Double Irreversibilities and Endogenous Learning

in Land Conversion Decisions

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Abstract

We show the effects of uncertainty, ecological and economic irreversibility, and endogenous learning on land preservation decisions for species conservation. We compare the optimal decisions of three types of decision makers: an experimenting decision maker who recognizes the endogeneity of information, a real options decision maker who ignores the endogeneity but recognizes the future availability of more information, and a nonlearning decision maker who ignores the possibility of future information altogether. While information endogeneity always leads to more land preservation, its effect is dampened by economic irreversibility but enhanced by ecological irreversibility. The prospect of future information itself may or may not lead to more preservation depending on the relative importance of the two irreversibilities. When ecological irreversibility dominates land preservation irreversibility, the experimenting decision maker preserves the largest amount of land, followed by the real options decision maker, and the nonlearning decision maker.

Keywords: biodiversity, active experimentation, irreversibility

1 Introduction

There is a great deal of concern that human actions are leading to increased rates of extinction of species. As one response, the U.S. Congress passed the Endangered Species Act (ESA) in 1973 which, among other things, requires federal agencies to undertake programs for the conservation of endangered and threatened species. Land preservation is an important component of these programs, primarily because the chief reason cited for the increasing rates of species extinction is the loss of habitat [Maser, 1999; McNeely, 1988; Barbier et al., 1994]. The United States Fish and Wildlife Service (FWS), Environmental Protection Agencies (EPA), the United States Department of Agriculture (USDA) and other federal agencies are actively involved in protecting and restoring habitats through purchase and contract of land. In addition, private organizations such as the Nature Conservancy, Ducks Unlimited and the Audubon Society are also engaged in this endeavor.

However, land preservation comes at a cost, including the direct cost of converting and maintaining land for preservation as a wildlife habitat and the opportunity cost in terms of forgone returns from alternative uses of the land area. Land preservation can also be costly to reverse, and the preservation decisions often have to be made under conditions of uncertainty regarding the efficacy of land as a wildlife habitat. On the other hand, the species can be irreversibly lost if insufficient habitat is established.[1]

We study the optimal land conversion decisions under uncertainty, endogenous learning, and ecological (i.e., species loss) and economic (i.e., land preservation) irreversibilities. In the presence of uncertainty about the efficacy of land as a wildlife habitat, the decision maker can learn about the efficacy through converting land into preservation areas and observing the resulting species population. More information can be generated as more land is preserved. However, the dual and opposing sources of irreversibilities complicate the role of learning and information, as well as the optimal decisions. If she preserves too much land, and the preserved land turns out to be ineffective, it will be costly to revert the land to its original use. But if she preserves too little land, not only

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[1] We interpret the ecological irreversibility in a broad sense. If the preservation effort is on only one of the many possible habitats of a certain species, the failure at this one habitat does not represent the complete loss of the species. In this case, we assume that once failed, it will be too expensive to re-introduce the species to this habitat. Then as far as this particular habitat is concerned, the loss is irreversible.

will she not learn much about the land's efficacy, but the species may also be irreversibly lost.

Given the complexities involved in making such dynamic decisions with endogenous information, optimal strategies may not always be chosen in real world resource and species management. In this paper, we consider and compare three types of decision makers, differing in their attitudes toward future information about the land's efficacy. An *experimenting decision maker (EDM)* recognizes that future information can be endogenously generated through land preservation; she is proactive in generating information and adaptive in responding to the new information. A *real options decision maker (RDM)* expects future new information but fails to recognize its endogeneity; she is not proactive but still is adaptive. A *non-learning decision maker (NDM)* ignores the existence of future information altogether in making current decisions, and represents passive management.[2] The difference between the first two types of decision makers captures the effects of information endogeneity, and that between the last two types highlights the effects of new information. Both effects in turn are influenced by the two irreversibilities.

The three types of decision makers studied in the paper are far more than theoretical curio. Rather, there are real world examples for each of them, and intensive debate has been carried out (and is still going on) regarding the pros and cons of each approach. For instance, one of the most important recent developments in ecosystem management under uncertainty is the concept of "adaptive management" (Holling, 1978), the central tenet of which is that management activities should be considered as experiments that generate data to be used in response to uncertainties, and such activities should be planned with explicit information goals. In the context of our model, an EDM takes precisely the adaptive management approach. Although widely recognized as the essential and efficient approach to resource management, adaptive management has only been sparsely applied.[3] Wilhere (2002) shows that in implementing habitat conservation plans (HCP),[4] managers often misinterpret the adaptive management approach, and consequently adaptive management is

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[2]The NDM may still (passively) respond to the new information in future periods, but she ignores this possibility in making the *current period* decision.

[3]Meretsky et al (2000) details a case study in which adaptive management is successfully applied to planned flooding on the Colorado River in Grand Canyon in 1996.

[4]A HCP forms the basis for a contract between a landowner and the US government in which the landowner takes conservation measures to protect a listed (i.e., endangered or threatened) species, in return for an Incidental Take Permit.

seldom applied. To some managers, "adaptive simply means a willingness to change" in response to new information. Further, the dominant paradigm in HCP is trial and error, or reactive learning. These management practices correspond to an NDM in our paper: the NDM responds to new information when it arises in the future but ignores its importance in planning *current* decisions.[5]

Another central concept concerning resource and environmental management is the precautionary principle,[6] based on the option value argument that under future learning and irreversible actions, current decisions should "take precaution" to allow room for future response to new information. Future information, however, is typically assumed to be exogenous. Thus, a precautionary manager ignoring information endogeneity corresponds to the RDM in our paper.[7]

Ignoring transaction and information costs, it is obvious that the EDM's approach is optimal. By comparing the decisions of the three types of decision makers, we are able to assess the direction and magnitude of the "errors" of the RDM and NDM. We find that there is a positive "experimentation effect:" the EDM preserves more land than the RDM in order to obtain more information. This effect is enhanced by the ecological irreversibility but dampened by the economic irreversibility. We also derive the "learning effect," corresponding to the difference in the first period decisions of the RDM and NDM. We show that the sign of the learning effect depends on the relative importance of the two irreversibilities. When species loss irreversibility is more important and dominates the economic irreversibility, the RDM preserves more land than the NDM. On the other hand, if the ecological irreversibility cannot preclude the economic irreversibility, the RDM may preserve less land than the NDM. Consequently, the welfare ranking between the RDM and NDM is less clearcut.

The literature on species conservation is increasingly recognizing the importance of economics and information. Ando et al. (1998) explicitly introduced economic considerations in species conservation. Expanding that work, Polasky et al. (2000) and Polasky and Solow (2001) studied the

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[5]Costello and Polasky (2004) also argues that most studies of habitat selection are static, and the few dynamic studies are deterministic.

[6]Its importance is highlighted in part by the existence of a separate entry for it in Wikipedia - we refer readers to Wikipedia for more discussion and for citations.

[7]Wilhere (2002) terms such a practice as the "diferred action" approach in HCP: "an ecosystem is not managed until after it is understood." However, it is precisely through active management that the ecosystem will be understood.

value of information in selecting preservation sites given a limited budget. Costello and Polasky (2004) considered dynamic site selection decisions. The literature has mostly ignored issues of irreversibilities, especially economic irreversibilities, learning and active experimentation.

Our paper is also related to the real options literature with double irreversibilities (Olson (1990), Kolstad (1996) and Pindyck (2000)). Olson (1990) considered a situation where developed environment is used together with another factor (e.g., capital) to produce a consumption good. The development of the environment is irreversible, so is the saving of capital for future production. In this case, more future information may not lead to less development of the environmental resource. Kolstad (1996) analyzes irreversible investment decisions to reduce a stock pollutant, and finds that more future information has ambiguous effects on investment. Our model differs from the literature in two aspects. First, we consider *endogenous* future information. Second, although the two irreversibilities in our model arise due to opposite decisions (two little or too much land preserved), they are affected by the random shocks in the same direction: both are more likely to be binding when the preserved land turns out to be less effective. In this case, the decision maker may regret the preservation (a binding economic irreversibility), and the species may be lost (binding ecological irreversibility). In Kolstad (1996), the two irreversibilities require opposite shocks to be binding: if the investment turns out to be effective, too much may have been invested, and in case of less effective investment, the stock of pollution may increase.

Our paper is closely related to the real options literature with endogenous information (Miller and Lad (1984), and Fisher and Hanemann (1987)), and the literature on active experimentation in decision making under uncertainty (Prescott (1972), Grossman et al. (1977), Mirman et al. (1993)), in particular Datta et al. (2002). The first group of papers focus on comparing the decisions of the EDM and one who adopts open-loop decisions, and finds that the comparison is ambiguous. Our paper is different in that all types of decision makers make closed-loop decisions in one aspect: they all modify their future decisions in response to whether the species has survived or not. The second group of papers focuses on the experimentation effect, and finds that the effect can be negative. Our paper builds on Datta et al (2002) but imposes more structure on it that enables us to analyze how the experimentation and learning effects are influenced by parameters of the model, specifically

the two irreversibilities.

The paper is organized as follows. We set up the land conversion model in Section 2, and analyze the optimal decisions of the three types of decision makers in Sections 3 and 4. In Section 5, we compare the decisions of the three types of decision makers and investigate the experimentation and learning effects and how they are influenced differently by the two irreversibilities. In Section 6, we focus on the EDM and study the effects of uncertainty and irreversibilities. We conclude the paper in Section 7.

2   A Model of Land Conversion

Consider a decision maker who converts land between agricultural use and serving as a wildlife habitat for a certain species.[8] We adopt the following conventions in describing land management decisions. Land is said to be *converted* between agriculture and wildlife habitat. Moreover, agricultural land is *preserved* into a wildlife habitat, while a habitat is *reverted* into agricultural use. If x is used to denote the amount of land converted, then land preservation is represented by $x > 0$ and reversion is represented by $x < 0$. Finally, the habitat is used to *conserve* the species.

There are two periods, and in each period the species population depends on the size of the habitat, the efficacy of land as a habitat, and a random exogenous shock. We assume that currently all land is in agricultural use. If x_1 acres of land are preserved in the current period (period one), the end of period population is

$$z_1(x_1, \theta, \epsilon_1) = f(x_1, \theta) + \epsilon_1, \tag{1}$$

where $\theta \in \{\theta^L, \theta^H\}$ denotes the land's efficacy with $\theta^H > \theta^L$, and $\epsilon_1 \in [0, \infty)$ is a random shock that may be due to climatic conditions, changes in the population of related species, alterations in other habitats for migratory species, etc. The random shock affects the species population but is independent of the efficacy of land. We assume that there exists $\bar{x} < \infty$ such that $f_x(\bar{x}, \theta) = 0$ for both values of θ: there is a finite maximum amount of land needed for species conservation.

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[8]For simplicity, we focus on one particular species, rather than a number of species. It could be a keystone or a threatened/endangered species. Protecting keystone species is often a priority for conservation efforts since the existence of numerous other species depends on them.

