: the fundraiser. The fundraiser's objective is to maximize contributions, and she has only one choice variable: the selection of a threshold, T, below which the contributions will be refunded. The fundraiser can commit to refund all contributions if the threshold is not met, and, in addition, she can commit to not accepting any other donations after the refund.

Once the fundraiser has chosen T, the n players simultaneously contribute to the public good, choosing $g_i^F(T) \in [0; w_i]$, where the superscript F refers to the fact that we are examining the full-commitment game. The corresponding provision level of the public good is:

$$G^F(T; \{g_i^F(T)\}_{i=1;\ldots;n}) = \begin{cases} \sum_{i=1}^n g_i^F(T) & \text{if } \sum_{i=1}^n g_i^F(T) \geq T \\ 0 & \text{otherwise:} \end{cases} \quad (1)$$

When the threshold is not met the contributions are refunded and no public good is provided.

Thus, the consumption of the private good is:

$$x_i^F(T;\{g_i^F(T)\}_{i=1,\ldots,n}) = \begin{cases} w_i - g_i^F(T) & \text{if } \sum_{i=1}^{n} g_i^F(T) \geq T \\ w_i & \text{otherwise:} \end{cases} \tag{2}$$

There is no monetary rebate and contributions in excess of the threshold are used to provide more of the public good.[6] In this event we assume that the production function is linear, however this assumption is without loss of generality.[7]

At the end of the game just described, each player receives utility according to a utility function, $U_i(x_i^F(T;\{g_i^F(T)\}_{i=1,\ldots,n}); G^F(T;\{g_i^F(T)\}_{i=1,\ldots,n}))$, which is continuous, strictly quasi concave, twice continuously differentiable, and monotonically increasing in both arguments. As is standard in the literature we assume that all players have complete information.

If there is no fundraiser, or if equivalently the threshold is set equal to zero, then the game is identical to the standard simultaneous contribution game, and a unique Nash equilibrium is guaranteed by assuming that both the private and the public goods are normal goods.[8] Denote the unique Nash equilibrium contribution profile when $T = 0$ by $\{g_i^*\}_{i=1,\ldots,n}$. In order to ensure that the equilibrium provision level remains unique even when $T > 0$, we will use the same refinement used by Bagnoli and Lipman (1989), namely the Undominated Perfect Equilibrium concept. This equilibrium concept eliminates dominated strategies and applies the notion of perfection to the resulting game. While perfection alone can rule out equilibria where the sum of contributions exceed the threshold, it is not sufficient to rule out the many inefficient Nash equilibria where the threshold is not reached. If however all dominated strategies are eliminated before applying perfection then these inefficient Nash equilibria can be ruled out.[9]

[6] Marks and Croson (1998) refer to this as a Utilization Rebate.

[7] This standard approach simply corresponds to having incorporated the production function into the utility function. Whether we solve the problem with individual utility functions of the form $V_i(x_i; f(\sum g_i))$ (for some production function $f(\cdot)$) or $U_i(x_i; \sum g_i)$ will not affect our results.

[8] See Bergstrom, Blume and Varian (1989).

[9] When there is a discrete unit of a public good, Bagnoli and Lipman (1989) show that efficiency may be secured through a mechanism where contributions in excess of the necessary funds are confiscated and insufficient contributions refunded (alternatively, a rebate of excess contributions may be offered, as long as increasing one's contribution never increases one's rebate more than one-for-one). They show that the

2.2 The Partial-Commitment Game

In the full-commitment game described above, the public good provision level is zero whenever the threshold is not met (i.e. $G^F = 0$). A more realistic assumption is that if the threshold is not met, then the donors are free to contribute as much as they want. That is, they can always contribute what they would have donated in the absence of the threshold. We refer to this game as the partial-commitment game. Let the individual contributions in this case be denoted by $g_i^P(T)$. The partial-commitment game is very similar to the full-commitment game. The only difference between the two, is that following a refund, there is an additional contribution stage of the game. This stage is identical to the standard simultaneous-move contribution game, and there is a unique continuation equilibrium where every player gives g_i^*.[10] The resulting public good provision level is:

$$G^P(T; \{g_i^P(T)\}_{i=1,\ldots,n}) = \begin{cases} \sum_{i=1}^n g_i^P(T) & \text{if } \sum_{i=1}^n g_i^P(T) \geq T \\ \sum_{i=1}^n g_i^* & \text{otherwise.} \end{cases} \quad (3)$$

2.3 Additional Assumptions

For the analysis of both the games described above we need to impose three more assumptions.

Assumption 1 $U_i(0; w_i + \sum_{j \neq i} g_j) \quad U_i(w_i; 0)$, $\forall \{g_j\}_{j \neq i}$, $\forall i$.

Assumption 2 There exists a $g_i > 0$ such that $U_i(w_i - g_i; g_i) > U_i(w_i; 0)$ $\forall i$.

unique Undominated Perfect Equilibrium of the game is one of efficient provision. Extending the problem to provision of multiple units complicates the analysis. In general, there are no Nash equilibria of the simultaneous contribution and refund game that implement efficient provision. However when preferences are quasi linear Bagnoli and Lipman show that efficiency is secured in a game where contributions are made sequentially. In particular contributions are raised for one unit at a time, and fundraising ends whenever donations are insufficient to pay for the next unit. In this case undominated perfect equilibria do not fully implement the efficient outcome, which is, on the other hand, fully implemented in successively undominated strictly perfect equilibria (obtained by successively eliminating dominated strategies and applying strict perfection to the resulting game). Although there is some experimental support for the one-unit case (see Bagnoli and McKee 1991), there is no experimental support for the multiple-unit case (Bagnoli, Ben-David and McKee (1992).

