

# GETTING AWAY WITH ROBBERY? PATENTING BEHAVIOR WITH THE THREAT OF INFRINGEMENT

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## Abstract

The paper examines the relationship between the innovator's patenting and patent breadth decisions as well as how these two decisions affect, and are affected by, the innovator's ability to enforce her patent rights. An important feature of the model is that the entrant may be able, by his choice of location in product space, to affect the innovator's decision to defend her patent. An interesting finding of the paper is that the innovator might find it optimal to patent her innovation and be indifferent between not having her patent infringed and not defending her patent under infringement. The paper also shows that the greater is the entrant's R&D effectiveness, the smaller is the innovator's incentive to patent her product. If patenting occurs, however, the greater is R&D effectiveness, the greater is the patent breadth that could be chosen without triggering infringement.

**Keywords:** patent breadth, entry deterrence, patent infringement, patent validity.

**JEL Classification Codes:** L20; L13; O34.

## 1. Introduction

Patents have been used over the last 500 years as means of protecting intellectual property. The decision to patent an innovation implies that patenting is perceived as generating more rents than when no protection is in place. With no patent protection, the innovator cannot affect the market entry and location decisions of potential entrants, although she may be able to hinder the generation of competing innovations by keeping her innovation a secret.<sup>1</sup> With patent protection, however, an innovator can strategically use the breadth of her patent to influence the entrant's decision to enter,

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<sup>1</sup> For an analysis of the innovator's decision to patent her innovation or to keep it a secret, see Horstmann et al. (1985), Waterson (1990), Aoki and Spiegel (2003), and Erkal (2005).

and if entry occurs, where in the innovation space the entrant will locate.<sup>2</sup> Thus, the decision to patent must be accompanied by a decision regarding the breadth of patent protection that should be claimed.

The question of whether to patent an innovation depends critically on the degree to which the patent provides the innovator with the ability to influence the entrant's entry and location behavior, since the patent, which is costly to obtain, will have little or no value if no change in behavior is induced. A critical aspect of the patent's ability to influence behavior is the degree to which the innovator is prepared to legally defend the patent if it is challenged or infringed.<sup>3</sup>

The purpose of this paper is to examine the relationship between the innovator's patenting and patent breadth decisions as well as how these two decisions affect, and are affected by, the innovator's ability to enforce her patent rights. On this last point, the paper examines whether it is possible for the innovator to affect the entrant's location decision (and thus the rents that can be captured by the patent) even when defending the patent under infringement is not optimal. To address the above issues, the paper develops a game theoretic model that examines the optimal patenting behavior of an incumbent innovator who has generated a patentable product innovation and who is faced with potential entry by an entrant supplying a superior quality product. The incumbent/innovator has to decide whether she should patent her innovation and, if so, what patent breadth should be claimed. If her patent is infringed, the incumbent also has to decide whether she should invoke a trial to defend the patent. An important feature of the model is that the entrant may

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<sup>2</sup> See Yiannaka and Fulton (2006) for a model of strategic patent breadth. In general, a large patent breadth makes it harder for potential entrants to enter in the patentee's market with non-infringing innovations. At the same time, however, a broad patent affects the innovator's ability to enforce and/or defend her patent rights as a large patent breadth may make the patent more prone to invalidation as courts are more likely to uphold narrow rather than broad patents (Waterson 1990), thus, shortening the effective patent life. In addition, broad patents may invite more infringement and patent validity challenges (Merges and Nelson 1990, Lerner 1994, Lanjouw and Schankerman 2001, Harhoff and Reitzig 2004).

<sup>3</sup> Crampes and Langinier (2002) examine the patentee's optimal reaction in the case of infringement – to go to court, to settle or to accept entry – without considering, however, the decision to patent or the patent breadth decision.

be able, by his choice of location in product space, to affect the incumbent's decision to defend her patent.

The results of the model show that the innovator will never find it optimal to seek patent protection when the potential entrant's R&D costs are relatively small, when the effect that patent breadth has on the patent being found invalid during an infringement trial is relatively large and when the innovator's trial costs are relatively large and her monopoly profits are relatively small. Under these conditions the patent has no impact on the entrant's behavior and patenting is not desirable. When the above conditions do not obtain, however, a patent, combined with the optimal patent breadth, can affect the entrant's entry and location decisions and patenting may be optimal for the innovator. When patenting is optimal, the smaller are the entrant's R&D costs, the greater is the patent breadth that could be chosen without triggering infringement. This result occurs because the greater is the entrant's R&D effectiveness, the further away from the incumbent the entrant can locate in the product space; the outcome is increased product differentiation, less competition and thus higher profits for both players. Moreover, there are certain conditions – specifically when the entrant's R&D and trial costs are large, the effect that patent breadth has on the patent being found invalid during an infringement trial is small and the innovator's trial costs are small and her monopoly profits are large – when the innovator can use the breadth of her patent to deter market entry.

An important finding of the paper is that, when entry deterrence is not possible, the innovator might find it optimal to patent her innovation even if the patent would not be defended if it were to be violated. This result, which is more likely to occur when the entrant's R&D effectiveness is relatively high, occurs because, by choosing to patent her innovation, the incumbent can induce the entrant to choose a location in product space that, even though it infringes the patent, is still advantageous for the incumbent (i.e., it is further away from the incumbent's location than

the location chosen under no patent protection). Under this case, the entrant, knowing that his location decision affects the incumbent's decision to invoke a trial, strategically chooses a location that will not be challenged by the incumbent. The possibility that an incumbent might patent an innovation, even though she would not legally enforce it, depends critically on the fact that the patent is potentially enforceable; it is this potential enforceability that induces the entrant to choose a location that will not be challenged (i.e., without this enforceability, the entrant would simply locate at his most preferred location).

The infringement and no trial outcome results in a situation where the incumbent chooses a patent breadth that induces the entrant to locate at the edge of the patent breadth. Similar behavior is found in the model of Yiannaka and Fulton (2006); however, litigation is not endogenous in their model and a trial always occurs upon infringement.

The rest of the paper is organized as follows. Section 2 describes the theoretical development of the patenting decisions model (i.e., the decision to patent, the patent breadth decision and the decision to invoke a trial under infringement), section three provides the analytical solution of the patenting game and section four concludes the paper.

## **2. The patenting decisions model**

### **2.1 Model assumptions**

Our model builds upon the model developed by Yiannaka and Fulton (2006) who study the optimal patent breadth decision when under infringement a trial always takes place. In addition to examining the innovator's optimal patent breadth decision, our model considers the innovator's decision to patent and her decision to invoke a trial when her patent is infringed. The patenting decisions are modeled in a sequential game of complete and perfect information between two agents; an incumbent innovator who has invented a patentable drastic product innovation and a potential entrant. At the beginning of the game the incumbent's product has already been

generated.<sup>4</sup> The incumbent decides whether to seek patent protection, how broad of a protection to claim and whether to defend her patent when infringement occurs; the entrant decides whether to enter the incumbent's market and, if entry occurs, where to locate in a vertically differentiated product space. To keep the focus on the innovator's patenting and patent breadth decisions we assume that the regulator (e.g., Patent Office) always grants the patent as claimed; thus, the regulator is not explicitly modeled.<sup>5</sup> It is further assumed that each of the agents produces at most one product for which no other substitute exists, consumers buy one unit of either the incumbent's or the entrant's product and the entrant does not patent his product since further entry is not considered. Both agents are risk neutral and maximize profits.

The incumbent's product is of quality  $q_p$  and provides consumers with utility

$U_p = V + \lambda q_p - p_p$ , where  $V$  is a base level of utility,  $\lambda$  is a differentiating consumer attribute uniformly distributed with unit density  $f(\lambda) = 1$  in the interval  $\lambda \in [0,1]$ , and  $p_p$  is the product's price. The entrant's production process is assumed to be deterministic and if the entrant enters his product is a superior product with quality  $q_e > q_p$ , that provides consumers with utility

$U_e = V + \lambda q_e - p_e$ , where  $p_e$  is the price of the entrant's product. It is assumed that  $V$  is large enough so that  $V \geq p_i \forall i = p, e$  and  $U_i \geq 0$  and  $U_i > U_j$  so the market is always served by at least one product.