We also assume that $f_x > 0$, $f_{xx} < 0$, $f(x, \theta^H) > f(x, \theta^L)$, $f_x(x, \theta^H) > f_x(x, \theta^L)$, for $x \in [0, \bar{x})$, and $f(0, \theta) = 0$ for both values of θ: more land improves the species population at a decreasing rate, and as θ rises, the population increases and the marginal "productivity" of land in conserving the species increases. Thus, a higher θ indicates that the land is more effective as a habitat. We assume that $f_x(0, \theta)$ is sufficiently large for both values of θ, i.e., the marginal productivity of land is sufficiently high when there is no land in preservation. This condition guarantees that some land will be preserved in period one. Finally, $f(0, \theta) = 0$ and $f_x > 0$ implies that $f(\cdot)$ is always positive when $x > 0$. Since ϵ is distributed on $[0, \infty)$, the species population is never negative.

The decision maker does not know the value of θ at the beginning of period one. She does know that $\theta = \theta^H$ with probability B_1. The land's efficacy θ remains constant across the time periods, but the decision maker's belief about $\theta = \theta^H$ in period two, B_2, may change as new information arrives overtime. The random shocks ϵ_t, $t = 1, 2$, are i.i.d. with density function $g(\cdot)$ on $\mathcal{R}_+$ that is differentiable, with mean $\bar{\epsilon}$. The random variables θ and ϵ_t are independent.

Decisions on habitat preservation are characterized by widespread uncertainties, especially about how successful the habitat will be. While biologists have made significant progress in identifying "suitable" plots for certain species, it is much harder to quantify *how* suitable a plot might be, especially when evaluating spatially connected or fragmented plots (Polasky et al. (2000) and Polasky and Solow (2002)). Another related uncertainty is regarding species incidence, i.e., the number of species found on a plot of land, which is also an indicator of the plot's efficacy.

To capture the irreversibility in the loss of species, we adopt the Safe Minimum Standard (SMS) approach[9] and assume that if the period one population z_1 falls below a certain threshold level $\bar{z}$, the species becomes extinct and hence the population in the second period is zero.[10] If the species survives, the end of second period population again depends on the total habitat area, the land's

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[9]The SMS approach to conservation was first proposed by Ciriacy-Wantrup [1964]. Subsequently there has been an active interest in this approach [Bishop, 1978; Farmer and Randall, 1998; Castle et al., 1996].

[10]Many species also have minimum-habitat requirements in addition to a threshold population. With such requirements, there could be increasing returns from land preservation. We assue this away to streamline the analysis. One possible approach to incorporate such requirements is to interpret variable x_1 in our model as land preservation over and above the minimum habitat.

efficacy, and the exogenous shock. That is,

$$z_2(x_1 + x_2, \theta, \epsilon_2, z_1) = \begin{cases} f(x_1 + x_2, \theta) + \epsilon_2 & \text{if } z_1 \geq \bar{z} \\ 0 & \text{if } z_1 < \bar{z} \end{cases} \qquad (2)$$

Notice that the period one population z_1 affects z_2 only indirectly through influencing the survival of the species: given survival, z_2 is independent of z_1. Period one land preservation x_1, however, directly contributes to z_2 in addition to helping survival of the species.

Habitat preservation can involve several kinds of costs, including land conversion cost, maintenance cost, and the opportunity cost from foregone agricultural profit. Some of these costs such as the conversion cost are sunk. For simplicity and clarity of the model we focus on per acre opportunity cost, c, the profit from agriculture in one period. Then the per period cost associated with x acres of land in preservation is cx.

We assume that land preservation is costly reversible: it costs τx to revert x amount of preserved land back to agriculture.[11] For example, if preservation involves converting agricultural land to wetlands, reverting the wetlands back to agriculture can incur significant costs. A preservation area may further spur related activities such as an eco-tourism industry that are costly to reverse. We will assume that currently no land is in preservation, and in a two period model, land reversion is relevant only in period two. Then if $\tau \geq c$, land preservation decisions are absolutely irreversible.[12] To allow for costly reversibility, we assume that $0 \leq \tau \leq c$ with $\tau = 0$ and $\tau = c$ representing two polar cases of perfect reversibility and absolute irreversibility. Parameter τ thus represents the degree of economic (or land preservation) irreversibility.

The benefit derived from preservation is an increasing function of the species population: $U(z) = \gamma z + U_0$, where γz measures the use value and U_0 is the existence value. The benefit function is invariant overtime with constant marginal use value. Our results will remain the same if the use value is concave in z.

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[11] We are implicitly assuming away fixed costs of reversion. Zhao and Zilberman (1999, 2001) show that when fixed costs are allowed, the degree of irreversibility is determined to a large part by the size of the fixed costs. Mason (2001) also assumes fixed costs in studying costly shut-down and reopening of mines.

[12] For infinite periods, the relevant comparison is between τ and a fraction of c, with the fraction depending in part on the discount rate.

The decision maker can learn about θ from period one results. Given land preserved in the first period x_1 and prior belief B_1, she knows that the density function of z_1 is

$$h(z_1, x_1) = B_1 g(z_1 - f(x_1, \theta^H)) + (1 - B_1) g(z_1 - f(x_1, \theta^L)). \tag{3}$$

At the beginning of the second period she observes z_1, and can update her belief about θ using the Bayes rule. In particular, observing z_1, the posterior probability of $\theta = \theta^H$ is

$$B(z_1, x_1) = \frac{B_1 g(z_1 - f(x_1, \theta^H))}{h(z_1, x_1)}. \tag{4}$$

Given x_1, if z_1 turns out to be large, the decision maker should infer that the land is more likely to be effective. To guarantee this intuitive condition, we assume that $g(\cdot)$ satisfies the strict monotone likelihood ratio property (MLRP): $\frac{g'(\epsilon)}{g(\epsilon)}$ is strictly decreasing in ϵ. This property is satisfied by most of the commonly used density functions. From (4), this condition implies that $B_z(z_1, x_1) > 0$ when $h(z_1, x_1) > 0$.

Equation (4) indicates that the posterior belief and how much the decision maker learns about θ are influenced by x_1. If the decision maker wishes, she may preserve more land in period one in order to have better information about θ in period two. Following Datta et al (2002), we call this type of decision maker an *experimenting decision maker* (EDM). As we discussed earlier, in the real world not all decision makers recognize and respond to the endogeneity of future information, or even the existence of future information, in their decision making process. Based on how they handle future information, we consider two more categories of decision makers: a *real options decision maker* (RDM) and a *nonlearning decision maker* (NDM). The former recognizes that there will be more information about θ in future periods, but ignores the fact that she can influence the amount of future information by changing x_1. She treats the future signals z_1 as being entirely exogenous, and thus acts in the real options framework of Arrow and Fisher (1974) and Dixit and Pindyck (1994). A NDM, when making her decision on x_1, completely ignores the possibility that there will be new information at all in period two, and acts as if $B_2 = B_1$. This type of decision maker is the most "static" in nature in handling future information. Note that the difference among the three types of decision makers pertains to their first period decisions. In the second period, even the NDM *may* respond to new information after observing z_1. What is lacking for her is that she

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does not recognize this future possibility when choosing x_1 now.

Common to the three types of decision makers is that they all recognize the dynamic link between the two periods in every aspect other than information, e.g., in species population and survival. That is, they all solve the following optimization problem:

$$\max_{x_1} \quad J(x_1) + \delta E_{z_1 \geq \bar{z}} V(x_1, B_2) - \delta \tau x_1 H(\bar{z}, x_1), \tag{5}$$

subject to (1), where

$$J(x_1) = B_1 \gamma f(x_1, \theta^H) + (1 - B_1) \gamma f(x_1, \theta^L) + \gamma \bar{\epsilon} - c x_1 + U_0 \tag{6}$$

is the expected first period payoff,

$$V(x_1, B_2) = \max_{x_2 \geq -x_1} B_2 \gamma f(x_1 + x_2, \theta^H) + (1 - B_2) \gamma f(x_1 + x_2, \theta^L) + \gamma \bar{\epsilon} + U_0$$
$$- c(x_1 + x_2) + \tau x_2 1(x_2 < 0) \tag{7}$$

is the expected second period payoff given belief B_2 if the species survives, $\delta \in [0, 1]$ is the discount factor, $E_{z_1 \geq \bar{z}}$ represents expectation on z_1 over the range $z_1 \geq \bar{z}$ with the density function $h(z_1, x_1)$, and $H(\cdot)$ is the cumulative distribution function of $h(\cdot)$. The last term in the objective function (5) represents the expected cost of reverting all preserved land back to agriculture when the species does not survive. (The assumption of $\tau \leq c$ implies that such reversion is optimal.) In (7), $1(\cdot)$ is an indicator function taking the value of one if its argument is true and zero otherwise. If the decision maker decides to revert land back to agriculture, he can only revert up to the preserved amount in period one: $x_2 \geq -x_1$.

The three types of decision makers differ in belief B_2. For the EDM, $B_2 = B(z_1, x_1)$, which is given in (4), and we denote the corresponding optimal solutions as $(x_1^*, x_2^*(x_1^*, z_1))$, where x_2^* describes the optimal period two land conversion *if the species survives*. For the NDM, $B_2 = B_1$, and the only new information in period two is whether or not the species has survived. If not, the preserved land is reverted back to agriculture, and if yes, the optimal decision depends only on x_1 and not on z_1. We denote the optimal solutions as $(\tilde{x}_1, \tilde{x}_2(\tilde{x}_1))$. The RDM forms her belief in period two through an "exogenous" process, which we will fully define in Section 4. Let $\hat{x}_1$ be the corresponding optimal first period solution. If the species survives, the decision maker makes further conversion decisions in response to a signal $\hat{z}_1$ that is provided exogenously. We denote the

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optimal second period decision (if the species survives) as $\hat{x}_2(\hat{x}_1, \hat{z}_1)$.

Since we assumed that $f_x(0, \theta)$ is sufficiently high and no land is preserved initially, the optimal first period decision always involves $x_1 > 0$ for all types of decision makers. Thus the land preservation irreversibility constraint applies only to the second period.

3 The Experimenting Decision Maker

We now analyze the EDM's optimization problem. In period two, if $z_1 < \bar{z}$, the species are extinct and all preserved land should be reverted back to agriculture. If the species survives, the necessary conditions on x_2 in (7) are

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\begin{aligned}
& x_2^* > 0, \text{ s.t. } Q(x_1 + x_2^*, B(z_1, x_1)) = c, && \text{if } Q(x_1, B(z_1, x_1)) \geq c \\
& x_2^* < 0, \text{ s.t. } Q(x_1 + x_2^*, B(z_1, x_1)) = c - \tau, && \text{if } Q(x_1, B(z_1, x_1)) \leq c - \tau && (8) \\
& x_2^* = 0 && \text{if } Q(x_1, B(z_1, x_1)) \in (c - \tau, c),
\end{aligned}
$$

where $Q(x_1 + x_2^*, B(z_1, x_1)) = B(z_1, x_1) \gamma f_x(x_1 + x_2^*, \theta^H) + (1 - B(z_1, x_1)) \gamma f_x(x_1 + x_2^*, \theta^L)$ measures the expected marginal benefit of preserved land. Based on the new belief in period two, $B(z_1, x_1)$, if this marginal benefit from x_1 alone exceeds the marginal cost c, more land is preserved in period two. If the marginal benefit is lower than $c - \tau$, the rate of cost saving from land reversion, then some preserved land should be reverted back to agriculture. Since $f_x(0, \theta)$ is sufficiently large, the constraint $x_2 \geq -x_1$ is never binding: it is never optimal to revert *all* preserved land if the species survives. If the marginal benefit $Q(x_1, B(z_1, x_1))$ falls between $c - \tau$ and c, no land is converted, and "too much" land is kept in preservation due to land preservation irreversibility.