[10] Equivalently, one could describe the partial-commitment game exactly like the full-commitment game, but with the individual refund being equal to $g_i^P(T) - g_i^*$, every time the threshold is not met.

Assumption 1 implies that, independent of the size of the contributions of the other players, each consumer weakly prefers consuming none of the public good to consuming none of the private good. Assumption 2 requires that individuals care enough about the public good to make a positive contribution in the event that nobody else does.[11]

The next assumption is a behavioral one:

Assumption 3 If an individual is indifferent between two donations and if his choice is pivotal to reaching a threshold, then he chooses the contribution that reaches the threshold:

1. If $U_i(w_i - g_i^F; g_i^F + \sum_{j \neq i} g_j^F) = U_i(w_i; 0)$ for some $\{g_j\}_{j \neq i}$, then g_i^F is chosen against any g_i'' such that $g_i'' + \sum_{j \neq i} g_j < T$.
2. If $U_i(w_i - g_i^P; g_i^P + \sum_{j \neq i} g_j) = U_i(w_i - g_i^*; \sum_{j=1}^n g_j^*)$ for some $\{g_j\}_{j \neq i}$, then g_i^P is chosen against any g_i'' such that $g_i'' + \sum_{j \neq i} g_j < T$.

3 Equilibrium

This section characterizes the equilibria of the two games. The basic properties of the two games are very similar, and it is convenient to start with the simpler one. Hence we first consider the full-commitment game, where the fundraiser can commit to rejecting contributions that are made following a refund. Then we examine the more realistic case where the fundraiser accepts the donations that are made after the refunds.

3.1 Full Commitment

Suppose that the fundraiser can commit to refunding insufficient funds and not accepting subsequent donations. Then the production of the public good is given by the discontinuous function $G^F(\cdot;\cdot)$. The truncated production function is illustrated in Figure 1, where T^* is the fundraiser's chosen threshold.

Let $\underline{T} = \underline{G} = \sum_{i=1}^n g_i^*$ denote the contribution level that corresponds to the traditional simultaneous contribution level. Let $\overline{G}^F$ denote the positive contribution level at which all

[11] $U_i(x_i; G) = x_i^{\frac{1}{2}}(a + G)^{\frac{1}{2}}$, with $w_i > a > 0\ \forall i$, is an example of a utility function that satisfies these two assumptions.

contributors are exactly indifferent between meeting the threshold and getting none of the public good, i.e., $\overline{G}^F \equiv \sum_{i=1}^n g_i > 0$ such that

$$U_i(w_i - g_i; \sum_{i=1}^{n} g_i) = U_i(w_i; 0) \quad \forall i \tag{4}$$

We first show that there exists a contribution level $\overline{G}^F$ where (4) holds for all i. Then we show that the contribution level $\overline{G}^F$ is unique.

Lemma 1 There exists $\overline{G}^F \equiv \sum_{i=1}^n g_i > 0$ such that $U_i(w_i - g_i; \sum_{i=1}^n g_i) = U_i(w_i; 0) \;\; \forall i$.

Proof. Let us first characterize the set of n indifference curves $U_i(w_i - g_i; g_i + \sum_{j \neq i} g_j) = U_i(w_i; 0)$. Figure 2 shows an example of contributor i's indifference curve. The sum of contributions by others, $G_{-i} = \sum_{j \neq i} g_j$; is on the vertical axis, and i's contribution, g_i, on the horizontal axis. Monotonicity and strict quasi concavity of $U_i(\cdot;\cdot)$ imply that i's indifference curve bounds a strictly convex set, and is U-shaped. For all $G_{-i} > 0$ and $g_i > 0$, the indifference curve $U_i(w_i - g_i; G_{-i} + g_i) = U_i(w_i; 0)$ is an increasing and concave function, $g_i = f_i(G_{-i})$. Continuity of U_i implies that f_i is continuous in G_{-i}. Assumption 1 implies that $f_i(G_{-i})$ is bounded above by w_i. By Assumption 2 we know that there exists a $g_i = f_i(0) > 0$. For sufficiently small $\epsilon > 0$, let $A = \{y \in R^n : \epsilon \leq y_i \leq w_i; i = 1, \ldots, n\}$. A is clearly a compact and convex set. By Assumption 2 we know that the functions $g_i = f_i(G_{-i})$ are continuous functions from A into itself. Hence, by Brouwer's fixed point theorem, there must exist a fixed point $\{g_i\}_{i=1,\ldots,n}$, such that $U_i(w_i - g_i; \sum_{i=1}^n g_i) = U_i(w_i; 0) \; \forall i$. This fixed point contribution profile results in a total provision level $\overline{G}^F > 0$. QED.

Lemma 2 There is a unique positive contribution level $\overline{G}^F \equiv \sum_{i=1}^n g_i > 0$ such that $U_i(w_i - g_i; \sum_{i=1}^n g_i) = U_i(w_i; 0) \;\; \forall i$.