To simplify the notation without affecting the qualitative nature of the model, the quality of the incumbent's product  $q_p$  is set equal to zero (i.e.,  $q_p = 0$ ). As a result, the entrant's quality  $q_e$  is interpreted as the difference in quality between his product and that of the incumbent, or more

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<sup>4</sup> Even though the innovator's R&D investment decision that led to the generation of the innovation is important, to keep the analysis tractable, this decision is not considered here.

<sup>5</sup> As in Yiannaka and Fulton (2006) who point to failures in the patent granting process and suggest that the innovator cannot always rely on the Patent Office for help in refining her patent claims, we examine the innovator's patenting behavior when she has no feedback from the Patent Office.

generally as the distance the entrant has located away from the incumbent. The above imply that  $q_e \in (0,1]$  – i.e., the maximum distance the entrant can locate away from the patentee is normalized to equal one.

The consumer who is indifferent between the two products has a  $\lambda$  value denoted by  $\lambda^*$ , where  $\lambda^*$  is determined as follows:  $V \geq p_i \forall i = p, e$

$$(1) \quad U_p = U_e \Rightarrow \lambda^* = \frac{p_e - p_p}{q_e}$$

Since each consumer consumes one unit of the product of her choice, the demand for the products produced by the incumbent and the entrant are given by  $y_p = \lambda^*$  and  $y_e = 1 - \lambda^*$ , respectively.

The incumbent's decision to patent the innovation implies patenting costs denoted by  $z$ , where  $z > 0$ , that are assumed to be independent of patent breadth. This assumption is in line with our assumption that the Patent Office always grants the patent as claimed. At the beginning of the game the incumbent's R&D costs, denoted by  $F_p$ , are sunk. The entrant's R&D costs of developing the higher quality product are given by  $F_e(q_e)$ , where  $F_e'(q_e) > 0$ ,  $F_e''(q_e) > 0$  and  $F_e(q_p) = 0$ . The above imply that it is increasingly costly for the entrant to locate away from the incumbent in the one-dimensional product space (i.e., to produce the better quality product) and the filing of a patent by the incumbent provides the entrant with knowledge of how to produce the incumbent's product (i.e., the assumption of perfect information disclosure by the patent is made). An important assumption of the model is that, in the absence of patent protection, reverse engineering of the product innovation is possible and costless. The formulation  $F_e = \beta \frac{q_e^2}{2}$  ( $\beta \geq \frac{4}{9}$ ) is used for the entrant's R&D costs; the restriction on the parameter  $\beta$  ensures that the quality chosen by the entrant,  $q_e$ , is bounded between zero and one. Once the R&D costs are incurred, production of the

products by both the incumbent and the entrant occur at zero marginal cost and neither the incumbent nor the entrant find it optimal to relocate once they have chosen their respective qualities (i.e., relocation is prohibitively costly).

The patent breadth is denoted by  $b$  where  $b \in (0,1]$ ; it determines the area in the one-dimensional product space that the patent protects. Patent breadth values close to zero indicate protection of the patented innovation only against duplication. It is assumed that patent claims define an exact border of protection – i.e., the patent system being modeled is a fencepost patent system where infringement will always be found when an entrant locates within the incumbent's claims, unless the entrant proves that the patent is invalid (Cornish 1989). The implication of assuming a fencepost patent system is that the probability that infringement is found (given that the entrant has located at  $q_e \leq b$  distance away from  $q_p$ ) does not depend on how close the entrant has located to the incumbent and it is equal to the probability that the validity of the patent will be upheld.<sup>6</sup> Thus, the fencepost patent system implies that the events that the patent is found to be infringed and that the patent is found to be invalid can be treated as mutually exclusive and exhaustive.

The probability  $\mu(b)$  that the patent will be found to be valid, or equivalently that infringement will be found, is assumed to be inversely related to patent breadth – i.e.,  $\mu'(b) < 0$ . This assumption is based on evidence from the literature that shows that, the broader is the patent protection, the harder it is to establish validity since the harder it is to show novelty, nonobviousness and enablement (Cornish 1989, Miller and Davis 1990). In addition, courts tend to uphold narrow patents and invalidate broad ones (Waterson 1990, Cornish 1989, Merges and

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<sup>6</sup> Under a signpost patent system where the patent claims are interpreted using the doctrines of equivalents and reverse equivalents, claims provide an indication of protection and the entrant's location when he infringes the patent becomes important in determining whether infringement will be found. Thus, under a signpost system, the closer the entrant locates to the incumbent the easier it is to prove infringement using the doctrine of equivalents while infringement may be found even when the entrant locates outside the incumbent's claims using the doctrine of reverse equivalents.

Nelson 1990).<sup>7</sup> Given the above, the probability that the patent will be found valid is given by  $\mu(b) = 1 - \alpha b$ . The validity parameter  $\alpha$ ,  $\alpha \in (0, 1)$ , reflects the degree that patent breadth affects patent validity. For any given patent breadth, the greater is the validity parameter  $\alpha$ , the greater is the probability that the patent will be found invalid.

When the entrant locates at a distance  $q_e \leq b$  away from  $q_p$  the patent is infringed and the incumbent must decide whether to invoke an infringement trial or not. It is assumed that the filing of an infringement lawsuit by the incumbent is always met with a counterclaim by the accused infringer that the patent is invalid – a common defense of accused infringers (Cornish 1989, Merges and Nelson 1990). Note that given our assumption of perfect information, the incumbent costlessly identifies infringement as soon as it occurs. This further implies that the incumbent suffers no losses in profits due to infringement and thus the case where the courts award infringement damages to the incumbent is not considered. The legal costs incurred during the infringement trial/validity attack by the incumbent and the entrant are denoted by  $C_p^T$  and  $C_e^T$ , respectively, and are assumed to be sunk and independent of the breadth of protection and of the entrant's location.<sup>8</sup> Finally, to keep the analysis tractable and the focus on the interplay between the decision to patent, the patent breadth decision and the decision to invoke a trial under infringement, our model does not consider the possibility of settlement or licensing.

The patent breadth game consists of five stages. In the first stage of the game, the incumbent decides whether to seek patent protection or not. If the incumbent decides not to patent her

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<sup>7</sup> Note that, since further entry is not anticipated in our model our analysis and results are not affected by whether the entire patent is invalidated during the infringement/validity trial or only certain claims are found to be invalid (i.e., the patent breadth is narrowed).