When $x_2^* \neq 0$, comparative statics on (8) indicates that

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\frac{\partial x_2^*(x_1, z_1)}{\partial z_1} \propto B_z(z_1, x_1) \left[\gamma f_x(x_1 + x_2^*, \theta^H) - \gamma f_x(x_1 + x_2^*, \theta^L) \right]. \tag{9}
$$

Conditions $f_x(x, \theta^H) > f_x(x, \theta^L)$ and $B_z > 0$ then imply that $x_2^*(x_1, z_1)$ increases in z_1: given x_1, if the species population z_1 is higher, $B(z_1, x_1)$ is higher, i.e., the land is believed to be more effective and thus more land should be preserved or less land should be reverted. Following Zhao and Zilberman (1999), we know from (8) and (9) that there exist two critical levels of z_1, denoted as $z_1^e(x_1)$ and $\underline{z}_1^e(x_1)$, so that $x_2^* > 0$ if $z_1 > z_1^e(x_1)$, $x_2^* < 0$ if $z_1 < \underline{z}_1^e(x_1)$, and $x_2^* = 0$ if $z_1 \in (\underline{z}_1^e, z_1^e)$.

In other words, if the species population z_1 is sufficiently high (so that $B(z_1, x_1)$ is sufficiently high), more land should be preserved. Conversely, some land is reverted if z_1 is too low.

For each x_1, the unique levels $\underline{z}_1^e$ and z_1^e, with $\underline{z}_1^e \le z_1^e$, are implicitly given by

$$Q(x_1, B(z_1^e, x_1)) = c, \quad Q(x_1, B(\underline{z}_1^e, x_1)) = c - \tau. \tag{10}$$

Note that $\underline{z}_1^e$ is decreasing in τ, the degree of economic irreversibility: land reversion becomes less likely as it is more costly to do so. If $\tau = c$, $\underline{z}_1^e$ becomes nonbinding and no land will be reverted.

Applying the implicit function theorem to (8) and (10), we find that $dz_1^e(x_1)/dx_1 > 0$ (or $d\underline{z}_1^e(x_1)/dx_1 > 0$) if and only if $\partial x_2^*(x_1, z_1^e)/\partial x_1 < 0$ (or $\partial x_2^*(x_1, \underline{z}_1^e)/\partial x_1 < 0$): if higher x_1 leads to less preservation (or more reversion) in period two, it will also reduce (or raise) the likelihood that there is preservation (or reversion). Thus, x_1 affects the level and likelihood of preservation (or reversion) in the same direction.

To see how $x_2^*(x_1, z_1)$ depends on x_1, note from (8) that for $x_2^* \ne 0$,

$$\frac{\partial x_2^*(x_1, z_1)}{\partial x_1} \propto [B(z_1, x_1) \, \gamma f_{xx}(x_1 + x_2, \theta^H) + (1 - B(z_1, x_1)) \, \gamma f_{xx}(x_1 + x_2, \theta^L)]$$
$$+ B_x(z_1, x_1) \gamma \left[f_x(x_1 + x_2, \theta^H) - f_x(x_1 + x_2, \theta^L) \right] \tag{11}$$

Thus, x_1 affects the second period conversion in two ways. The first two terms in the first set of square brackets on the right hand side capture the traditional *substitution effect*: as x_1 rises, less preserved land is needed in period two due to decreasing returns ($f_{xx} < 0$). However, with active learning, there is an additional effect: belief $B(z_1, x_1)$ also changes as x_1 rises. The last term captures this *information effect*. Since $f_x(x, \theta^H) > f_x(x, \theta^L)$, the direction of the information effect depends on the sign of B_x. If $B_x(z_1, x_1) > 0$, i.e., if observing z_1, the decision maker believes that the land is more effective as x_1 gets higher, the information effect is positive and implies that more preserved land is needed. It works against the substitution effect, and if strong enough, may lead to $\frac{\partial x_2^*(x_1, z_1)}{\partial x_1} > 0$. However, if $B_{x_1} < 0$, the information effect is negative and works in the same direction as the substitution effect.

Now we analyze the sign of $B_x(z_1, x_1)$. Intuitively, as x_1 increases, there are two factors influencing the beliefs about θ. First, since z_1 is fixed, higher x_1 implies that more land was used to obtain the same species population z_1. Then it must be that the land is not as effective as when

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x_1 is low. This factor tends to make B_x negative. However, when x_1 rises, the decision maker also learns more about θ in period two. To see this, note that in (1), population z_1 is affected by two uncertainties, that of land efficacy θ and that of exogenous shock ϵ_1. As x_1 increases, $f_x(x, \theta^H) > f_x(x, \theta^L)$ implies that variations in θ lead to more variations in z_1, or variations in ϵ_1 are relatively less important in affecting z_1. Thus, observing z_1 and x_1, the decision maker should be more confident in making inferences about θ when x_1 is higher. If indeed the land is more effective than previously believed, that is, if $B_2 = B(z_1, x_1) > B_1$, then more learning will more likely reveal the true (high) value of θ. This factor tends to make B_x positive. In contrast, if the land is indeed less effective than previously believed, i.e., if $B_2 < B_1$, then more learning leads to a lower B_2, or $B_x < 0$. The overall effect depends on the relative magnitude of these effects, specified in the following proposition (proved in the Appendix).

Proposition 1 If $B(z_1, x_1) \leq B_1$, then $g'(z_1 - f(x_1, \theta^H)) \geq 0$ and $B_x(z_1, x_1) < 0$. If $g'(z_1 - f(x_1, \theta^H)) < 0$, then $B(z_1, x_1) > B_1$ and the sign of $B_x(z_1, x_1)$ is ambiguous.

If $z_1^e(x_1) \leq \bar{z}$, the critical population level for species survival, the economic irreversibility is *precluded* by ecological irreversibility: the decision maker in period one knows that if the species survives, more land will be preserved in period two. This scenario is possible: if the decision maker preserved little land in period one but the species still survived, she may infer that the land is likely to be extremely effective and always choose to preserve more in period two. If $z_1^e(x_1) > \bar{z}$, both ecological and economic irreversibilities exist: if the species survives but the population is relatively but not extremely low, i.e., $z_1 \in (\underline{z}_1^e, z_1^e)$, the posterior belief of $\theta = \theta^H$ is not sufficiently high and no land is further converted. Of course, if $\bar{z} < \underline{z}_1^e$, it is possible that the species survives, and some land is reverted back to agriculture.

We can write the second period payoff as (cf. (5))

$$W^e(x_1) = \int_{\bar{z} - f(x_1, \theta^H)}^{\infty} V(x_1, B(f(x_1, \theta^H) + \epsilon, x_1)) B_1 g(\epsilon)(d)\epsilon$$
$$+ \int_{\bar{z} - f(x_1, \theta^L)}^{\infty} V(x_1, B(f(x_1, \theta^L) + \epsilon, x_1))(1 - B_1) g(\epsilon)(d)\epsilon \qquad (12)$$
$$- \int_0^{\bar{z}} \tau x_1 h(z_1, x_1) dz_1,$$

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where the first two terms on the right hand side measure the expected value if the species survives, and the last term measures the expected cost of land reversion when the species becomes extinct. The EDM's maximization problem at the beginning of the first period is

$$\max_{x_1} [J(x_1) + \delta W^e(x_1)]. \tag{13}$$

4 Real Options and Nonlearning Decision Makers

We first describe the period two belief of the RDM, following the logic in Datta et al (2002). To help understand the information updating process of this decision maker, suppose, hypothetically, that she hires an information service that conducts the experiment for her. In particular, the service preserves y acres of land in period one, and after observing the resulting species population

$$\hat{z}_1 = f(y, \theta) + \epsilon_1, \tag{14}$$

reports back to the RDM with the new belief $B(\hat{z}_1, y)$ according to (4). When the RDM chooses her preservation level x_1, she treats the level of experimentation by the service, y, as exogenously fixed.

Thus, the second period decision for the RDM is structurally the same as that of the EDM, except that now the signal is considered exogenous. She updates her belief after observing signal $\hat{z}_1$ and makes the decision on x_2 accordingly. If the species survives, the second period value function for the RDM is $V(x_1, B(\hat{z}_1, y))$, where $V(\cdot)$ is given in (7). The necessary conditions for $\hat{x}_2$ are similar to (8), from which we obtain, for $\hat{x}_2 \neq 0$,

$$\frac{\partial \hat{x}_2(x_1, \hat{z}_1)}{\partial \hat{z}_1} \propto \frac{\partial B(\hat{z}_1, y)}{\partial \hat{z}_1} \left[\gamma f_x(x_1 + \hat{x}_2, \theta^H) - \gamma f_x(x_1 + \hat{x}_2, \theta^L) \right] > 0$$
$$\frac{\partial \hat{x}_2(x_1, \hat{z}_1)}{\partial x_1} \propto B(\hat{z}_1, y)\gamma f_{xx}(x_1 + \hat{x}_2, \theta^H) + (1 - B(\hat{z}_1, y))\gamma f_{xx}(x_1 + \hat{x}_2, \theta^L) < 0. \tag{15}$$

Similar to (9), as signal $\hat{z}_1$ is higher, the RDM believes that it is more likely that the land is efficient, and thus preserves more land or reverts less land. Unlike (11), as x_1 rises, there is only the substitution effect since the RDM's belief $B(\hat{z}_1, y)$ is independent of x_1. Thus, less land will be preserved (or more land reverted) in period two.

Similar to $z_1^e(x_1)$ and $\underline{z}_1^e(x_1)$, we define the critical levels of $\hat{z}_1$ between which no land is con-

verted, or land preserved in period one becomes irreversible. The two critical levels are denoted as $z_1^r(x_1, y)$ and $\underline{z}_1^r(x_1, y)$:

$$Q(x_1, B(z_1^r, y)) = c, \quad Q(x_1, B(\underline{z}_1^r, y)) = c - \tau. \tag{16}$$

From these equations, we know $\frac{\partial z_1^r(x_1, y)}{\partial x_1} > 0$ and $\frac{\partial \underline{z}_1^r(x_1, y)}{\partial x_1} > 0$: the more land is preserved in period one, the more likely that no land is further preserved (or land will be reverted) in period two.

The distribution of species population z_1 and the signal $\hat{z}_1$ are perfectly correlated: they are due to the same stochastic variables ϵ_1 and θ. Given θ, the species survives when $z_1 \geq \bar{z} - f(x_1, \theta)$, in which case the observed signal is $\hat{z}_1 = f(y, \theta) + \epsilon$. Thus, given the exogenous level of experimentation y, the expected second period payoff for the RDM is

$$W^r(x_1, y) = \int_{\bar{z} - f(x_1, \theta^H)}^{\infty} V(x_1, B(f(y, \theta^H) + \epsilon, y)) B_1 g(\epsilon) d\epsilon$$
$$+ \int_{\bar{z} - f(x_1, \theta^L)}^{\infty} V(x_1, B(f(y, \theta^L) + \epsilon, y))(1 - B_1) g(\epsilon) d\epsilon \tag{17}$$
$$- \int_0^{\bar{z}} \tau x_1 h(z_1, x_1) dz_1.$$

The RDM's optimization problem in period one is

$$\max_{x_1} [J(x_1) + \delta W^r(x_1, y)]. \tag{18}$$

The optimal solution $\hat{x}_1$ depends on y: the RDM's optimal decision in period one is affected by how much information she expects to receive in the future. However, it is the RDM's own action $\hat{x}_1$ that generates the future information: the optimal solution must satisfy $y = \hat{x}_1(y)$. The decision maker expects to respond to the information; she simply does not realize its endogeneity.