Proof. By Lemma 1, we know that there is at least one contribution level $\overline{G}^F = \sum_{i=1}^n g_i > 0$ such that $U_i(w_i; 0) = U_i(w_i - g_i; \sum_{i=1}^n g_i) \; \forall i$: Suppose there are two such levels, and denote them by $\overline{G}$ and $\overline{G}'$. Assume, without loss of generality, that $\overline{G} < \overline{G}'$: Call $\{g_i\}_{i=1,\ldots,n}$ the contribution profile such that $\sum_{i=1}^n g_i = \overline{G}$ and (4) holds; call $\{g_i'\}_{i=1,\ldots,n}$ the contribution profile such that $\sum_{i=1}^n g_i' = \overline{G}'$ and (4) holds. We will use the notation of the previous proof, i.e., $g_i = f_i(G_{-i}) \; \forall i$ and $g_i' = f_i(G'_{-i}) \; \forall i$.

Strict quasi concavity and monotonicity of the utility function imply that $f_i' > 0$ and $f_i'' < 0$. Hence, if there exist two positive contribution levels, $\overline{G}$ and $\overline{G}'$, which everyone views as good as $U_i(w_i; 0)$, then monotonicity implies that $g_i' > g_i$ and $G'_{-i} > G_{-i}$. Concavity implies that

$$\frac{g_i}{G_{-i}} > \frac{g_i'}{G'_{-i}}; \ \forall i.$$

This implies that each individual gives a smaller share of the total provision level at $\overline{G}'$ than they do at $\overline{G}$:

$$\frac{g_i}{\overline{G}} > \frac{g_i'}{\overline{G}'}; \ \forall i.$$

Summing over i we immediately see the contradiction:

$$\sum_{i=1} \frac{g_i}{\overline{G}} = 1 > \sum_{i=1} \frac{g_i'}{\overline{G}'} = 1.$$

It is not possible to have a $\overline{G}' > \overline{G}$ satisfying (4) if $\overline{G}$ satisfies it. QED.

In order to characterize the equilibrium of the full-commitment game we first determine the undominated perfect equilibrium (UPE) of the subgame that follows the announcement of a threshold. In particular, it is necessary to characterize the continuation equilibrium profile $\{g_i(T)\}_{i=1,\ldots,n}$ and the corresponding provision function $G^F(T; \{g_i(T)\}_{i=1,\ldots,n})$. Once it has been determined how contributions change with the threshold, then the fundraiser's optimal threshold at the first stage can be determined.

Lemma 3 For every T there is a unique UPE provision level $G^F(T)$.

1. If $T \leq \underline{G}$; then the UPE contribution profile is $\{g_i^*\}_{i=1,\ldots,n}$ and the UPE provision level is $G^F(T) = \sum_{i=1}^n g_i^* = \underline{G}$;
2. If $T \in (\underline{G}; \overline{G}^F]$; then all the UPE contribution profiles $\{g_i^F(T)\}_{i=1,\ldots,n}$ are such that $\sum_{i=1}^n g_i^F(T) = T$, and hence $G^F(T) = T$;
3. If $T > \overline{G}^F$ then $G^F(T) = 0$.

Proof.

1. Since the threshold constraint does not bind when $T \leq \underline{G}$, the equilibrium contribution will be identical to that of a game with no threshold. Due to the normality assumption, the Nash equilibrium without a threshold is unique, and $g_i^F(T) = g_i^* \; \forall i$. Hence, $G^F = \sum_{i=1}^n g_i^F(T) = \sum_{i=1}^n g_i^* = \underline{G}$ for any $T \leq \underline{G}$. Obviously, this unique Nash equilibrium is also the unique UPE.

2. If $T \in (\underline{G}, \overline{G}^F]$ then $G^F(T) = T$. Let us first show that there exists a Nash equilibrium where T is reached. If $T \leq \overline{G}^F$, it follows that there must exist contribution profiles $\{g_i(T)\}_{i=1,\ldots,n}$ such that $\sum_{i=1}^n g_i(T) = T$ and $U_i(w_i - g_i(T); T) \geq U_i(w_i; 0) \; \forall i$ (with strict inequality if $T < \overline{G}^F$). No player i has a profitable deviation $g_i' < g_i(T)$ from any such profile, since such deviation would entail no provision of public good. Similarly there are no deviations with $g_i' > g_i(T)$ for any i. This can be seen by noting that when $G > \underline{G}$ it must be the case that $\frac{\partial U_i}{\partial G} \leq \frac{\partial U_i}{\partial x_i}$.

   Next, we show that $G^F(T) = T$ is the unique UPE provision level. Since no donor will contribute a $g_i' > g_i(T)$ there cannot be equilibria where the threshold is exceeded. Also there can not be UPE where the threshold is not reached. As in Bagnoli and Lipman (1989), there are inefficient Nash equilibria where the good fails to be provided. However, as in their analysis, only equilibria that reach the threshold are UPE.[12]

3. In order to have $\sum_{i=1}^n g_i^F(T) > \overline{G}^F$ we would need $U_i(w_i; 0) < U_i(w_i - g_i^F(T); T)$ for some i, however by construction of $\overline{G}^F$ this is not possible. Thus, for this range of thresholds there does not exist an equilibrium where contributions reach the threshold. QED.

The findings of Lemma 3 are summarized in Figure 3. When the threshold is less than the standard Nash equilibrium level, it has no effect on the contribution level. Once the threshold is past this level, total contributions increase in a one-to-one fashion until the threshold surpasses $\overline{G}^F$, at which point the provision level drops to 0. When $T \in (\underline{G}, \overline{G}^F]$ all equilibria result in a contribution level that exactly equals the threshold. Note that in general the contribution profile is unique only when $T = \overline{G}^F$.