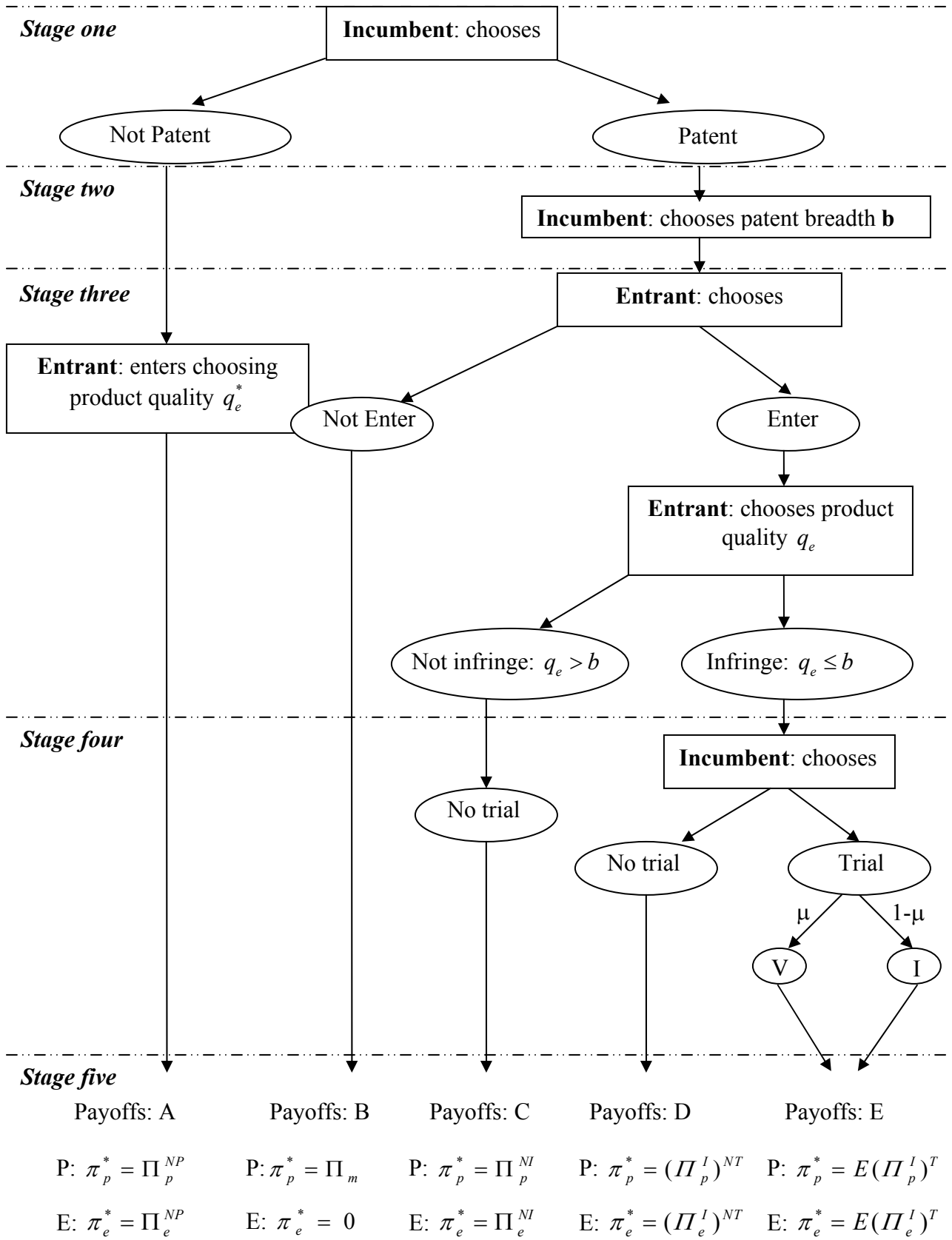
<sup>8</sup> By assuming that legal costs are sunk we exclude the possibility of the courts awarding legal fees to either party. In some cases, if infringement is found to be wilful, the court may require that the infringer pays damages up to three times greater than the actual losses due to infringement, opponent's legal costs and court costs (Lerner 1995, Crampes and Langinier 2002). To keep the analysis simple, the possibility of wilful infringement is not examined. Note that, given our assumption of perfect information, the entrant knows in our model whether he has infringed the patent or not (i.e., whether he has located within the incumbent's claims), and thus, the assumption that infringement is not wilful implies that when the entrant infringes the patent he believes that the patent is invalid and thus not infringed.

innovation then, given the assumption of possible and costless reverse engineering, the entrant enters at his most preferred location and he and the incumbent compete in prices at the last stage of the game and earn duopoly profits  $\Pi_e^{NP}$  and  $\Pi_p^{NP}$ , respectively. If the incumbent decides to patent her innovation then at the second stage of the game she decides on the patent breadth,  $b$ , claimed. In the third stage of the game, a potential entrant observes the incumbent's product and the breadth of protection granted to it and chooses whether or not to enter the market. If the entrant does not enter he earns zero profits while the incumbent operates as a monopolist in the last stage of the game and earns monopoly profits  $\Pi_m$ . If the entrant enters, he does so by choosing the quality  $q_e$  of his product relative to that of the incumbent. This decision determines whether the entrant infringes the patent or not, as well as whether the incumbent will invoke a trial in the case the patent is infringed.

If the entrant chooses a quality greater than the patent breadth claimed by the incumbent (i.e.,  $q_e > b$ ), then no infringement occurs, and he and the incumbent compete in prices in the last stage of the game and earn duopoly profits  $\Pi_e^{NI}$  and  $\Pi_p^{NI}$ , respectively. If the entrant locates inside the patent breadth claimed by the incumbent (i.e.,  $q_e \leq b$ ), the patent is infringed and the incumbent needs to decide whether to invoke a trial or not. This decision is made in the fourth stage of the game. The payoffs for the incumbent and the entrant when the entrant chooses  $q_e \leq b$  and the incumbent chooses not to invoke a trial are  $(\Pi_p^I)^{NT}$  and  $(\Pi_e^I)^{NT}$ , respectively. If the incumbent invokes a trial then the validity of the patent is examined. With probability  $\mu(b)$ , the patent is found to be valid (i.e., infringement is found), the entrant is not allowed to market his product and the incumbent operates as a monopolist in the last stage of the game (this follows from our assumption that relocation is prohibitively costly). With probability  $1 - \mu(b)$ , the patent is found to be invalid, and the entrant and the incumbent compete in prices. The payoffs for the incumbent and the entrant

when the entrant chooses  $q_e \leq b$  and the incumbent invokes a trial are  $E(\Pi_p^I)^T$  and  $E(\Pi_e^I)^T$ , respectively. Figure 1 illustrates the extensive form of the game outlined above.

The solution to this game is found by backward induction. The fifth stage of the game in which the incumbent and the entrant – when applicable – compete in prices is examined first, followed by the fourth stage in which the incumbent makes her trial decision, the third stage in which the entrant makes his entry decision, the second stage in which the incumbent makes her decision regarding patent breadth and finally the first stage in which the incumbent decides whether to patent her innovation or not.



**Figure 1.** The patenting game

### 3. The solution of the patenting game

#### 3.1 Stage 5 – The pricing decisions

In the fifth stage of the game, two cases must be considered depending on whether the entrant chooses to enter or to not enter the market. In the absence of entry, the incumbent will charge

$p_p = V$  and earn monopoly profits  $\Pi_m = V - F_p$ . If entry occurs, the problem facing duopolist  $i$  is

to choose price  $p_i$  to maximize profit  $\pi_i = p_i y_i$  ( $i = p, e$ ), where  $y_p = \frac{P_e - P_p}{q_e}$  and

$y_e = \frac{q_e + P_p - P_e}{q_e}$ . Recall that the R&D costs,  $F_p$  and  $F_e$  for the incumbent and the entrant,

respectively, are assumed to be sunk at this stage in the game and thus are not included in the profit expression. The Nash equilibrium in prices, as well as the resulting outputs and profits, are given by:

$$(2) \quad \text{Incumbent:} \quad p_p^* = \frac{q_e}{3}, \quad y_p^* = \frac{1}{3}, \quad \pi_p^* = \frac{q_e}{9}$$

$$(3) \quad \text{Entrant:} \quad p_e^* = \frac{2q_e}{3}, \quad y_e^* = \frac{2}{3}, \quad \pi_e^* = \frac{4q_e}{9}$$

The entrant has the higher quality product and is able to charge the higher price. Profits for both the incumbent and the entrant are increasing in the quality chosen by the entrant,  $q_e$ , as the greater is the difference in quality between the two products, the less intense is competition at the final stage of the game.<sup>9</sup>

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<sup>9</sup> This is a well-established result in the product differentiation literature in simultaneous games; when competitors first simultaneously choose their locations in the product space and then compete in prices they choose maximum differentiation to relax competition in the pricing stage that would curtail their profits (Lane 1980, Motta 1993, Shaked and Sutton 1982).

### 3.2 Stage 4 – The incumbent’s trial decision

As illustrated in Figure 1, under patenting, the entrant’s location decision (his quality choice  $q_e$ ) will determine whether the patent will be infringed and whether in the case of infringement a trial will take place. When the entrant infringes the patent, the incumbent needs to decide whether to invoke an infringement trial or not. Given the quality chosen by the entrant, the incumbent will invoke a trial when the patent is infringed as long as her expected profits when a trial takes place,  $E(\Pi_p^I)^T$ , are greater than her profits when a trial does not take place,  $(\Pi_p^I)^{NT}$ , i.e.,

$E(\Pi_p^I)^T > (\Pi_p^I)^{NT}$ . When the incumbent invokes a trial her expected profits are given by:

$$(4) \quad E(\Pi_p^I)^T = \mu\Pi_m + (1 - \mu)\pi_p - C_p^T = (1 - ab)\Pi_m + ab\frac{q_e}{9} - C_p^T$$

Equation (4) demonstrates that at trial infringement will be found (or equivalently the validity of the patent will be upheld) with probability  $\mu$ , the entrant will not be allowed in the market and the incumbent will have a monopoly position. Conversely, with probability  $1 - \mu$ , infringement will not be found, the entrant will be allowed to market his product and the incumbent and the entrant will operate as duopolists.