Therefore, the RDM's problem is (18) subject to the constraint of $\hat{x}_1(y) = y$. Since the objective function in (18) is strictly concave in x_1 (due to MLRP of $g(\cdot)$ and the concavity of $f(\cdot, \theta)$), we know for any given y, there is a unique solution $\hat{x}_1(y)$. This function defines a mapping from $[0, \bar{x}]$ onto itself, and the decision maker's optimal decision is the fixed point of the mapping. Since $\hat{x}_1(y)$ is continuous and $\bar{x}$ is finite, we know such fixed points exist. The fixed point is unique if $\hat{x}_1(y)$ is monotone in y.

At any optimal solution $\hat{x}_1$, $y = \hat{x}_1$ and thus the signal equals the observed species population $\hat{z}_1 = z_1$. From (10) and (16), we know the critical levels $z_1^e(\hat{x}_1) = z_1^r(\hat{x}_1, \hat{x}_1)$ and $\underline{z}_1^e(\hat{x}_1) = \underline{z}_1^r(\hat{x}_1, \hat{x}_1)$.

But since $y \neq x_1$ for any $x_1 \neq \hat{x}_1$, we know $z_1^e(x_1) \neq z_1^r(x_1, \hat{x}_1)$ and $\underline{z}_1^e(x_1) \neq \underline{z}_1^r(x_1, \hat{x}_1)$ for $x_1 \neq \hat{x}_1$.

The NDM's second period payoff, if the species survives, is $V(x_1, B_1)$, which is independent of z_1. Her expected second period payoff is

$$
\begin{aligned}
W^n(x_1) = & \int_{\bar{z}-f(x_1, \theta^H)}^{\infty} V(x_1, B_1) B_1 g(\epsilon) d\epsilon + \int_{\bar{z}-f(x_1, \theta^L)}^{\infty} V(x_1, B_1)(1-B_1) g(\epsilon) d\epsilon \\
& - \int_0^{\bar{z}} \tau x_1 h(z_1, x_1) dz_1.
\end{aligned}
\tag{19}
$$

and her first period decision is

$$
\max_{x_1} J(x_1) + \delta W^n(x_1).
\tag{20}
$$

Since the NDM's optimal decision $\tilde{x}_2$ is independent of z_1, she expects three possible conversion patterns: $\tilde{x}_2 > 0$, $\tilde{x}_2 = 0$, and $\tilde{x} < 0$. The RDM has incentive to preserve a limited amount of land in period one because the preserved land would be costly to revert back in period two if the species does not survive. More land can always be preserved later after knowing that the species has survived. This economic irreversibility pushes for a low x_1 and more preservation if the species survives, or a positive $\tilde{x}_2$. On the other hand, the decision maker may also wish to preserve a large amount of land in period one in order to make sure that the species survives. But if the species survives, too much land may already have been preserved and some land may need to be reverted back. That is, the ecological irreversibility pushes for $\tilde{x}_2 \leq 0$. The following Proposition shows that which scenario occurs depends on the relative importance of the two irreversibilities.

Proposition 2 *Let $x_1^m = \arg\max J(x_1)$ be the preservation level that maximizes the expected first period benefit only. Then $\tilde{x}_1 > x_1^m$, so that in period two $\tilde{x}_2 \leq 0$ if and only if*

$$
\left[V(x_1^m, B_1) + \tau x_1^m \right] \phi(\bar{z}, x_1^m) > \tau H(\bar{z}, x_1^m),
\tag{21}
$$

where

$$
\begin{aligned}
\phi(\bar{z}, x_1^m) = & \frac{d}{dx_1} \int_{\bar{z}}^{\infty} h(z, x_1^m) dz \\
= & B_1 g(\bar{z} - f(x_1^m, \theta^H)) f_x(x_1^m, \theta^H) + (1 - B_1) g(\bar{z} - f(x_1^m, \theta^L)) f_x(x_1^m, \theta^L)
\end{aligned}
\tag{22}
$$

measures the marginal increase in the probability of species survival as x_1 rises.

The proof is in the Appendix. If the NDM preserves one more unit of land above x_1^m, the survival probability of the species is marginally increased by $\phi(\bar{z}, x_1^m)$, with the marginal benefit

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of $V(x_1^m, B_1) + \tau x_1^m$. (The benefit includes τx_1^m because if the species does not survive, cost τx_1^m will be incurred to revert land back to agriculture.) Thus, the left hand side of (21) represents the marginal gain from reducing the probability of species extinction when more land is preserved. However, if the species does not survive (with probability $H(\bar{z}, x_1^m)$), economic irreversibility implies that the decision maker will have to incur marginal cost of τ to revert the land back to agriculture. Therefore, the right hand side of (21) represents the marginal cost due to land reversion if the species does not survive. More land than x_1^m should be preserved in period one if and only if the marginal expected benefit of avoiding the ecological irreversibility overcomes the marginal cost of the economic irreversibility.

Note that if $\tilde{x}_1 < x_1^m$, the optimal second period conversion is $\tilde{x}_2 = x_1^m - \tilde{x}_1$, so that the total land preserved in the two periods equals x_1^m. If $\tilde{x}_1 > x_1^m$, there are two possibilities: $\tilde{x}_2 < 0$ if $Q(x_1^m, B_1) < c - \tau$ and $\tilde{x}_2 = 0$ if $Q(x_1^m, B_1) \geq c - \tau$.

5 Comparison of the Decisions

In this section, we compare the optimal first period decisions of the three types of decision makers, We know the EDM obtains the efficient decision and the other two decision makers' choices are suboptimal. The questions we address are (i) relative to the EDM's decision x_1^*, do the two suboptimal choices lead to more or less land to be preserved in the first period? and (ii) whether or not we can welfare rank the two suboptimal choices. We limit the comparison to first period decisions because *ex post*, after observing the species population, we expect all types of decision makers to be responsive to the new information (reflected in $B(z_1, x_1)$) when presented to them.[13] What distinguishes the three types of decision makers is their *ex ante* treatment of the *ex post* information. In other words, while all decision makers are "rational" in response to "current" new information, they might be irrational in planning current actions in response to future information: they might ignore the fact that future information is related to current actions, or ignore the existence of future information altogether.

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[13] As we discussed earlier, even the NDM in implementing habit conservation plans still react to new information.

Following Freixas [1981] and Datta et al. (2002), we define the *experimentation effect* as the difference between the optimal first period choices of the EDM and RDM. Since the RDM understands all the dynamic implications of present action except for its effect on future beliefs, any deviation of her action from that of the EDM is due to her ignorance of the endogeneity of future information. We first derive the intuitive result that the EDM preserves more land in period one in order to obtain more information. We then illustrate how the experimentation effect is influenced by the two irreversibilities. We show that the economic irreversibility tends to reduce and the ecological irreversibility tends to enhance the experimentation effect.

We define the *learning effect* as the difference between the first period preservation decisions of the RDM and NDM. This effect captures how the ability to learn (indicated by updating of beliefs) affects the first period choices. We explicitly separate out the effects of the economic and ecological irreversibilities, show how they work with or against each other, and illustrate factors that affect their magnitudes. The difference between the decisions of the NDM and EDM is then the sum of the experimentation and learning effects.

5.1 The Experimentation Effect

As shown in Datta et al. (2002), one difficulty in comparing x_1^* and $\hat{x}_1$ is that there may be multiple solutions to both the EDM and RDM's optimization problems. The objective function of the EDM in (13) may not be concave, because of the non-concavity of the value of information (Radner and Stiglitz, 1984). As we discussed earlier, there may be multiple fixed points $\hat{x}_1(y) = y$ in (18). Datta et al. (2002) defined a weaker concept comparing the minimum and maximum selections from the set of maximizers of (13) and (18), which we will adopt here. Specifically, we concentrate on the comparison of the marginal benefits of x_1 for the two types of decision makers. If the marginal benefit is higher everywhere for the EDM, we say that this decision maker preserves more land in period one than the RDM, and the experimentation effect is positive.

Taking the derivative of (13) and (18) with respect to x_1 and evaluating their difference, we know the experimentation effect is captured by $W_x^e(x_1) - W_x^r(x_1, x_1)$. However, from (12) and (17), the only difference between the two decision makers is that the EDM recognizes that the decision

x_1 changes the level of experimentation y:

$$W_x^e(x_1) = W_x^r(x_1, y)\Big|_{y=x_1} + W_y^r(x_1, y)\Big|_{y=x_1}. \tag{23}$$

Therefore, the experimentation effect equals $W_y^r(x_1, x_1)$.

Result 1 *The experimentation effect $W_y^r(x_1, x_1)$ is positive. As a result, $x_1^* > \hat{x}_1$: compared with the RDM, the EDM preserves more land in period one in order to generate more information.*

This Result is a direct consequence of Blackwell's theorem (Blackwell, 1951, 1953). For the EDM, a higher y generates more informative signals, and thus leads to higher expected payoffs in the second period.

The expected value of active experimentation depends on the expected changes in belief B when more land is preserved in period one, subject to the two irreversibility constraints. Since the species population z_1 is higher as x_1 increases, we see from (1) that the marginal change in $B(z_1, x_1)$ given the value of θ is

$$\frac{dB(z_1(x_1, \theta), x_1)}{dx_1} = B_z(z_1, x_1)f_x(x_1, \theta) + B_x(z_1, x_1). \tag{24}$$

Let

$$\Omega(z, x_1) = \int_z^\infty \left[\left(\frac{dB(z_1(x_1, \theta^H), x_1)}{dx_1} \right) B_1 g(z_1 - f(x_1, \theta^H)) \right.$$
$$\left. + \left(\frac{dB(z_1(x_1, \theta^L), x_1)}{dx_1} \right) (1 - B_1) g(z_1 - f(x_1, \theta^L)) \right] dz_1 \tag{25}$$

be the expected change in $B(z_1, x_1)$ when x_1 increases, subject to the condition that $z_1 \geq z$. We can establish that

Proposition 3 *The expected change in belief $\Omega(z, x_1)$ satisfies the following properties:*

(i) $\Omega(0, x_1) = 0$ and $\Omega(\infty, x_1)$ for all x_1; and

(ii) $\Omega(z, x_1) \geq 0$ for all $z \geq 0$. Further, it is strictly positive if there exist values of $z_1 \leq z$ with a strictly positive measure satisfying $h(z_1, x_1) > 0$.

When $z = 0$, $\Omega(z, x_1)$ measures the expected change in the posterior as x_1 increases. Since the expected posterior equals the prior, we know $\Omega(0, x_1) = 0$. From (25), it is obvious that $\Omega(\infty, x_1) = 0$. If $z > 0$, $\Omega(z, x_1)$ measures the expected change in the posterior conditional on the

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fact that the species population $z_1 \geq z$. The cases when z_1 is too low are excluded, resulting in $\Omega(z, x_1) > 0$.