[12] Assumption 3 secures the positive contribution profile even when $T = \overline{G}^F$. Without this assumption there would be a large number of equilibria that all fail in providing the public good.

It is now straightforward to solve the first stage of the game and determine the equilibrium threshold choice, T^*:

Proposition 1 If the fundraiser is able to commit to refund contributions and end the campaign when the threshold is not reached, then she will always choose $T^* = \overline{G}^F$. The ensuing equilibrium is Pareto inferior to the continuation equilibria of any other $T < \overline{G}^F$.

Proof. The fundraiser's objective is to maximize total contributions. Hence, by monotonicity of $G^F(T)$ when $T \in [\underline{G}; \overline{G}^F]$, the fundraiser will choose $T^* = \overline{G}^F$.

We know from Lemma 2 that there is a unique $\overline{G}^F$ such that everybody is indifferent between the situation where none of the public good is provided and giving g_i^F such that $\sum_{i=1}^n g_i^F = \overline{G}^F$, i.e., $U_i(w_i - g_i^F(\overline{G}^F); \overline{G}^F) = U_i(w_i; 0)\ \forall i$: Obviously there does not exist an equilibrium where a consumer receives a utility level less than $U_i(w_i; 0)$. In the case where $T > \overline{G}^F$ none of the public good is provided, so $U_i = U_i(w_i; 0)\ \forall i$: However, when $T < \overline{G}^F$ the UPE secure provision and a utility level of $U_i(w_i - g_i^F(T); T) \geq U_i(w_i; 0)\ \forall i$, with strict inequality for some i. Hence, any of the continuation UPE that arises when $T < \overline{G}^F$ is Pareto superior to the equilibrium contribution profile at $\overline{G}^F$.[13] QED.

The equilibrium threshold $T^* = \overline{G}^F$ and the contribution profile $\{g_i^F(T^*)\}_{i=1;:::;n}$ constitute the unique UPE of the overall game. That is, there is no coordination problem – everyone gives their most and everyone is pivotal to the public good being provided. The contribution-maximizing fundraiser's choice of $\overline{G}^F$ is inefficient, but, in sharp contrast to past results on the private provision of public goods, the inefficiency arises as a result of over provision rather than under provision of the public good.

Given that the fundraiser does not choose an efficient outcome, one may wonder wether a benevolent planner can secure an efficient provision of the public good by committing to a threshold. Let $\{g_i^{**}\}_{i=1;:::;n}$ denote a Pareto efficient contribution profile. Knowing that the Nash equilibrium of the simple simultaneous game is one of under provision and that $\{g_i^F(\overline{G}^F)\}_{i=1;:::;n}$ is one of over provision, we also know that $\underline{G} < \sum_{i=1}^n g_i^{**} < \overline{G}^F$. Now the question is whether it is possible to induce a $\{g_i^{**}\}_{i=1;:::;n}$ by setting a threshold and offering a refund when contributions fall short of this goal. Generally the answer is no, because for any

[13] The fundraiser's utility is obviously not counted in the welfare comparison.

$T \in (\underline{G}, \overline{G}^F)$ there exist multiple equilibria, and the planner cannot secure any one of these outcomes simply by setting T.[14] Furthermore, since the efficient provision level generally depends on the contribution profile, it is not necessarily the case that the planner will choose $T = \sum_{i=1}^n g_i^{**}$. One exception arises when preferences are quasi linear ($U_i = x_i + V_i(G)$).

Proposition 2 If all individuals have quasi-linear utility functions, then there is a unique Pareto efficient provision level and a benevolent planner can secure this outcome by letting $T = \sum_{i=1}^n g_i^{**}$.

Proof. If a contribution profile $\{g_i^{**}\}_{i=1,...,n}$ is Pareto efficient then the Samuelson condition must hold, i.e. evaluated at such a profile $\sum_{i=1}^n \frac{\partial U_i / \partial G}{\partial U_i / \partial x_i} = 1$. In the case where preferences are quasi linear, the Samuelson condition reduces to $\sum_{i=1}^n \frac{\partial V_i}{\partial G} = 1$, and is independent of the distribution of wealth across individuals. Hence any contribution profile where $G = \sum_{i=1}^n g_i^{**}$, results in the Pareto efficient provision level of the public good. QED.

Recall that Bagnoli and Lipman (1989) focused on the case where subjects have quasi-linear preferences. Given this type of preferences they show that the UPE is efficient in the case where there is one discrete unit of the public good. Unfortunately this refinement is insufficient at securing efficient provision when there are multiple discrete units of the public good, and subjects freely contribute to the good. In our case where the fundraiser artificially truncates the production function, the problem is however similar to that of a discrete unit and as a result UPE is a sufficient refinement to secure efficient provision.

[14] A welfare-maximizing planner who has the power to choose his preferred mechanism would obviously prefer to set individual thresholds rather than an aggregate threshold. Individual thresholds would, under complete information, allow the social planner to obtain the efficient contribution profile. On the other hand, a contribution-maximizing fundraiser would never benefit from substituting aggregate thresholds with individual ones.

3.2 Partial Commitment

Let us now analyze the case in which the fundraiser accepts voluntary donations after the refund. In this case the consequence of not meeting the threshold is no longer zero provision of the public good, but rather a contribution level that is identical to the Nash contribution level with no threshold, as displayed in Equation (3).