When the incumbent does not invoke a trial her profits are given by:

$$(5) \quad (\Pi_p^I)^{NT} = \pi_p^* = \frac{q_e}{9}$$

Equation (5) shows that when the incumbent does not invoke a trial when infringement occurs she shares the market with the entrant and realizes duopoly profits which depend on the entrant’s choice of location in the quality product space.

Given the above the incumbent will invoke a trial when her patent is infringed if:

$$(6) \quad E(\Pi_p^I)^T > (\Pi_p^I)^{NT} \Rightarrow q_e < 9\left(\Pi_m - \frac{C_p^T}{1 - ab}\right)$$

Equation (6) shows that the incumbent's decision on whether to invoke a trial when her patent is infringed is affected by the entrant's location decision. We denote the quality that makes the incumbent indifferent between invoking and not invoking a trial by  $\bar{q}_e$ , i.e.,  $\bar{q}_e = 9(\Pi_m - \frac{C_p^T}{1-ab})$ ,  $\bar{q}_e \in (0, I)$  and assume that when the incumbent is indifferent she will choose to not invoke a trial. Since infringement occurs when  $q_e \leq b$ ,  $\bar{q}_e$  is defined for patent breadth values such that  $\bar{q}_e \leq b$ .

**Definition 1.** Let  $\bar{b} \in (0, I)$  be the patent breadth that satisfies the condition  $\bar{q}_e = b$ . The patent breadth  $\bar{b} \in (0, I)$  is equal to the maximum quality that, when chosen by the entrant, infringes the patent and makes the incumbent indifferent between invoking and not invoking a trial.

It can be easily shown that  $\bar{b} = \frac{1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}$  is the patent breadth that satisfies the  $\bar{q}_e = b$  condition (see Appendix A<sub>1</sub> for a proof).

Given the above,  $\bar{q}_e \in (0, I)$  is defined for patent breadth values in the interval  $b \in [\bar{b}, I]$  and is decreasing in patent breadth at an increasing rate, i.e.,  $\frac{\partial \bar{q}_e}{\partial b} < 0$ ,  $\frac{\partial^2 \bar{q}_e}{\partial b^2} < 0$ . Thus, the greater is the patent breadth chosen, the smaller can be the quality chosen by the entrant that will infringe the patent without invoking a trial. Figure 2 below illustrates the relationship between the quality chosen by the entrant,  $q_e$ , and the incumbent's decision to invoke a trial for any given patent breadth choice,  $b$ .

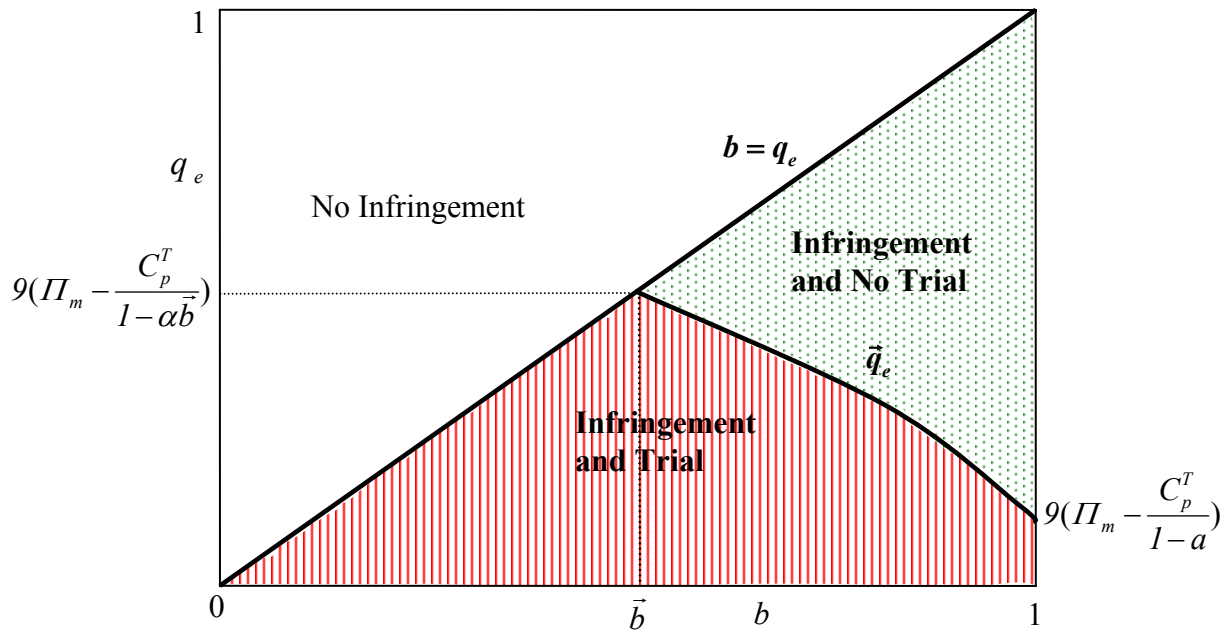


Figure 2. The incumbent's trial decision

As depicted in Figure 2, as long as the entrant chooses a product quality  $q_e > b$  the patent is not infringed. When the entrant chooses a product quality  $q_e$  such that  $\bar{q}_e < q_e \leq b$  (i.e., a quality to the right of locus  $\bar{q}_e$  and below the locus  $b = q_e$ ) the patent will be infringed but the incumbent will not invoke a trial. This outcome is depicted by the dotted area in Figure 2. When the entrant chooses a product quality  $q_e$  such that  $q_e \leq b$  and  $q_e < \bar{q}_e$  (i.e., a quality to the left of locus  $\bar{q}_e$  and below the locus  $b = q_e$ ) the patent will be infringed and the incumbent will invoke a trial. This outcome is depicted by the horizontally hatched area in Figure 2. As the monopoly profits that can be earned by the incumbent (under no market entry or when the patent is infringed and its validity is upheld during trial) increase, the locus  $\bar{q}_e$  shifts upward and the more likely it becomes that a trial will take place under infringement (the infringement and trial area becomes larger). As the incumbent's trial costs increase, the locus  $\bar{q}_e$  shifts downward and the less likely it is that the incumbent will find it optimal to invoke a trial under infringement (the infringement and trial area becomes smaller).

It is important to point out that when  $\bar{b} \rightarrow 1$ , the locus  $\bar{q}_e$  is above the locus  $q_e = b$   $\forall q_e \in (0, 1)$  and  $b \in (0, 1]$  and  $q_e \leq \bar{q}_e \forall q_e \in (0, 1]$  which imply that invoking a trial when the patent is infringed ( $q_e \leq b$ ) is always an optimal strategy for the incumbent, regardless of the quality chosen by the entrant.<sup>10</sup> This case emerges when  $\Pi_m \geq \frac{1}{9} + \frac{C_p^T}{1-\alpha}$ . Also note that, when  $\bar{b} \rightarrow 0$  the locus  $\bar{q}_e$  is below the locus  $q_e = b \forall q_e \in (0, 1)$  and  $b \in (0, 1]$  and  $q_e > \bar{q}_e \forall q_e \in (0, 1]$  which imply that invoking a trial when the patent is infringed ( $q_e \leq b$ ) is never an optimal strategy for the incumbent, regardless of the quality chosen by the entrant; this case emerges when  $\Pi_m \leq C_p^T$ . In this latter case, however, it is straightforward to show that, as long as the patenting costs are positive, the incumbent will not have an incentive to take a patent. This is so because, given our model assumptions of complete and perfect information and costless and possible reverse engineering, if the entrant knows that regardless of his quality choice a trial will never take place, he will always find it optimal to locate at his most preferred location,  $q_e^*$ , (where he locates under no patent protection) regardless of the patent breadth chosen.<sup>11</sup>

**Result 1.** *A  $\bar{q}_e \in [\bar{b}, 1)$  that, when chosen by the entrant, infringes the patent and makes the incumbent indifferent between invoking and not invoking an infringement trial exists when the*

*condition  $C_p^T < \Pi_m < \frac{1}{9} + \frac{C_p^T}{1-\alpha}$  is satisfied.*

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<sup>10</sup> The case where under infringement a trial always occurs regardless of the entrant's product quality choice has been examined by Yiannaka and Fulton (2006) and is not considered here.