Next we show how the experimentation effect is influenced by the two irreversibilities. Specifically, the experimentation effect can be expressed as (derived in the Appendix)

$$
\begin{aligned}
W_y^r(x_1, y)\Big|_{y=x_1} &= -\int_{\bar{z}}^{\infty} V_B(x_1, B(z_1, x_1)) d\Omega(z_1, x_1) \\
&= V_B(x_1, B(\bar{z}, x_1))\Omega(\bar{z}, x_1) + \int_{\bar{z}}^{\max\{\bar{z}, \underline{z}_1^e(x_1)\}} \Omega(z_1, x_1) dV_B(x_1, B(z_1, x_1)) \quad (26) \\
&\quad + \int_{\max\{\bar{z}, z_1^e(x_1)\}}^{\infty} \Omega(z_1, x_1) dV_B(x_1, B(z_1, x_1)).
\end{aligned}
$$

To understand the first equality in (26), note from (25) that, as x_1 increases, $-d\Omega(z_1, x_1)$ measures the marginal increase in the expected posterior $B(z_1, x_1)$. Thus, $-V_B d\Omega$ measures the marginal value of this information change when the species population is z_1. Information has value only when the species survives, or when $z_1 \geq \bar{z}$.

Further, the value of information V_B depends on whether the economic irreversibility is binding, i.e., whether $\underline{z}_1^e < z_1 < z_1^e$. This effect is highlighted in the second equality in (26), which is obtained from the first equality by integration by parts. The first term evaluates information value V_B at $z = \bar{z}$, and the second and third terms capture the additional value when $z > \bar{z}$. Suppose $z_1^e > \underline{z}_1^e > \bar{z}$. Then the additional value is non-zero (positive) only when $z_1 > z_1^e$ and $z_1 < \underline{z}_1^e$, i.e., only when the additional information leads to land conversion in period two, or when the decision maker can respond to the new information. When $\underline{z}_1^e < \bar{z} < z_1^e$, the second term on the right hand side of (26) equals zero: if $z_1 \in (\bar{z}, z_1^e)$ so that the species survives but too much land has been preserved, economic irreversibility (i.e., $\tau > 0$) precludes reverting land back to agriculture. In the extreme, if $\bar{z} > z_1^e$, the economic irreversibility is precluded by the ecological irreversibility: if the species survives, more land will be preserved and economic irreversibility becomes irrelevant.

Economic irreversibility, if the species survives, is represented by the existence of $z_1^e(x_1) > \max\{\underline{z}_1^e(x_1), \bar{z}\} > 0$. As z_1^e rises and/or $\underline{z}_1^e$ decreases, the last two terms in (26) decrease, reducing W_y^r. Thus, economic irreversibility reduces the incentive to experiment. On the other hand, taking the derivative of (26) with respect to $\bar{z}$, we know

$$
\frac{\partial W_y^r(x_1, y)|_{y=x_1}}{\partial \bar{z}} = V_B(x_1, B(\bar{z}, x_1)) \frac{\partial \Omega(\bar{z}, x_1)}{\partial \bar{z}}.
$$

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Since $\Omega(0, x_1) = 0$ and $\Omega(z_1, x_1) \geq 0$ for $z_1 > 0$, we know $\partial\Omega/\partial\bar{z} \geq 0$ in the neighborhood of $\bar{z} = 0$. Thus, at least when $\bar{z}$ is low, a higher $\bar{z}$, or a higher degree of ecological irreversibility, raises W_y^r. In summary,

Result 2 *(i) Suppose $z_1^e > \bar{z}$. As z_1^e rises and/or $\underline{z}_1^e$ decreases, i.e., as the economic irreversibility constraint becomes increasingly binding, the second period benefit of more information about θ decreases, reducing the magnitude of the experimentation effect $W_y^r(x, y)|_{y=x}$.*
(ii) At low degrees of ecological irreversibility (specifically in the neighborhood of $\bar{z} = 0$), increasing the degree of ecological irreversibility raises the magnitude of the experimentation effect.

We can also understand Result 2 in the following way. As shown in Result 1, the decision maker preserves more land in order to experiment. However, a higher x_1 means that the economic irreversibility constraint is more likely to be binding. Thus, when economic irreversibility becomes more significant, the incentive to experiment goes down. On the other hand, a higher x_1 helps overcome the possibility of species loss. The incentive to experiment and to preserve the species are consistent with each other. Consequently, more restrictive ecological irreversibility helps increase the EDM's incentive to preserve land in period one.

Results 1 and 2 then imply that as the economic irreversibility becomes more severe (i.e., as τ increases), the additional land preserved by the EDM relative to that of the RDM, $x_1^* - \tilde{x}_1$, decreases. The reverse is true when the ecological irreversibility increases at low levels.

5.2 The Learning Effect

The learning effect is represented by $W_x^r(x_1, y)\big|_{y=x_1} - W_x^n(x_1)$. From (17) and (19), we can show that

$$
\begin{aligned}
W_x^r(x_1, x_1) - W_x^n(x_1) = \int_{\bar{z}}^{\infty} & \Big[V_x(x_1, B(z_1, x_1)) - V_x(x_1, B_1)\Big] h(z_1, x_1) dz_1 \\
& + \Big[V(x_1, B(\bar{z}, x_1)) - V(x_1, B_1)\Big] \phi(\bar{z}, x_1),
\end{aligned}
\tag{27}
$$

where $\phi(\cdot)$ is given in (22). (The derivation of (27) is in the Appendix.)

The sign of the effect depends on whether or not the economic irreversibility constraint is always binding for the NDM, i.e., whether $x_1 > x_1^m$ or not. We analyze the intuitive situation when x_1 is

such that more land is preserved if the species survives, i.e., from Proposition 2, when $x_1 \leq x_1^m$. The case when $x_1 > x_1^m$ is easier to analyze and is presented in the Appendix.

When $\tilde{x}_2(x_1) \geq 0$, the economic irreversibility constraint for the NDM is precluded by the ecological irreversibility: more land needs to be preserved if the species survives. From (7), we know for the NDM, $V_x(x_1, B_1) = 0$ for $z_1 \geq \bar{z}$, as the second period conversion x_2 is always positive for the NDM if the species survives. For the RDM, we know from (7) and (17) that $V_x(x_1, B(z_1, x_1)) = 0$ when the economic irreversibility is not binding, i.e., when $z_1 > z_1^r(x_1, x_1)$ or $z_1 < \underline{z}_1^r(x_1, x_1)$: this can be shown from the first order condition on x_2 when $x_2 \neq 0$. Further, $V_x < 0$ when $x_2 = 0$ or when $\underline{z}_1^r(x_1, x_1) \leq z_1 \leq z_1^r(x_1, x_1)$: when the irreversibility constraint is binding, the decision maker would have wanted to reduce the area in preservation, and further increase in x_1 only reduces the second period payoff. Thus, the first term on the right hand side of (27) reduces to

$$\int_{\max\{\bar{z}, \underline{z}_1^r(x_1, x_1)\}}^{\max\{\bar{z}, z_1^r(x_1, x_1)\}} V_x(x_1, B(z_1, x_1)) h(z_1, x_1) dz_1 \leq 0, \tag{28}$$

and the inequality is strict if $z_1^r > \bar{z}$.

This negative effect is due to the economic irreversibility that is relevant only for the RDM. She recognizes that there will be more information about θ, and she prefers "flexibility" in order to utilize the new information. Given land preservation irreversibility, the way to increase the flexibility in the second period is to reduce the current land preservation: if the land turns out to be effective, more land can always be preserved, and if the land tuns out to be ineffective, there is no much regret. Notice that the term in (28) becomes more negative as z_1^r rises and/or $\underline{z}_1^r$ decreases, or as economic irreversibility becomes more severe. When there is no ecological irreversibility, i.e., when $\bar{z} = 0$, $\phi(0, x_1) = 0$ and the entire learning effect in (27) reduces to (28), a standard result in the irreversible investment literature.

The next proposition shows the sign of the second term on the right hand side of (27).

Proposition 4 *Suppose x_1 is such that $\tilde{x}_2(x_1) \geq 0$. Then $B(z_1^r(x_1, x_1), x_1) \leq B_1$. Further,*

(i) if $\bar{z} \leq z_1^r(x_1, x_1)$, the term $V(x_1, B(\bar{z}, x_1)) - V(x_1, B_1)$ in (27) is negative.

(ii) If $\bar{z}$ is sufficiently high so that $B(\bar{z}, x_1) > B_1$, the last term in (27) is positive.

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The last term in (27) measures the difference in the two decision makers' incentives to raise x_1 in order to reduce the likelihood of species extinction. Both types of decision makers face the same possibility of a binding ecological irreversibility. A marginal increase in x_1 helps reduce the probability of extinction by $\phi(\bar{z}, x_1)$. However, the two decision makers view differently the marginal benefit of a smaller probability of extinction. At $z_1 = \bar{z}$, the RDM's belief about θ is given by $B(\bar{z}, x_1)$ while that of the NDM is given by B_1. If $\bar{z} < z_1^r(x_1, x_1)$, i.e., if the ecological irreversibility is mild, $B(\bar{z}, x_1) < B_1$. As a result, the RDM expects a lower payoff from reducing the probability of extinction, and consequently preserves less land in period one. In this case, the economic irreversibility is necessarily binding if the ecologic irreversibility is binding.

If the ecological irreversibility becomes sufficiently severe so that $B(\bar{z}, x_1) > B_1$ (in this case, $\bar{z} > z_1^r(x_1, x_1)$), economic irreversibility is precluded by ecological irreversibility. Only the last term remains on the right hand side of (27), and this term is positive. In this case, the RDM preserves less land. Finally, we do not need to evaluate the learning effect for all x_1 since there is a unique solution in the NDM's optimization problem. We only need to evaluate (27) at $x_1 = \tilde{x}_1$. In summary, we know

Result 3 *Suppose $\tilde{x} \leq x_1^m$ so that $\tilde{x}_2(\tilde{x}_1) \geq 0$. That is, the NDM plans to preserve more land in period two if the species survives.*

(i) If $\bar{z} < z_1^r(\tilde{x}_1, \tilde{x}_1)$, i.e., when both irreversibilities are relevant at the NDM's optimal decision, the learning effect in (27) is negative and $\hat{x}_1 < \tilde{x}_1$: the NDM preserves more land than the RDM.

(ia) In this case, the two irreversibilities work in the same direction: they both reduce the RDM's first period preservation relative to that of the NDM.

(ii) If ecological irreversibility is sufficiently severe so that $B(\bar{z}, \tilde{x}_1) > B_1$ and thus economic irreversibility is precluded by ecological irreversibility (i.e., $\bar{z} > z_1^r(x_1, x_1)$), the learning effect is positive and $\hat{x}_1 > \tilde{x}_1$: the NDM preserves less land than the RDM.