Let $\overline{G}^P \equiv \sum_{i=1}^{n} g_i$ such that

$$U_i(w_i - g_i; \sum_{i=1}^{n} g_i) = U_i(w_i - g_i^*; \underline{G}) \quad \forall i. \tag{5}$$

Existence and uniqueness is easily extended to the partial-commitment game. The only difference relative to Lemma 1 is that individual indifference curves intersect at $\{g_i^*\}_{i=1,\dots,n}$, where $\frac{\partial U_i / \partial G}{\partial U_i / \partial x_i} \leq 1$. For given $G_{-i} > G^*_{-i}$, the set of points that individual i views as being indifferent to $(w_i - g_i^*; \underline{G})$ is a monotonically increasing and concave function $g_i = f_i(G_{-i})$, with $g_i > g_i^*$. Hence, if we redefine the set A to be $A = \{y \text{ in } R^n : \epsilon + g_i^* \leq y_i \leq w_i$ for $i = 1, \dots, n\}$, then it is easy to see that $\overline{G}^P$ exists. Similarly one can easily extend Lemma 2 to show that $\overline{G}^P$ is unique.

In determining the equilibria of this case it is important to note that the only difference between partial commitment and full commitment is in the value of the outside option, i.e., the utility experienced when the threshold is not met. A small decrease in an individual's contribution will trigger the refund and bring the total contribution level down to the simultaneous contribution level with no threshold. It is therefore apparent that the character of the results found for full commitment carries over to the case of partial commitment.

Lemma 3':With partial commitment, for every T there is a unique UPE provision level $G^P(T)$ such that

1. For $T \leq \underline{G}$, $G^P(T) = \sum_{i=1}^{n} g_i^P(T) = \underline{G}$;
2. For $T \in (\underline{G}, \overline{G}^P]$, $G^P(T) = \sum_{i=1}^{n} g_i^P(T) = T$;
3. For $T > \overline{G}^P$, $G^P(T) = \underline{G}$.

Similar to the full-commitment game, the fundraiser will choose the threshold that results in the highest provision level. As shown in Lemma 3', $G^P(T)$ increases up to the point where

$T = \overline{G}^P$, and thus the fundraiser will indeed choose $T^* = \overline{G}^P$: Since $U_i(w_i - g_i(T); T) \geq U_i(w_i - g_i^*; \underline{G})$ for all $T \in (\underline{G}; \overline{G}^P]$, a contribution-maximizing fundraiser will choose a threshold such that the resulting contribution level is inefficiently large.

All the above guarantees the following general result:

Theorem 1 *If a contribution-maximizing fundraiser has access to either partial or full commitment, then the public good is over provided in any Undominated Perfect Equilibrium.*

4 Population Size

An interesting question is how an increase in population size affects the provision of the public good. The classical private provision of public goods model, where agents derive utility solely from their consumption of the private and public good, predicts that the average contribution approaches zero as the population increases. Andreoni (1988) shows that as the population gets larger the total provision of the public good remains finite, and that the fraction of the population contributing to the good declines monotonically with n. If preferences are identical, only the wealthiest individuals will contribute in the limit.[15]

An interesting feature of our model is that it presents a case where individuals contribute to the public good even as the economy gets large. In fact the model predicts that each and every individual who cares about the public good always makes a contribution in equilibrium. The reason is that in both the full- and partial-commitment games the threshold results in provision levels that exceed that of the outside option, and that each contributor must be indifferent between reaching the threshold, and receiving a refund. Hence, independent of n; it must be the case that every individual is making a strictly positive contribution.

The intuition is shown graphically in Figure 4. In the full-commitment and partial-commitment games, the fundraiser can extract the highest possible contribution level from the consumer. This implies that the new equilibrium results in higher contributions than the outside option, but will be along the same indifference curve as the outside option. Since the indifference curve crosses the outside option bundle from above (be it $(w_i; 0)$ or $(w_i - g_i^*; \underline{G})$

[15] As pointed out by Andreoni (1988) this prediction makes the Red Cross, the Salvation Army and PBS logical impossibilities.

and because preferences are strictly convex, all consumers will make contributions to the good when $T = \overline{G}$.

Furthermore, in the full-commitment case it is also possible to determine how an increase in n affects the total and individual contribution levels.

Proposition 3 In the full-commitment game the individual and total contribution increase with n.

Proof. Suppose $n = n'$ and $T' = \sum_{i=1}^{n} g_i^F(n')$; where $g_i^F(n')$ is such that $U_i(w_i - g_i^F(n'); T') = U_i(w_i; 0)\ \forall i$. Suppose an additional individual j is added to the population, such that $n = n''$. First, we note that by monotonicity and assumption 1 it will be the case that for any positive threshold (T'') $g_j^F(n'') > 0$ for $U_j(w_j - g_j^F(n''); T'') = U_j(w_j; 0)$. Second, we want to show that it is possible for the fundraiser to choose a $T'' > T'$. Suppose $T'' = T'$, then given that $g_j^F(n'') > 0$, it must be the case that some contributor k decreases his contribution and therefore $U_k(w_k - g_k^F(n''); T') > U_k(w_k - g_k^F(n'); T') = U_k(w_k; 0)$, hence the fundraiser can increase his threshold past T'.

With a $T'' > T'$ we note that $U_i(w_i - g_i^F(n'); T'') > U_i(w_i - g_i^F(n'); T') = U_i(w_i; 0)$, $\forall i \neq j$. Therefore the threshold T'' is set at a level where each individual i's contribution $g_i^F(n'') > g_i^F(n')$; $i = 1; \ldots; n'$. QED.