<sup>11</sup> Note that this is not necessarily true when reverse engineering is possible and costly because the entrant's optimal location choice  $q_e^*$  will be different under patenting where the information about the incumbent's product is public knowledge and under no patenting where the entrant has to incur a cost to obtain this information.

### 3.3 Stage 3 – The entrant’s location decision

As illustrated in Figure 1, two cases must be considered regarding the entrant’s location decision depending on whether the incumbent has patented her innovation or not. The latter case is considered first.

#### 3.3.1 No patent protection

With no patent protection the entrant can freely locate at any point in the quality product space. Given our assumption of possible and costless reverse engineering, the entrant cannot be deterred from entering the market under no patent protection; at the very least, the entrant can locate at  $q_e = q_p = 0$ , share the market with the incumbent and realize zero profits. Let  $q_e^*$  be the optimal quality the entrant chooses under no patent protection, where  $q_e^*$  solves the following problem:

$$(7) \quad \max_{q_e} \Pi_e = \pi_e - F_e = \frac{4q_e}{9} - \beta \frac{q_e^2}{2}$$

Optimization of equation (7) yields the optimal quality  $q_e^*$

$$(8) \quad q_e^* = \frac{4}{9\beta}$$

Equation (8) gives the entrant’s most preferred location and indicates that the less costly it is to produce the better quality product (i.e., the smaller is  $\beta$ ), the further away from the incumbent the entrant locates.

The relationship between the entrant’s most preferred location,  $q_e^*$ , and patent breadth  $\bar{b}$  determines the incumbent’s optimal patenting strategy as discussed in the following proposition and corollary.

**Proposition 1.** *The incumbent will never find it optimal to seek patent protection when she cannot use patent breadth to influence the entrant’s location decision, i.e., when  $q_e^* > \bar{b}$ .*

**Proof.** When  $q_e^* > \bar{b}$  the entrant will always choose to locate at her most preferred location,  $q_e^*$ , and the incumbent will not invoke a trial when the patent is infringed. However, knowing that when  $q_e^* > \bar{b}$ , regardless of the patent breadth chosen, she won't be able to enforce/defend her patent rights, the incumbent will not seek patent protection. Thus, for positive patenting costs, when  $q_e^* > \bar{b}$  a patent will not be sought by the incumbent.

It is straightforward to show that the condition  $q_e^* > \bar{b}$  is satisfied for R&D cost values,  $\beta$ ,

such that 
$$\beta < \frac{8\alpha}{9(1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})} = \beta_0 \text{ where } \beta_0 > \frac{4}{9}$$

$\forall \alpha \in (0,1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha})$  (see Appendix A<sub>2</sub>). Corollary 1 summarizes the

relationship between the parameters that give rise to the no patent protection condition,  $\beta < \beta_0$ .

**Corollary 1.** *The greater is the entrant's R&D effectiveness (i.e., the smaller is  $\beta$ ), the validity parameter,  $\alpha$ , and the incumbent's trial costs,  $C_p^T$ , and the smaller are the monopoly profits,  $\Pi_m$ , the more likely it is that the incumbent will not find it optimal to seek patent protection.*

The proof for corollary 1 is given in Appendix A<sub>2</sub>.

The intuition behind the results in corollary 1 is as follows. The greater is the entrant's R&D effectiveness (i.e., the smaller is  $\beta$ ), the further away from the incumbent the entrant finds it optimal to locate (see equation (8)) while the greater is the validity parameter and the incumbent's trial costs, the harder it becomes for the incumbent to defend her patent under infringement. Finally, the smaller are the incumbent's monopoly profits, the smaller is the incumbent's incentive to patent since these profits can be realized only under patent protection (either when entry can be deterred or when infringement is found during an infringement trial).

### 3.3.2 Patent protection ( $q_e^* \leq \bar{b}$ )

Given the result in proposition 1, a necessary condition for patent protection to be an optimal strategy for the incumbent is that  $q_e^* \leq \bar{b}$ . The condition  $q_e^* \leq \bar{b}$  is satisfied for R&D cost values,  $\beta$ , such that  $\beta \geq \beta_0$ . Figure 3 below illustrates the combinations of values of the entrant's R&D effectiveness,  $\beta$ , and the monopoly profits that can be captured by the incumbent,  $\Pi_m$ , for which patent protection will and will not be sought.

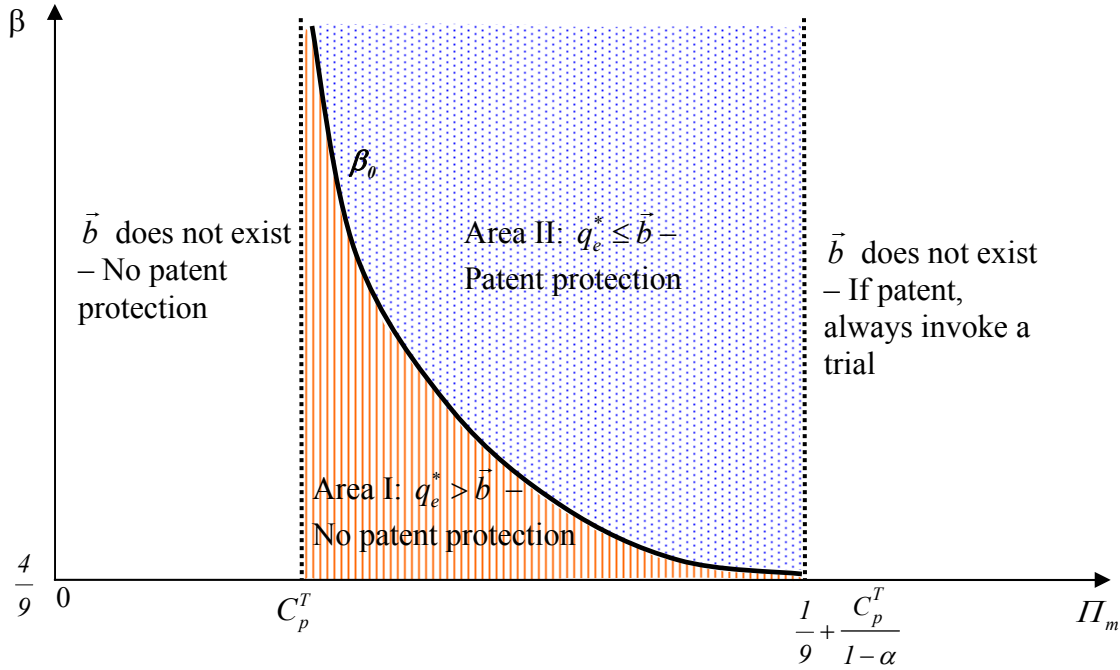


Figure 3. Combinations of  $\beta$  and  $\Pi_m$  values for which patent protection will and will not be sought.

As depicted in Figure 3, when  $\Pi_m \rightarrow C_p^T$ ,  $\beta_0 \rightarrow \infty$  while when  $\Pi_m \rightarrow \frac{1}{9} + \frac{C_p^T}{1-\alpha}$ ,

$\beta_0 \rightarrow \frac{4}{9} \forall \alpha \in (0,1)$  and  $C_p^T \geq 0$ ; thus,  $\beta_0 > \frac{4}{9} \forall \alpha \in (0,1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha})$ . For

combinations of  $\beta$  and  $\Pi_m$  values to the right of locus  $C_p^T$  and below the locus  $\beta_0$  (Area I in

Figure 3),  $\bar{b}$  exists,  $q_e^* > \bar{b}$  and the incumbent will not find it optimal to seek patent protection. For

combinations of  $\beta$  and  $\Pi_m$  values to the left of locus  $\frac{1}{9} + \frac{C_p^T}{1-\alpha}$  and to the right of locus  $\beta_0$  (Area

II in Figure 3),  $\bar{b}$  exists,  $q_e^* \leq \bar{b}$ , the incumbent might find it optimal to seek patent protection and the entrant's location decision will determine whether the incumbent will find it optimal to invoke a trial in the case of infringement.<sup>12</sup> Note that Figure 3 provides an illustration of corollary 1. For

instance, as the validity parameter,  $\alpha$ , increases, the locus  $\frac{1}{9} + \frac{C_p^T}{1-\alpha}$  shifts to the right, the locus  $\beta_0$

shifts upwards and the no patent protection area becomes larger.