6 Effects of Uncertainty and Irreversibilities

By comparing the first period decisions of the three types of decision makers, we obtain the effects of *anticipating* future information and of knowing the endogeneity of such information. We showed that these effects depend on the two irreversibilities. Another way to assess the effects of learning (or uncertainty about θ) and irreversibilities is to focus on the fully rational decision maker, the EDM, and study how her optimal decision x_1^* changes as the degrees of uncertainty and irreversibilities vary. Not surprisingly the effects are ambiguous in general due to the nonlinearities in payoff functions and in the Bayesian learning functions. We restrict ourselves to analytically tractable scenarios to illustrate the factors behind the effects. Specifically, to evaluate the effects of learning and uncertainty, we compare x_1^* with the optimal decision when the EDM has *nothing to learn*, i.e., she knows the value of θ *a priori*. To evaluate the effects of the economic and ecological irreversibilities, we study how x_1^* changes in response to τ and $\bar{z}$ in the neighborhood of no irreversibility at all, i.e., $\tau = \bar{z} = 0$.[14] The results might still be ambiguous even for these presumably simple comparisons, and we will make further functional form assumptions when necessary.

6.1 The Effects of Uncertainty

Consider the situation when the EDM knows the value of θ, and for comparison purposes, let the value of θ be equal to the expected value $\bar{\theta} = B_1 \theta^H + (1 - B_1)\theta^L$. Let the optimal decision be $\bar{x}_1^*$, determined from the following optimization problem (cf. (5)):

$$
\begin{aligned}
\bar{x}_1^* = \arg\max_{x_1} \quad & \bar{J}(x_1; \bar{\theta}) + \delta \bar{V}(x_1; \bar{\theta})(1 - \bar{H}(\bar{z}, x_1; \bar{\theta})) - \delta \tau x_1 \bar{H}(\bar{z}, x_1; \bar{\theta}) \\
\text{s.t.} \quad & z_1(x_1, \epsilon_1; \bar{\theta}) = f(x_1, \bar{\theta}) + \epsilon_1, \\
& \bar{J}(x_1; \bar{\theta}) = \gamma f(x_1, \bar{\theta}) + \gamma \bar{\epsilon} - c x_1 + U_0, \\
& \bar{V}(x_1; \bar{\theta}) = \max_{x_2 \geq -x_1} \gamma f(x_1 + x_2, \bar{\theta}) + \gamma \bar{\epsilon} + U_0 - c(x_1 + x_2) + \tau x_2 1(x_2 < 0),
\end{aligned}
\tag{29}
$$

where $\bar{H}(\bar{z}, x_1; \bar{\theta}) = H(\bar{z} - f(x_1, \bar{\theta}))$ is the probability that the species will not survive by the end of period one.

Note that (29) is also the optimization problem for the NDM under full information about θ

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[14]We thank an anonymous referee for suggesting this line of analysis.

(cf. (20)): since there is nothing to learn about θ, the EDM and NDM behave in the same fashion: they both believe that $\theta = \bar{\theta}$ in both periods. Thus,

Remark 1 *The optimal decisions of the EDM and NDM are the same when there is no uncertainty about θ.*

The Remark greatly simplifies our analysis of the uncertainty effects for the EDM, because it is much easier to analyze the effects of uncertainty for the NDM than for the EDM. From the previous section, we know that when there is uncertainty about θ, the EDM and NDM's decisions, x_1^* and $\tilde{x}_1$, differ by the experimentation and learning effects. To compare x_1^* and $\bar{x}_1^*$, we only need to find out the *uncertainty effect*: the difference between $\tilde{x}_1$ and $\bar{x}_1^*$.

The uncertainty effect naturally depends on the curvature of the payoff function, in particular the curvature of $f_x(x, \theta)$. If it is concave in θ, uncertainty is likely to reduce the optimal preservation and the converse is true if it is convex in θ. From the $\bar{J}(\cdot)$ and $\bar{V}(\cdot)$ functions in (29), we see that this property of $f_x(\cdot)$ directly carries over to $\bar{J}(\cdot)$ and $\bar{V}(\cdot)$: if $f_x(x, \theta)$ is concave (or convex) in θ, so are $\bar{J}(x_1; \theta)$ and $\bar{V}(x_1; \theta)$ in θ.

The uncertainty effect, however, also interacts with the random shock ϵ: in the objective function in (29), both $V(\cdot)$ and x_1 interact with $H(\cdot; \bar{\theta})$. For many commonly used distributions, $H(\cdot)$ is typically both convex and concave. For instance, if ϵ is normally distributed, even if $f(\cdot)$ is linear in θ, $H(\cdot)$ is convex in θ for high values of θ and concave for low values of θ. The uncertainty effect is then ambiguous, depending on the relative magnitudes of the concavity and convexity forces.

We thus focus on a special case, where $f_x(x_1, \theta)$ is *linear* in θ, and ϵ is uniformly distributed so that $H(\cdot)$ is also *linear*.[15] It is straightforward to verify that in this case, the derivative of the objective function in (29) with respect to x_1 is convex in θ.

Result 4 *When $f_x(x_1, \theta)$ is linear in θ and ϵ is uniformly distributed, the uncertainty effect is positive: the NDM preserves more land in period one under uncertainty than under certainty about θ.*

---

[15]The assumption of uniform distribution implies that there is an upper bound in the support of ϵ. The spirit of our results still holds true in this case, although the analysis becomes more complicated because learning is discontinuous: for some z_1 values there is no learning about θ and for other values there could be complete information about θ.

Combining Results 1, 3, and 4, we can identify the difference between x_1^* and $\bar{x}_1^*$ and the overall effects of knowing θ. Under the linearity assumptions in Result 4, both the uncertainty and experimentation effects imply that the EDM preserves more land under uncertainty than under full knowledge about θ. This difference is further enhanced by the learning effect when the ecological irreversibility is severe but might be weakened when the ecological irreversibility is mild.

6.2 The Effects of Irreversibilities

Since the decision maker's first period payoff $J(x_1)$ is independent of the two irreversibilities $\bar{z}$ and τ, from (13), we only need to evaluate the irreversibilities' effects on $W_x^e(x_1)$. From (23), we know W_x^e can be decomposed into $W_x^r(x_1, y)|_{y=x_1}$ and $W_y^r(x_1, y)|_{y=x_1}$, and W_y^r is given in (26). From the W^r expression in (17), we obtain (through taking derivatives and changing variables of integration)

$$
\begin{aligned}
W_x^r(x_1, y)|_{y=x_1} = & \int_{\bar{z}}^{\infty} V_x(x_1, B(z_1, x_1)) h(z_1, x_1) dz_1 + V(x_1, B(\bar{z}, x_1)) h(\bar{z}, x_1) \\
& - \frac{d}{dx_1} \int_0^{\bar{z}} \tau x_1 h(z_1, x_1) dz_1.
\end{aligned}
\tag{30}
$$

Suppose initially there is no irreversibility whatsoever: $\tau = \bar{z} = 0$, and now consider the effects of increasing τ and $\bar{z}$ respectively. A standard result in real option theory is that, when future information is exogenous, the optimal decision depends on the decision maker's risk attitude. In other words, the effects depend again on the curvature of the payoff and Bayesian updating equations. As in studying the effects of uncertainty, we make linearity assumptions to streamline the analysis and to obtain sharp intuition.

Result 5 *Suppose there is no ecological irreversibility, $\bar{z} = 0$, and suppose that $f_{xx}(x, \theta^H) = f_{xx}(x, \theta^L)$, i.e., θ does not affect the* slope *of the marginal productivity of land f_x. Then $\frac{\partial W_x^e(x_1; \tau)}{\partial \tau} < 0$ for all $\tau \geq 0$: increasing economic irreversibility ambiguously reduces the level of first period land preservation.*

This Result (proved in the Appendix) is intuitive: when there is no ecological irreversibility, the "cost" of preserving more land in order to learn about θ (in addition to foregone profit c), arises from the economic irreversibility constraint: if too much land is preserved, it is costly to revert it back. Note that without the curvature assumption about $f_{xx}(\cdot)$, the result would not be clear-cut

because uncertainty about θ will affect how much land is converted in period two. Then if learning changes this uncertainty, it will also change the future conversion decisions, and this will interact with the effects of a higher reversion cost τ.

To study the effects of the ecological irreversibility $\bar{z}$, we assume away the economic irreversibility: $\tau = 0$. From (26) and (30), substituting in $\tau = 0$, and noting that $V_x(\cdot) = 0$ when $\tau = 0$,[16] we get

$$\frac{\partial W_x^e(x_1; \bar{z})}{\partial \bar{z}} = \frac{\partial [V(x_1, B(\bar{z}, x_1)) h(\bar{z}, x_1)]}{\partial \bar{z}} + V_B(x_1, B(\bar{z}, x_1)) \frac{\partial \Omega(\bar{z}, x_1)}{\partial \bar{z}}.$$

Without making functional form assumptions, we are able to obtain a clear-cut result in the neighborhood of $\bar{z} = 0$. In this case, $h(\bar{z}, x_1)$ is increasing in $\bar{z}$ at $\bar{z} = 0$: $h(0, x_1) = 0$ because the species population cannot be negative, and $h(z, x_1) \geq 0$ as $z > 0$. Further, from Proposition 3, we know $\Omega(\bar{z}, x_1)$ is also increasing in $\bar{z}$ at $\bar{z} = 0$: $\Omega(0, x_1) = 0$ while $\Omega(z, x_1) \geq 0$ when $z > 0$. Since both $V(x_1, \cdot)$ and $B(\cdot, x_1)$ are increasing, we know W_x^e is increasing in $\bar{z}$:

Result 6 *Suppose there is no economic irreversibility: $\tau = 0$. Then in the neighborhood of $\bar{z} = 0$, or when the degree of ecological irreversibility is mild, increasing the degree of ecological irreversibility raises the first period land preservation level.*

7   Conclusions and Discussions

In this paper, we compare the optimal land preservation decisions of three types of decision makers, distinguished by their attitudes towards future information and learning. The EDM is fully rational and makes the efficient choices, while the RDM errs by not recognizing the endogeneity of future information and the NDM simply ignores future information altogether. Many real world decisions are made similar to those of the NDM and RDM, while the approach of EDM is advocated in ecosystem management and is increasingly being adopted.

We find that the RDM always preserves less land than the EDM. The difference is captured by an experimentation effect, the magnitude of which is affected by the economic and ecological

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[16]Without any irreversibility in land preservation, land preserved in the first period can be freely reverted in the second period. The first order condition of x_2 in (7) immediately implies that $V_x = 0$.

irreversibilities. The difference decreases as it becomes more costly to revert preserved habitat back to agriculture, and is likely to increase as the species is harder to conserve (i.e., as it requires a larger minimum population to survive).

As we discussed earlier, many real world decisions in habitat preservation resemble that of a NDM. We find that the NDM may or may not preserve less land than the EDM: the difference in their decisions is captured by the summation of the experimentation effect and a learning effect, with the sign and magnitude of the learning effect depending on the relative importance of the two irreversibilities. The learning effect is positive so that the NDM preserves less land than the RDM and thus also than the EDM, if the species is hard to conserve and preserved land is easy to revert back to agriculture. If to the contrary the species is relatively easy to conserve but it is costly to revert preserved land, the NDM may preserve more land than the RDM, and possibly even more than the EDM.

Although we do not conduct a formal welfare analysis, we know that the solution of the EDM represents the socially optimal land preservation decisions. The ranking of the welfare between the RDM and NDM, two second best cases, is not clear-cut, depending on the sign of the learning effect. If, as we discussed above, the learning effect is positive so that the RDM preserves more land than the NDM, we obtain an unambiguous ranking: the RDM obtains a higher payoff than the NDM. In this case, recognizing future information alone will improve social welfare.