In the full-commitment game the outside option is unaffected by the size of the population, and it is therefore only necessary to examine how an increase in n affects $\overline{G}^F$. If we add one more contributor, this individual obviously has to make a contribution for (4) to hold. This increase in G_{-i} weakly increases everyone's contribution. Hence, in the full-commitment case total contributions are increasing with n.

It is somewhat more difficult to determine what happens when n increases in the partial-commitment game. The reason is that an increase in n weakly increases the attractiveness of the outside option. As the population increases, the individual contribution following a refund weakly decreases, and the fundraiser is no longer able to extract the same amount of surplus from the individuals who contribute following a refund. Therefore it is possible that an increase in n results in a decrease in the contributions by these individuals. This potential decrease implies that we also are unable to characterize the change in contributions for those who are non-contributors following a refund. Naturally if the increase in population leaves the set of contributors following a refund unchanged, then the outside option is unaffected and Proposition 3 extends to the partial-commitment game.[16] That is, in this particular case individual contributions increase with n.

5 Contributing Fundraiser and Credibility

In the above analysis we have assumed that the person choosing the threshold is someone who only cares about total contributions. However, in some cases it might be more reasonable to assume that the person choosing the threshold cares about the public good, and therefore is a potential contributor. One example may be the director of the charity's board. Let us denote this individual by D and assume that she has a utility function $U_D(x_D; G_D)$. In choosing the threshold, T^D, the director will aim at maximizing her objective function subject to her budget constraint and the individual rationality constraints of the other contributors. Let us assume that the director has partial commitment, hence the set of individual rationality constraints are such that all other contributors will make contributions $g_i(T^D)$ as long as $U_i(w_i - g_i(T^D); \sum_{-i} g_i(T^D)) \geq U_i(w_i - g_i^*; \underline{G})$. The question of interest is whether the director's preferred threshold differs from that of a donation maximizing fundraiser, and in particular whether the resulting equilibrium may result in over provision.

If the director had the power to choose a particular contribution profile, then she would pick a profile such that the individual rationality constraint binds for all other contributors. That is she would choose the point where her indifference curve is tangent to the set of outside option indifference curves for everyone else. Let us denote this contribution level by $\overline{G}^D$ and the director's resulting utility level by $\overline{U}_D$. Obviously this particular contribution profile is Pareto efficient.

Not surprisingly the director cannot secure this contribution profile by simply setting a threshold of $\overline{G}^D$. Rather this threshold will result in a continuum of equilibria that all have

[16] Recall that Andreoni (1988) showed that when the population is sufficiently large and preferences are identical then only the wealthiest individuals contribute to the public good. In our model this implies that an increase in the population which does not affect the size of the wealthiest class, will leave the outside option constant, and result in an increase in individual contributions towards the threshold.

contribution levels that meet the threshold, and where everyone is no worse off than in the no threshold scenario. Given that there exists a continuum of equilibria, it is not clear what the director's preferred threshold will be.

If we assume the possible equilibria that may result at a given threshold are equally likely, then the director will indeed maximize his expected utility by choosing $T^D = \overline{G}^D$. The reason is that independent of the chosen threshold she will reach a minimum utility of $U_D(w_D - g_D^*; \underline{G})$. As she moves closer to $\overline{G}^D$, less mass gets attributed to the lower utility levels, and equilibria that result in a higher utility level become likely, with the maximum utility $\overline{U}_D$ being reached at a threshold of $\overline{G}_D$. Now given a threshold of $\overline{G}^D$ there exists at least one equilibrium which is Pareto efficient. Namely the director's preferred contribution profile. However in general the other equilibria will not be Pareto efficient and the resulting equilibrium may be one of either over or under provision of the public good.[17]

When the threshold-determining individual cares about the public good, the preferred threshold is one that results in a higher social welfare than when no threshold is chosen, however the resulting equilibrium may still be one of over provision of the public good.

Another important question is when we may expect fundraisers to use a threshold strategy. There are two variables that certainly affect the likelihood and the effectiveness of such a strategy. First of all, it is easier to commit to a threshold if the charity has established credibility with the donors and there is going to be repeated interaction between them.[18] If the game is not a repeated game, or, in general, if a charity does not have any credibility, full commitment is impossible. Partial commitment may be easier to obtain since the promise

[17]One case in which the all the equilibria will result in the Pareto efficient provision level is the case where everyone has quasi-linear preferences.

[18]Assuming that the fundraiser's discount factor is sufficiently large, one can demonstrate that there is a sustainable equilibrium where contributors give up to the threshold. For simplicity, let us first examine the case where contributors have quasi-linear preferences, such that there is a unique Pareto efficient provision level. In this case a welfare-maximizing fundraiser chooses the Pareto efficient provision level as the threshold. If the contributions reach the threshold the donors will get payoffs that exceed the payoffs they achieve in the absence of the threshold, as well as their minimax payoffs. Since this outcome is in the convex hull of feasible and individual-rational outcomes, the outcome is sustainable in an infinitely repeated game (Fudenberg and Maskin (1986)). If we instead consider a contribution-maximizing fundraiser, then by continuity there must be a maximum threshold between the efficient one and $\overline{T}$, such that the same folk theorem applies.