Under patent protection and anticipating the incumbent's behavior concerning trial given  $q_e$ , the entrant must choose one of four options – Not Enter; Enter and Not Infringe the Patent; Enter, Infringe the Patent and Induce a Trial; or Enter, Infringe the Patent and Not Induce a Trial. For any given patent breadth,  $b$ , the entrant will choose the option that generates the greatest profit.

The outcome of the Not Enter option is straightforward – the entrant earns zero profits. The outcomes of the other three options depend on a number of factors, including patent breadth, R&D costs and trial costs. The benefits and costs associated with the Enter and Not Infringe option are examined first, followed by an examination of the benefits and costs associated with the Enter and Infringe option. The examination of the Enter and Infringe option consists of the examination of the Enter, Infringe and Not Induce a Trial and the Enter, Infringe and Induce a Trial options. Once the

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<sup>12</sup> Recall, from the analysis of the incumbent's trial decision, that when the monopoly profits are smaller than or equal to the incumbent's trial costs,  $\bar{q}_e$  (and thus  $\bar{b}$ ) does not exist; the incumbent never finds it optimal to invoke a trial under infringement and she, thus, never finds it optimal to seek patent protection. This case emerges for  $\beta$  and  $\Pi_m$  values to the left of locus  $C_p^T$  in Figure 3. When the monopoly profits are greater than or equal to  $\frac{1}{9} + \frac{C_p^T}{1-\alpha}$ ,  $\bar{b}$  does not exist; the incumbent might find it optimal to seek patent protection, in which case, she always finds it optimal to invoke a trial under infringement. This case emerges for  $\beta$  and  $\Pi_m$  values to the right of locus  $\frac{1}{9} + \frac{C_p^T}{1-\alpha}$  in Figure 3.

net benefits of each option are formulated, the most desirable option for the entrant is determined for any given patent breadth.

### 3.3.2.1 Entry with no infringement ( $q_e > b$ )

When the entrant wishes to enter without infringing the patent, he must choose a quality that is greater than the patent breadth – i.e.,  $q_e > b$ . Note that patent breadth will only be binding as long as  $q_e^* \leq b$ ; when  $q_e^* > b$ , patent breadth does not affect the location chosen by the entrant since the entrant can choose his optimal quality without fear of infringement.<sup>13</sup> Since  $q_e^*$  is the entrant's optimal quality, an increase in quality beyond  $q_e^*$  results in a reduction in profits. Thus, when the entrant wishes to enter and not infringe the patent he will choose his quality  $q_e^{NI}$  as follows:

$$(9) \quad q_e^{NI} = \begin{cases} q_e^* & \text{if } q_e^* > b \\ b+e & \text{if } q_e^* \leq b \end{cases} \quad \text{where } e \rightarrow 0$$

This quality choice yields profits of:

$$(10) \quad \Pi_e^{NI} = \begin{cases} \frac{4q_e^*}{9} & \text{if } q_e^* > b \\ \frac{4}{9}(b+e) - \frac{\beta}{2}(b+e)^2 & \text{if } q_e^* \leq b \end{cases}$$

**Result 2.** *When the entrant faces a binding patent breadth and he wishes to not infringe the patent, the entrant's profits under entry and no infringement are decreasing in  $q_e^{NI}$  at an increasing rate*

*for all  $q_e > q_e^*$ , i.e.,  $\frac{\partial \Pi_e^{NI}}{\partial b} < 0$ ,  $\frac{\partial^2 \Pi_e^{NI}}{\partial b^2} < 0$ .*

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<sup>13</sup> Note that, since patenting occurs only when  $\beta \geq \beta_0 > \frac{4}{9}$ , when  $q_e^* = 1$  (which occurs when the entrant's R&D costs are minimum, i.e.,  $\beta = \frac{4}{9}$ ), the incumbent will not seek patent protection.

### 3.3.2.2 *Entry with infringement ( $q_e \leq b$ )*

When the entrant decides to enter and infringe the patent he must determine whether to induce the incumbent to invoke a trial or not. The entrant's optimal strategy depends on which of the above two options generates greater profits. The entrant's profits under infringement and trial are determined below followed by an examination of the entrant's profits under infringement and no trial.

#### *The Entrant's Profits under Infringement and Trial*

Recall that during an infringement trial there is a probability  $\mu = 1 - \alpha b$  that infringement will be found (i.e., the validity of the patent will be upheld) and a probability  $1 - \mu = \alpha b$  that infringement will not be found (i.e., the patent will be revoked). If infringement is found during trial, the entrant is not allowed to market his product and the patentee earns monopoly profits. If infringement is not found during trial, the entrant is allowed to market his product and the patentee and the entrant operate as duopolists. The optimal quality chosen by the entrant under infringement and trial is determined by solving:

$$(11) \quad \max_{q_e} E(\Pi_e^I)^T = (1 - \mu) \cdot \pi_e - F_e - C_e^T = \alpha b \frac{4q_e}{9} - \beta \frac{q_e^2}{2} - C_e^T$$

and is given by:

$$(12) \quad (q_e^I)^T = \frac{4\alpha b}{9\beta}$$

This quality choice<sup>14</sup> yields profits of:

$$(13) \quad E(\Pi_e^I)^T = \frac{8\alpha^2 b^2}{81\beta} - C_e^T$$

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<sup>14</sup> Note that, the optimal quality under infringement and trial satisfies the condition  $(q_e^I)^T < b \Rightarrow \alpha < \frac{9}{4}\beta \quad \forall \alpha \in (0,1)$

and  $\beta > \beta_0 > \frac{4}{9}$ .

**Result 3.** *When the entrant infringes the patent knowing that he will face an infringement trial, he finds it optimal to locate at a distance proportional to the breadth of the patent and his most preferred location (i.e.,  $(q_e^I)^T = \alpha b q_e^*$ ).*

Under infringement and trial there is uncertainty as to whether the entrant will be able to continue in the market; thus, the entrant ‘underlocates’ to reduce the R&D costs, which are incurred with certainty.

**Result 4.** *The entrant’s profits under infringement and trial are increasing in patent breadth,  $b$ , at*

*an increasing rate, i.e.,  $\frac{\partial E(\Pi_e^I)^T}{\partial b} > 0$ ,  $\frac{\partial^2 E(\Pi_e^I)^T}{\partial b^2} > 0$ .*

Result 4 follows from the fact that, the greater is patent breadth,  $b$ , the greater is the probability that infringement will not be found at trial (i.e., that the patent will be found invalid and will be revoked).

*The Entrant’s Profits under Infringement and No Trial*

This case considers the situation where the choice of the entrant’s most preferred quality  $q_e^*$  results in patent infringement and trial and the entrant wishes to infringe but not induce a trial. In this case, which can occur only for patent breadth values  $b \in [\bar{b}, I]$  (see Figure 2), the entrant would maximize profits by choosing a quality  $(q_e^I)^{NT}$  that is the closest possible to his most preferred quality  $q_e^*$  and ensures that the incumbent does not invoke a trial. Thus, to maximize his profits under the infringement and no trial outcome, the entrant will choose the quality  $(q_e^I)^{NT} = \bar{q}_e$  (we assume that when the incumbent is indifferent between invoking and not invoking a trial she will choose to not invoke a trial).