We also compare the EDM's decision under uncertainty and irreversibilities with the cases when no such uncertainty exists, when preserved land can be costlessly reverted, or when the species will not be lost. These effects of uncertainty and irreversibilities depend on the curvature of the payoff function and the distribution function of the random shock. Under a broad set of linearity conditions, we find that the EDM preserves more land under uncertainty, and less land when it is costly to revert preserved land. When the species loss risk is mild, the EDM preserved more land as it is easier for the species to be lost.

Our results are derived under a number of simplifying assumptions, chief among which is the one in (2) that the first period population z_1, once the species survives, does not affect the second period population z_2 directly. In reality, if the species does survive, the second period population

should depend on z_1 as well. The simplest way to directly link z_1 to z_2 is to add a transition equation, say $\Pi(z_1)$, on the right hand side of (2) when the species survives. Then all of our major conclusions still hold: each decision maker preserves more land in period one due to the added benefit $\gamma\Pi(z_1)$, but the relative magnitudes of their decisions remain unchanged. However, if the transition equation also depends on x_2 and θ, e.g., $\Pi(z_1, x_2, \theta)$, the analysis is complicated by the interaction of the three variables. If $\Pi(\cdot)$ is similar to $f(\cdot)$ so that a higher θ raises the marginal benefit of x_2, the major results in Section 5, i.e., those on the learning and experimentation effects, still hold: these results depend on the differences in the expected benefit of x_2 and introducing $\Pi(\cdot)$ does not qualitatively change these differences (although quantitatively the differences will change). However, major results in Section 6, which depend on the linearity assumptions, will not hold unless we make appropriate linearity assumptions about $\Pi(\cdot)$.

Appendix

Proof of Proposition 1. From the formula of $B(z_1, x_1)$ in (4), we know

$$B_x(z_1, x_1) \propto \frac{g'(z_1 - f(x_1, \theta^L))}{g(z_1 - f(x_1, \theta^L))} f_x(x_1, \theta^L) - \frac{g'(z_1 - f(x_1, \theta^H))}{g(z_1 - f(x_1, \theta^H))} f_x(x_1, \theta^H). \tag{31}$$

The MLRP of $g(\cdot)$ and the fact that $z_1 - f(x_1, \theta^L) > z_1 - f(x_1, \theta^H)$ imply

$$g'(z_1 - f(x_1, \theta^L))/g(z_1 - f(x_1, \theta^L)) < g'(z_1 - f(x_1, \theta^H))/g(z_1 - f(x_1, \theta^H)).$$

Note also that $0 < f_x(x_1, \theta^L) < f_x(x_1, \theta^H)$. Thus, if $g'(z_1 - f(x_1, \theta^H)) \geq 0$, then $g'(z_1 - f(x_1, \theta^L))$ can either be positive or negative, and we always have $B_x < 0$. However, if $g'(z_1 - f(x_1, \theta^H)) < 0$, then $g'(z_1 - f(x_1, \theta^L)) < 0$ and the sign of B_x is ambiguous.

The MLRP also implies that $g(\cdot)$ is unimodal: $g'(\epsilon) > 0$ when ϵ is to the left of the modal point, and $g'(\epsilon) < 0$ when ϵ is to the right of the modal point. Thus, when $g'(z_1 - f(x_1, \theta^H)) < 0$ (and thus $g'(z_1 - f(x_1, \theta^L)) < 0$), both $z_1 - f(x_1, \theta^H)$ and $z_1 - f(x_1, \theta^L)$ are to the right of the modal point. The fact that $z_1 - f(x_1, \theta^H) < z_1 - f(x_1, \theta^L)$ implies that $g(z_1 - f(x_1, \theta^H)) > g(z_1 - f(x_1, \theta^L))$. But from (4), we know $B(z_1, x_1) \geq B_1$ if and only if $g(z_1 - f(x_1, \theta^H)) \geq g(z_1 - f(x_1, \theta^L))$. That is, when $g'(z_1 - f(x_1, \theta^H)) < 0$, we must have $B(z_1, x_1) > B_1$. Therefore, $B(z_1, x_1) \leq B_1$ implies

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that $g'(z_1 - f(x_1, \theta^H)) \geq 0$ and subsequently $B_x < 0$.

When $B(z_1, x_1) > B_1$, the sign of $g'(z_1 - f(x_1, \theta^H))$ can be either positive or negative, which subsequently determines the sign of B_x. ∎

Proof of Proposition 2. From the first order condition of (20), $J_x(\tilde{x}_1) + \delta W_x^n(\tilde{x}_1) = 0$, and the concavity of $J(\cdot)$ and $W^n(\cdot)$, we know $\tilde{x}_1 > x_1^m$ if and only if $W_x^n(x_1^m) > 0$.

From (3) and changing variables of integration from z to ϵ, we know the probability of species survival is

$$\int_{\bar{z}}^{\infty} h(z, x_1) dz = \int_{\bar{z} - f(x_1, \theta^H)}^{\infty} B_1 g(\epsilon) d\epsilon + \int_{\bar{z} - f(x_1, \theta^L)}^{\infty} (1 - B_1) g(\epsilon) d\epsilon. \tag{32}$$

Substituting this into (19) while observing that $\epsilon \geq 0$, we know

$$\begin{aligned}
W^n(x_1) =& V(x_1, B_1) \int_{\bar{z}}^{\infty} h(z_1, x_1) dz_1 - \tau x_1 \int_0^{\bar{z}} h(z_1, x_1) dz_1 \\
=& V(x_1, B_1) \left[\int_{\bar{z} - f(x_1, \theta^H)}^{\infty} B_1 g(\epsilon) d\epsilon + \int_{\bar{z} - f(x_1, \theta^L)}^{\infty} (1 - B_1) g(\epsilon) d\epsilon \right] \\
& - \tau x_1 \left[\int_0^{\bar{z} - f(x_1, \theta^H)} B_1 g(\epsilon) d\epsilon + \int_0^{\bar{z} - f(x_1, \theta^L)} (1 - B_1) g(\epsilon) d\epsilon \right],
\end{aligned}$$

where the first equality follows from (19), and the second equality is obtained from substituting in (32). Taking derivative with respect to x_1 and changing variables of integration from ϵ back to z_1, we obtain

$$W_x^n(x_1) = V_x(x_1, B_1)(1 - H(\bar{z}, x_1)) - \tau H(\bar{z}, x_1) + [V(x_1, B_1) + \tau x_1] \phi(\bar{z}, x_1).$$

Comparing (7) and (6), we know $V_x(x_1^m, B_1) = 0$, and thus (21) immediately follows. ∎

Proof of Result 1. From (17), we know the second period decision of the RDM is a standard static maximization problem with information exogenously given by y. Further, the exogenous signals are more informative as y increases. Blackwell's theorem (Blackwell (1951, 1953), and Kihlstrom (1984)) then implies that the expected payoff $W^r(x_1, y)$ increases as y rises. ∎

Proof of Proposition 3. From (25), it is obvious that $\Omega(\infty, x_1) = 0$. We next show $\Omega(0, x_1) = 0$ for all x_1. On the right hand side of (25), we change variables of integration from z_1 to $\epsilon = $

$z_1 - f(x_1, \theta^H)$ and to $\epsilon = z_1 - f(x_1, \theta^L)$ for the two terms respectively, and obtain

$$\Omega(z, x_1) = \int_{z-f(x_1,\theta^H)}^{\infty} \frac{dB(f(x_1, \theta^H) + \epsilon, x_1)}{dx_1} B_1 g(\epsilon) d\epsilon$$
$$+ \int_{z-f(x_1,\theta^L)}^{\infty} \frac{dB(f(x_1, \theta^L) + \epsilon, x_1)}{dx_1} (1 - B_1) g(\epsilon) d\epsilon \tag{33}$$

However, since $B_1 = E_{z_1} B(z_1, x_1)$ for all x_1 (the prior equals the expectation of all possible posteriors), we know

$$B_1 = \int_0^{\infty} B(f(x_1, \theta^H) + \epsilon, x_1) B_1 g(\epsilon) d\epsilon + \int_0^{\infty} B(f(x_1, \theta^L) + \epsilon, x_1)(1 - B_1) g(\epsilon) d\epsilon, \quad \forall x_1$$

Taking the derivatives of both sides with respect to x_1, we obtain

$$0 = \int_0^{\infty} \frac{dB(f(x_1, \theta^H) + \epsilon, x_1)}{dx_1} B_1 g(\epsilon) d\epsilon + \int_0^{\infty} \frac{dB(f(x_1, \theta^L) + \epsilon, x_1)}{dx_1} (1 - B_1) g(\epsilon) d\epsilon$$
$$= \Omega(0, x_1)$$

where in the last equality, we used the condition that $g(\epsilon) = 0$ when $\epsilon < 0$.

Next we show that $\Omega(z, x_1) \geq 0$, adopting the approach of Datta et al. (2002) (their Lemma 2). In the EDM's problem, a higher x_1 means that the signal z_1 is more informative. Thus, from Kihlstrom (1984), we know

$$\int_0^{\infty} L(B(z_1, x_1)) h(z_1, x_1) dz_1 \quad \text{in increasing in } x_1 \tag{34}$$

for every continuous and convex function $L(\cdot)$. Consider first values of z_1 such that $h(z_1, x_1) > 0$ (e.g., when z_1 is high). Pick such a z_1, say $\bar{\bar{z}}_1$, and suppose $\Omega(\bar{\bar{z}}_1, x_1) < 0$. Then continuity of $\Omega(\cdot, x_1)$ means that $\Omega(z_1, x_1) < 0$ for all z_1 in the neighborhood I of $\bar{\bar{z}}_1$. Construct a function $L(B)$ such that $L''(B) > 0$ on $\{B(z_1, x_1) : z_1 \in I\}$ and $L''(B) = 0$ otherwise. Then

$$\int_0^{\infty} L(B(z_1, x_1)) h(z_1, x_1) dz_1$$
$$= \int_0^{\infty} L(B(z_1, x_1)) \left[(B_1 g(z_1 - f(x_1, \theta^H)) + (1 - B_1) g(z_1 - f(x_1, \theta^L)) \right] dz_1$$
$$= \int_0^{\infty} L(B(f(x_1, \theta^H) + \epsilon, x_1)) B_1 g(\epsilon) d\epsilon + \int_0^{\infty} L(B(f(x_1, \theta^L) + \epsilon, x_1))(1 - B_1) g(\epsilon) d\epsilon$$

where the last equation follows from changing variables of integration from z_1 to ϵ: $z_1 - f(x_1, \theta^H) = \epsilon$ for the first integral and $z_1 - f(x_1, \theta^L) = \epsilon$ for the second integral. Again, the lower bounds of

integration is written as zero for simplicity, without affecting the values. Thus,

$$\frac{\partial}{\partial x_1} \int_0^\infty L(B(z_1, x_1))h(z_1, x_1)dz_1$$

$$= \int_0^\infty L'(B)\frac{dB(f(x_1, \theta^H) + \epsilon, x_1)}{dx_1}B_1 g(\epsilon)d\epsilon + \int_0^\infty L'(B)\frac{dB(f(x_1, \theta^L) + \epsilon, x_1)}{dx_1}(1 - B_1)g(\epsilon)d\epsilon$$

$$= \int_0^\infty L'(B)\left(\frac{dB(z_1(x_1, \theta^H), x_1)}{dx_1}\right)B_1 g(z_1 - f(x_1, \theta^H))dz_1$$

$$+ \int_0^\infty L'(B)\left(\frac{dB(z_1(x_1, \theta^L), x_1)}{dx_1}\right)B_1 g(z_1 - f(x_1, \theta^L))dz_1$$

$$= - \int_0^\infty L'(B)d\Omega(z_1, x_1),$$

where the second equality is obtained after changing the variable of integration back to z_1. Integrating by parts, we get

$$\frac{\partial}{\partial x_1} \int_0^\infty L(B(z_1, x_1))h(z_1, x_1)dz_1$$

$$= - L'(B(z_1, x_1))\Omega(z_1, x_1)\Big|_0^\infty + \int L''(B(z_1, x_1))\frac{\partial B(z_1, x_1)}{\partial z_1}\Omega(z_1, x_1)dz_1$$

$$= \int_I L''(B)\frac{\partial B}{\partial z_1}\Omega(z_1, x_1)dz_1 < 0,$$

where the last equality follows from $\Omega(0, x_1) = 0$ and $\Omega(0, x_1) = 0$, and the last inequality follows since $L'' > 0$ on I and $B_z(z_1, x_1) > 0$. But the inequality violates (34). Thus we know $\Omega(z_1, x_1) \geq 0$. In fact, to assure that the expression in (34) is strictly increasing, we must have $\Omega(z_1, x_1) > 0$. ∎