of not collecting pledged contributions can be written on observable and veri¯able contracts, while the subsequent voluntary contributions cannot be stopped in any way. The second important variable is competition: if a charity is a monopolist in the provision of a given public good, then it is possible to set an ine±ciently large threshold. However when many charities compete to provide a given public good the threshold is what they compete about, and it will not be possible to succeed with a threshold of $\overline{G}$: In the example of the New Democratic Party from Canada given in the introduction, it appears that both of these requirements were satis¯ed: the fundraiser was a monopolist collector for the party with credibility given by the repeated-game nature of election campaigns. [19]

6 Thresholds through Competition

So far, we have characterized the properties of a contribution game where the presence of a strategic fundraiser, who can commit to enforce a minimum threshold, leads to over provision. The objective of this section is to demonstrate that a threshold may arise endogenously in the absence of a fundraiser, and that it too can result in over provision. In particular, competition for a public good, or for its location, among di®erent groups of individuals may generate a truncated production function similar to that generated by the threshold-setting fundraiser. We will display this similarity by examining a simple \competitive public good" provision game.

Consider a public good which has both local and global bene¯ts. A hospital, for instance, may bene¯t all citizens of a country, but it certainly increases the utility of the region where

[19] The logic of this result applies also to many other contexts. First, consider a team-production process, where moral hazard is the main source of a free-riding problem similar to the one experienced in the production of public goods. Consider for example a team of engineers trying to develop a new engine. There, once again, if the principal is able to commit to a minimum standard, credibly claiming that the engine will not be used (and hence the agents will not be compensated) if the minimum standard is not reached, then the principal should set this standard at the highest possible level (satisfying (4)), and the e®ort of each agent would become pivotal, so that the free-riding problem is eliminated and, as a consequence, the principal reaches his objective and the agents have a lower welfare than in the free-riding equilibrium without thresholds. An example of this procedure is given by Toyota, where it is often the case that very high standards result in employees excerting a high level of e®ort.

it is built more than any other region. Let us examine a small country with two regions, A and B, where the government is committed to building one hospital. To simplify the example we assume that the production technology is such that at least \$K is needed for the hospital's provision, and any additional funds causes an increase in either the quality or the size of the hospital, i.e. once \$K is contributed the hospital can take on a continuum of values. The government has pledged \$K, but wants the largest hospital possible and must also decide where to locate the hospital. We assume for simplicity that neither region is able to raise sufficient funds on their own.

There are of course many different ways in which the government can choose between these locations. One is to base the location decision on which of the two regions pledges the larger amount of additional funds to the construction of the hospital. There are two benefits to letting the two groups of citizens compete. First it helps determine which region values a local hospital the most, and second it may result in the construction of a larger and better hospital than would otherwise be possible. The particular strategy that we have in mind is the following: the government announces that it will build the hospital in the region where the sum of the pledges made by the local residents is higher, and that it will add \$K to the contributions made by the citizens in that region. Ties will be broken with a coin toss.

In each region, a simultaneous pledging game takes place after this announcement, and then the government observes the two totals and follows it's contingent plan. We will denote the citizens in region $j = A, B$ by $i = 1, \ldots, n$, and let the competing region be denoted by k. Citizens can be thought to have preferences of the form $U_i(x_i, G_j, G_k)$. For simplicity we assume that $U_i(x_i, G_j, G_k) = U_i(x_i, G_j, 0)$ and we will simply denote i's preferences by $U_i(x_i, G_j)$. Finally, in order to avoid limit arguments, assume that the smallest possible individual contribution is ϵ – say, one penny.

Now let $\bar{T}_j < K$ be the value of the total sum of pledges in region j that can be obtained by having everybody contribute to the point where they are all indifferent between getting nothing and getting $G_j = \bar{T}_j + K$, i.e.,

$$U_i(w_i - g_i, \bar{T}_j + K) = U_i(w_i, 0) \; \forall i \in j.$$

We know by Lemma 1 and 2 that $\bar{T}_j$ exists and is unique. Generically, $\bar{T}_A \neq \bar{T}_B$. Assume $\bar{T}_A > \bar{T}_B$, and that in the absence of competition the voluntary donations in region A are less than μ_B, i.e. $\sum_{i \in A} g_i^* < \mu_B$. Then, it is easy to show that:

Proposition 4 The unique Nash equilibrium outcome of the competitive public good provision game is one where region A gets a public good of value $\mu_B + \epsilon + K$. All the equilibrium profiles are such that (I) the residents of region A contribute a total of $\mu_B + \epsilon$ and (II) the residents of region B pledge a total of μ_B.

It can be easily checked that nobody has profitable deviations from any such profile.[20] The proof of uniqueness of the equilibrium outcome is equally trivial, hence it is omitted. It is also easy to see, given the analysis above, that the unique equilibrium outcome of the competitive public good provision game may result in over provision. Similar to the contribution-maximizing fundraiser, the endogenous threshold in this competitive provision problem may push people to contribute an inefficiently large amount.[21]

The competitive provision model analyzed here could be used to model campaign financing: consider for example a potential candidate who is trying to raise funds for her campaign, and interprets the two regions as being groups of people who would want this candidate to choose a specific type of policy as the campaign focus. For example, one group of contributors would like to see the candidate defending a right-to-bear-arms policy and the other group would like this same candidate to advocate pro-life policies. In this case the match with the competitive provision story is perfect. Even without commitment to thresholds, the candidate can ask for pledges from the two groups, similar to above, and then collect contributions only from the group pledging the most.