Obviously,  $(q_e^I)^{NT} = \bar{q}_e$  is the quality that maximizes the entrant's profits under infringement and no trial when  $q_e^* \leq \bar{q}_e$ . If  $q_e^* > \bar{q}_e$  (i.e., if the loci  $q_e^*$  and  $\bar{q}_e$  cross for  $b \in [\bar{b}, I]$ ), there exists a patent breadth  $\bar{b} \in (\bar{b}, I]$  such that  $\bar{b} : q_e^* = \bar{q}_e \Rightarrow \bar{b} = \frac{81\beta\Pi_m - 81C_p^T\beta - 4}{\alpha(81\beta\Pi_m - 4)}$  and for patent breadth values  $b \in [\bar{b}, I]$  the optimal quality chosen by the entrant under infringement and no trial is given by  $(q_e^I)^{NT} = q_e^*$ , since the incumbent does not find it optimal to invoke a trial (see Figure 4 below).

Given the above, the entrant's profits under infringement and no trial are given by:

$$(14) \quad (\Pi_e^I)^{NT} = \begin{cases} \frac{4q_e^*}{9} & \text{if } q_e^* > \bar{q}_e \\ \frac{4}{9}\bar{q}_e - \frac{\beta}{2}\bar{q}_e^2 & \text{if } q_e^* \leq \bar{q}_e \end{cases}$$

**Result 5.** *The entrant's profits under infringement and no trial when  $q_e^* \leq \bar{q}_e$  are increasing in*

*patent breadth,  $b$ , at a decreasing rate, i.e.,  $\frac{\partial(\Pi_e^I)^{NT}}{\partial b} > 0$ ,  $\frac{\partial^2(\Pi_e^I)^{NT}}{\partial b^2} < 0$ .*

The intuition behind result 5 is as follows. Recall that  $\bar{q}_e$  is decreasing in  $b$  at an increasing rate and at  $\bar{b}$ ,  $\bar{q}_e = \bar{b}$ . Thus, as  $b \in [\bar{b}, I]$  increases,  $\bar{q}_e$  becomes smaller, the closer to his most preferred location,  $q_e^*$ , the entrant can locate without inducing the patentee to invoke a trial, and the greater are the entrant's profits under infringement and no trial.

The entrant's quality choices for  $q_e^* \leq b$  and his profits under no infringement, infringement and trial and infringement and no trial are depicted in Figure 4. As illustrated in Figure 4, the entrant's optimal quality choice depends on the patent breadth chosen by the incumbent. Thus, as long as the incumbent chooses a patent breadth that is not binding (i.e., a  $b \in (0, b_0]$  where

$b_0 \in (0, \bar{b})$  and  $b_0 = q_e^* = \frac{4}{9\beta}$ ) or a patent breadth for which the optimal strategy is to not invoke a

trial under infringement (i.e., a  $b \in [\bar{b}, I]$ ), the entrant will always find it optimal to enter the market and locate at his most preferred location,  $q_e^*$ , without invoking a trial.<sup>15</sup> When the patent breadth chosen is such that  $b \in (b_0, \bar{b})$ , the entrant cannot locate at his most preferred location,  $q_e^*$ , without infringing the patent while the incumbent will always find it profitable to invoke a trial when the patent is infringed. In this case, the entrant will have to decide whether to enter and if entry occurs whether to infringe or not infringe the patent knowing that if he infringes a trial will always take place. Finally, when the patent breadth chosen is such that  $b \in [\bar{b}, \bar{b})$ , the entrant cannot locate at his most preferred location,  $q_e^*$ , without infringing the patent but he can, by his choice of location,  $q_e$ , in the quality product space affect whether the incumbent will invoke a trial or not when the patent is infringed.

For the profit curves depicted in Figure 4, if the incumbent chooses patent breadth  $b_1$  the entrant will find it optimal to choose the product quality,  $(q_e)^{NT} = b_1 + e$  that does not infringe the patent; if the incumbent chooses patent breadth  $b_2$  the entrant will find it optimal to choose the product quality  $(q_e^I)^T = \alpha b_2 q_e^*$ , that infringes the patent and induces the incumbent to invoke a trial while if the incumbent chooses patent breadth  $b_3$  the entrant will find it optimal to choose the product quality  $(q_e^I)^{NT} = \bar{q}_e(b_3)$ , that infringes the patent and induces the incumbent to not invoke a trial.

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<sup>15</sup> While the patent breadth  $b_0 \in (0, \bar{b})$  always exists when  $q_e^* \leq b$ , the patent breadth  $\bar{b} \in (\bar{b}, I]$  exists only when the loci  $q_e^*$  and  $\bar{q}_e$  cross. Note that, although Figure 4 depicts the case where  $\bar{b}$  exists, as will become evident below, the existence of  $\bar{b}$  is not necessary for our results.

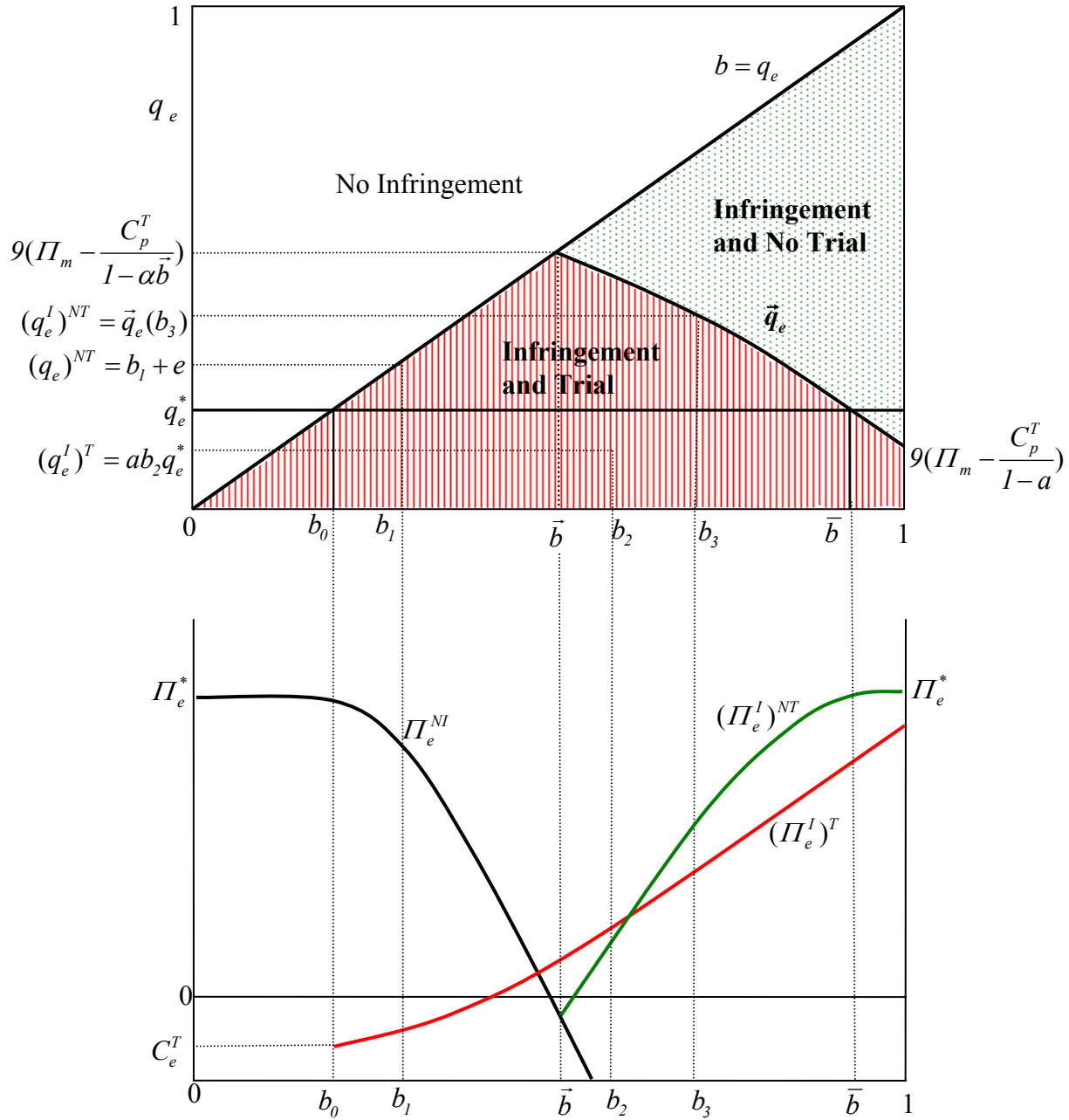


Figure 4. The entrant's quality choice and profits for  $q_e^* \leq \bar{b}$  and under no infringement, infringement and trial and infringement and no trial.