Proof of (26). Taking derivative of (17) with respect to y, we know

$$W_y^r(x_1, y)\Big|_{y=x_1} = \int_{\bar{z}-f(x_1, \theta^H)}^\infty V_B(x_1, B(f(x_1, \theta^H) + \epsilon, x_1))\left(\frac{dB(f(x_1, \theta^H) + \epsilon, x_1)}{dx_1}\right)B_1 g(\epsilon)d\epsilon$$

$$+ \int_{\bar{z}-f(x_1, \theta^L)}^\infty V_B(x_1, B(f(x_1, \theta^L) + \epsilon, x_1))\left(\frac{dB(f(x_1, \theta^L) + \epsilon, x_1)}{dx_1}\right)(1 - B_1)g(\epsilon)d\epsilon$$

$$= \int_{\bar{z}}^\infty V_B(x_1, B(z_1, x_1))\left(\frac{dB(z_1, x_1)}{dx_1}\right)B_1 g(z_1 - f(x_1, \theta^H))dz_1$$

$$+ \int_{\bar{z}}^\infty V_B(x_1, B(z_1, x_1))\left(\frac{dB(z_1, x_1)}{dx_1}\right)B_1 g(z_1 - f(x_1, \theta^L))dz_1$$

$$= - \int_{\bar{z}}^\infty V_B(x_1, B(z_1, x_1))d\Omega(z_1, x_1),$$

where the second inequality is obtained by changing the variables of integration from ϵ to $z_1 = f(x_1, \theta^H) + \epsilon$ and $z_1 = f(x_1, \theta^L) + \epsilon$ respectively, and the third equality follows from (25). Note that after the second equality, $dB(z_1, x_1)/dx_1$ is given in (24) and should not be taken as being

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$\partial B(z_1, x_1)/\partial x_1$. Integrating by parts, we get

$$W_y^r(x_1, x_1) = V_B(x_1, B(\bar{z}, x_1))\Omega(\bar{z}, x_1) + \int_{\bar{z}}^{\infty} \Omega(z_1, x_1)dV_B(x_1, B(z_1, x_1)).$$

From (7), $V_B = \gamma f(x_1 + x_2^*(x_1, z_1), \theta^H) - \gamma f(x_1 + x_2^*(x_1, z_1), \theta^L)$. Thus

$$dV_B = \gamma\Big[f_x(x_1 + x_2^*(x_1, z_1), \theta^H) - f_x(x_1 + x_2^*(x_1, z_1), \theta^L)\Big]\frac{\partial x_2^*(x_1, z_1)}{\partial z_1}dz_1 \geq 0. \tag{35}$$

From (8) and (10), we know when $\underline{z}_1^e(x_1) < z_1 < z_1^e(x_1)$, $x_2^*(x_1, z_1) = 0$ and thus $dV_B = 0$.
Substituting this in, we obtain (26). ∎

Proof of (27). Subtracting (19) from (17), we get

$$W^r(x_1, y) - W^n(x_1) = \int_{\bar{z}-f(x_1, \theta^H)}^{\infty} \Big[V(x_1, B(f(y, \theta^H) + \epsilon, y)) - V(x_1, B_1)\Big]B_1 g(\epsilon)d\epsilon$$

$$+ \int_{\bar{z}-f(x_1, \theta^L)}^{\infty} \Big[V(x_1, B(f(y, \theta^L) + \epsilon, y)) - V(x_1, B_1)\Big](1 - B_1)g(\epsilon)d\epsilon.$$

Taking derivatives with respect to x_1, and set $y = x_1$, we know

$$W_x^r(x_1, x_1) - W_x^n(x_1) = \int_{\bar{z}-f(x_1, \theta^H)}^{\infty} \Big[V_x(x_1, B(f(x_1, \theta^H) + \epsilon, x_1)) - V_x(x_1, B_1)\Big]B_1 g(\epsilon)d\epsilon$$

$$+ \int_{\bar{z}-f(x_1, \theta^L)}^{\infty} \Big[V_x(x_1, B(f(x_1, \theta^L) + \epsilon, x_1)) - V_x(x_1, B_1)\Big](1 - B_1)g(\epsilon)d\epsilon$$

$$+ \Big[V(x_1, B(\bar{z}, x_1)) - V(x_1, B_1)\Big]B_1 g(\bar{z} - f(x_1, \theta^H))f_x(x_1, \theta^H)$$

$$+ \Big[V(x_1, B(\bar{z}, x_1)) - V(x_1, B_1)\Big](1 - B_1)g(\bar{z} - f(x_1, \theta^L))f_x(x_1, \theta^L).$$

Changing the variables of integration from ϵ to $z_1 = f(x_1, \theta^H) + \epsilon$ in the first integration, and to $z_1 = f(x_1, \theta^L) + \epsilon$ in the second integration, we obtain (27). ∎

Proof of Proposition 4. Since $\tilde{x}_2(x_1) > 0$, the first order condition on $\tilde{x}_2$ (cf. (7)) implies that

$$B_1\gamma f_x(x_1, \theta^H) + (1 - B_1)\gamma f_x(x_1, \theta^L) > c. \tag{36}$$

Comparing (36) with (16) when $z_1 = z_1^r(x_1, x_1)$, we know $B(z_1^r, x_1) < B_1$ because the left hand side of (36) is increasing in B_1. Since $B(z_1, x_1)$ is increasing in z_1, if $z_1^r(x_1, x_1) > \bar{z}$, then $B(z_1^r, x_1) > B(\bar{z}, x_1)$. That is, $B(\bar{z}, x_1) < B_1$, which implies $V(x_1, B(\bar{z}, x_1)) - V(x_1, B_1) < 0$ since $V(x_1, \cdot)$ is increasing. Conversely, if $B(\bar{z}, x_1) > B_1$, the fact that $V(x_1, \cdot)$ is increasing immediately implies that the last term in (27) is positive. ∎

The learning effect when $x_1 \geq x_1^m$. When $x_1 \geq x_1^m$, $\tilde{x}_2(x_1) \leq 0$, and the sign of the learning effect in (27) is ambiguous. A sufficient condition for the effect to be positive is $B(\bar{z}, x_1) \geq B_1$. In this case, the second term on the right hand side of (27) is positive because $V(x_1, \cdot)$ is increasing. The first term equals

$$
\int_{\bar{z}}^{\max\{\bar{z}, \underline{z}_1^r(x_1, x_1)\}} (-V_x(x_1, B_1)) h_x(z_1, x_1) dz_1
$$
$$
+ \int_{\max\{\bar{z}, \underline{z}_1^r(x_1, x_1)\}}^{\max\{\bar{z}, z_1^r(x_1, x_1)\}} \left[V_x(x_1, B(z_1, x_1))|_{x_2=0} - V_x(x_1, B_1) \right] h_x(z_1, x_1) dz_1
$$
$$
+ \int_{z_1^r(x_1, x_1)}^{\infty} (-V_x(x_1, B_1)) h_x(z_1, x_1) dz_1,
$$

where in the first and third terms, we have used the condition $V_x(x_1, B(z_1, x_1)) = 0$ when $\tilde{x}_2 \neq 0$. Since $V_x(x_1, B) = B\gamma f_x(x_1, \theta^H) - (1 - B)\gamma f_x(x_1, \theta^L) - c$ is increasing in B, the second term in the above equation is positive. The first and third terms are positive since $V_x(x_1, B_1) < 0$ when $x_2 \geq 0$ is binding.

From this result and Result 3, we know that regardless of the magnitude of $\tilde{x}_1$, as long as the ecological irreversibility is sufficiently severe, i.e., $\bar{z}$ is sufficiently high so that $B(\bar{z}, \tilde{x}_1) \geq B_1$, the NDM always converts less land than the RDM. ∎

Proof of Result 5. Since there is no ecological irreversibility ($\bar{z} = 0$), combining (26) and (30), substituting in $\bar{z} = 0$, and noting that $h(0, x_1) = 0$ and $\Omega(0, x_1) = 0$ (Proposition 3), we obtain

$$
W_x^e(x_1; \tau) = \int_0^\infty V_x(x_1, B(z_1, x_1); \tau) h(z_1, x_1) dz_1 + \int_0^{z_1^e(x_1)} \Omega(z_1, x_1) dV_B(x_1, B(z_1, x_1))
$$
$$
+ \int_{z_1^e(x_1)}^\infty \Omega(z_1, x_1) dV_B(x_1, B(z_1, x_1))
$$

Both $h(\cdot)$ and $\Omega(\cdot)$ are independent of τ. From (35) and (9), we know $dV_B(\cdot)$ depends on τ through the dependence of $x_2^*(x_1, z_1; \tau)$ on τ. Consequently $dV_B/d\tau$ is proportional to $f_{xx}(x_1 + x_2^*(x_1, z_1), \theta^H) - f_{xx}(x_1 + x_2^*(x_1, z_1), \theta^L)$, which is zero from the linearity assumption. Thus, $dV_B/d\tau = 0$ and

$$
\frac{\partial W_x^e(x_1; \tau)}{\partial \tau} = \int_0^\infty \frac{\partial V_x(x_1, B(z_1, x_1); \tau)}{\partial \tau} h(z_1, x_1) dz_1 \leq 0, \tag{37}
$$

where the inequality follows because $\partial V_x / \partial \tau \leq 0$: as τ increases, it is harder to reverse x_1, reducing its marginal value. Specifically, $V_x = 0$ if $z_1 \geq z_1^e$, $V_x = -\tau$ if $z_1 \leq \underline{z}_1^e$, and $V_x \in (-\tau, 0)$ is

independent of τ when $z_1 \in (\underline{z}_1^e, z_1^e)$. ∎

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