7 Conclusion

The help of a welfare-maximizing social planer in some circumstances can secure efficient private provision of the public good. Unfortunately agents who actually take the role of mechanism designers are rarely trying to maximize the welfare of the game's participants. Rather, the goal of a fundraiser is probably more often one of contribution maximization.

[20] We are implicitly assuming that pledges are costless.

[21] Of course it is also possible to get over provision when the production technology is everywhere increasing. In this case the outside option simply becomes $U_i(w_i - g_i^*; \sum_{i \in j} g_i^*)$ rather than $U_i(w_i; 0)$:

Fundraisers with such motivations naturally prefer over provision and we have shown that they can secure such an outcome by commiting to refund donations if they do not reach a certain threshold. The commitment to refund is more likely to be credible if a fundraiser does not have competitors and if there is a positive probability that the same fundraiser will run a similar campaign in the future. However, competition may cause thresholds to arise endogenously, and in these cases the threshold is credible.

After a refund donors still have an incentive to contribute, and we suspect that few fundraisers will have the commitment ability to reject these voluntary and unconditional contributions. Even the weaker level of commitment results in over provision of the public good. If the extreme commitment level is also available, then the over provision is so large that contributors are worse o® than in the standard simultaneous contribution game without a threshold.

Interestingly in our model the total and individual contributions increase when the population increases, which is the exact opposite of what we predict in the absence of a threshold. Similarly, one can show that an increase in government donations results in crowding in, rather than the crowding out typically predicted when fundraisers' strategies are not taken into account. These comparative statics results suggest that one may be able to obtain greater predictive power for the altruistic model of private provision by explicitly considering fundraisers' strategies.

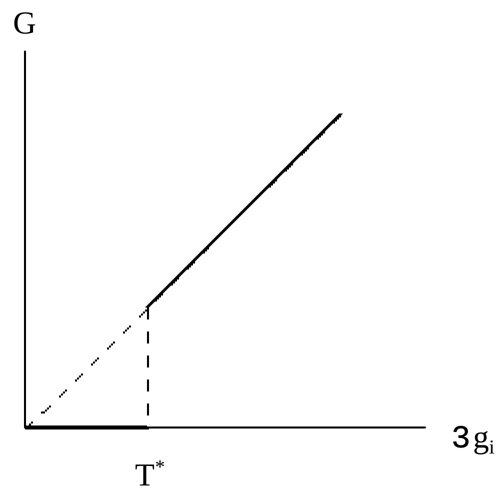

Figure 1: Truncated production of the public good.

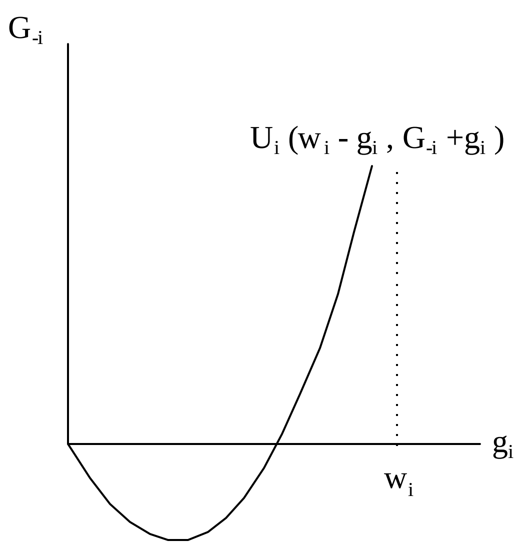

Figure 2: i's indi®erence curve

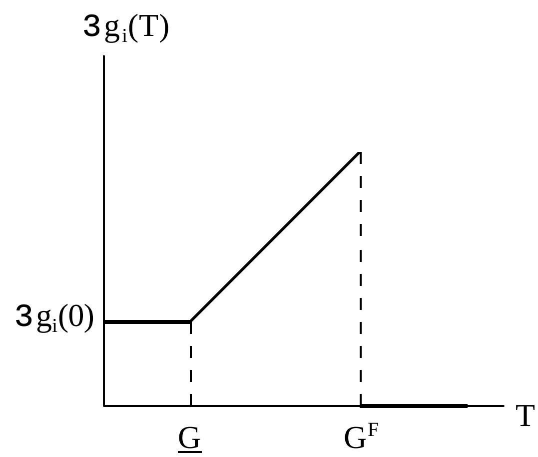

Figure 3: The e®ect of the threshold on donations.

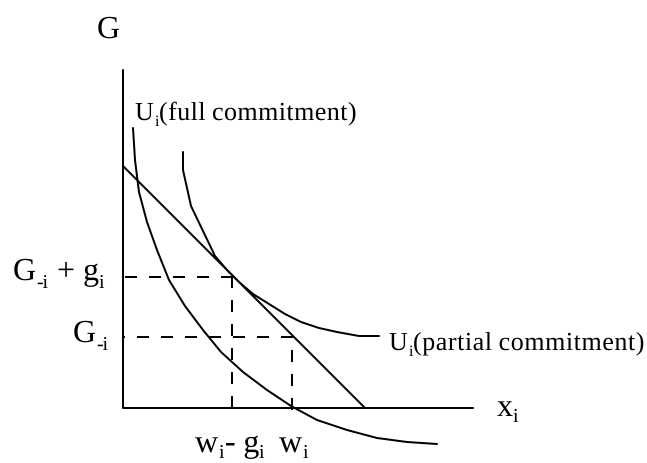

Figure 4: Indi®erence curves for partial and full commitment.

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