Note that, when the incumbent chooses the patent breadth  $\bar{b}$ , the entrant's profits under no infringement (where the entrant choose  $q_e^{NI} = \bar{b} + e$ ) and under infringement and no trial (where the entrant chooses  $(q_e^I)^{NT} = \bar{q}_e = \bar{b}$ ) are equal at the limit, i.e.,  $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b}) = (\Pi_e^I)^{NT}(\bar{b})$ .

**Result 6.** *When a patent breadth  $b \in [\bar{b}, I]$  is chosen by the incumbent, the entrant will never choose to not infringe the patent since the non infringement strategy is always dominated by the infringement and no trial strategy.*

The intuition behind result 6 is straightforward. As shown in result 2 and result 5, the entrant's profits under no infringement are decreasing in patent breadth while his profits under infringement

and no trial are increasing in patent breadth ( $\frac{\partial \Pi_e^{NI}}{\partial b} < 0$  and  $\frac{\partial (\Pi_e^I)^{NT}}{\partial b} > 0$ ), respectively. Since at

$$\bar{b} \lim_{\epsilon \rightarrow 0} \Pi_e^{NI}(\bar{b}) = (\Pi_e^I)^{NT}(\bar{b}) \text{ for any } b \in (\bar{b}, I], \Pi_e^{NI}(b) < (\Pi_e^I)^{NT}(b).$$

### **The entry/infringement decision**

Given that the entrant's quality choice depends on the incumbent's patent breadth decision, before we are able to determine the entrant's optimal strategy we must first examine whether there exist some critical patent breadth values that when chosen by the incumbent make the entrant indifferent between the alternative strategies that are available to him.

**Definition 2.** *Define,  $\tilde{b}$ , as the patent breadth that makes the entrant indifferent between not infringing the patent and infringing the patent and inducing a trial – i.e.,  $\tilde{b}$  solves*

$$\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b}) \text{ where } \tilde{b} \in (b_0, \bar{b}] \text{ when } \bar{b} \in (\bar{b}, I] \text{ exists and } \tilde{b} \in (b_0, I] \text{ when } \bar{b} \in (\bar{b}, I] \text{ does not exist.}$$

**Definition 3.** *Define,  $\tilde{\tilde{b}}$ , as the patent breadth that makes the entrant indifferent between infringing the patent and inducing a trial and infringing the patent and not inducing a trial – i.e.,  $\tilde{\tilde{b}}$  solves*

$$E(\Pi_e^I)^T(\tilde{\tilde{b}}) = (\Pi_e^I)^{NT}(\tilde{\tilde{b}}) \text{ where } \tilde{\tilde{b}} \in (\bar{b}, \bar{b}) \text{ when } \bar{b} \in (\bar{b}, I] \text{ exists and } \tilde{\tilde{b}} \in (\bar{b}, I] \text{ when } \bar{b} \in (\bar{b}, I] \text{ does not exist.}$$

The conditions under which  $\tilde{b}$  and  $\tilde{\tilde{b}}$  exist are derived in Appendix A<sub>3</sub>. We assume that when the entrant is indifferent between no infringement and infringement and trial he chooses to not infringe the patent while when he is indifferent between infringement and trial and infringement and no trial he choose to infringe and not induce a trial.

Scenario A: Entry deterrence

The entrant will not find it profitable to enter the market if there exists a patent breadth value  $\hat{b} \in (b_0, I]$  that when chosen by the incumbent makes the entrant's profits under no infringement, his expected profits under infringement and trial and his profits under infringement and no trial less than or equal to zero, i.e.,  $\Pi_e^{NI}(\hat{b}) \leq 0 \wedge E(\Pi_e^I)^T(\hat{b}) \leq 0 \wedge (\Pi_e^I)^{NT}(\hat{b}) \leq 0$ .

**Lemma 1.** *A patent breadth,  $\hat{b}$ , that deters entry always exists when at  $\bar{b}$*

$$\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) \leq 0 \wedge E(\Pi_e^I)^T(\bar{b}) \leq 0.$$

Note that, since at  $\bar{b}$   $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b})$  and from result 2, 4 and 5 we know that

$$\frac{\partial \Pi_e^{NI}}{\partial b} < 0, \frac{\partial E(\Pi_e^I)^T}{\partial b} > 0 \text{ and } \frac{\partial (\Pi_e^I)^{NT}}{\partial b} > 0 \quad \forall b \in [\bar{b}, I], \text{ respectively, then if}$$

$\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) \leq 0 \wedge E(\Pi_e^I)^T(\bar{b}) \leq 0$  is satisfied, a patent breadth value  $\hat{b} \in (b_0, I]$  (or a range of patent breadth values) exists, that, if chosen by the incumbent, can deter entry. This case is illustrated in Figure 5, panel (i).

Entry can also be deterred if at  $\bar{b}$  the entrant's profits under no infringement and his profits under infringement and no trial are less than or equal to zero, ( $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) \leq 0$ ), his expected profits under infringement and trial are positive at  $\bar{b}$  ( $E(\Pi_e^I)^T(\bar{b}) > 0$ ) and negative at  $\tilde{b}$

$(E(\Pi_e^I)^T(\tilde{b}) < 0)$ ; since  $\frac{\partial E(\Pi_e^I)^T}{\partial b} > 0$  the above imply that, if a  $\tilde{b}$  exists, it will be smaller than  $\vec{b}$ .

This case is illustrated in Figure 5, panel (ii).

**Proposition 2.** *A patent breadth  $\hat{b}$  that deters entry exists if and only if the entrant's trial costs,  $C_e^T$ , satisfy the condition  $C_e^T \geq \frac{512\alpha^2}{6561\beta^3} = \hat{C}_e^T$ . This condition implies that the entrant's R&D costs are such that  $\beta \geq 2\beta_0$ .*

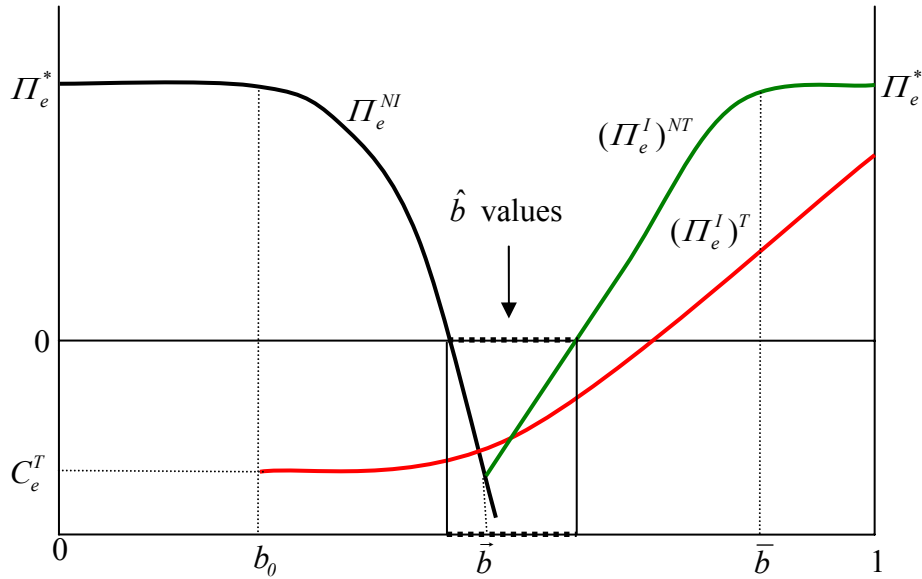
**Proof.** A proof of proposition 2 is given in Appendix A4.

The relationship between the parameters that give rise to the entry deterrence conditions in proposition 2 is summarized in corollary 2.

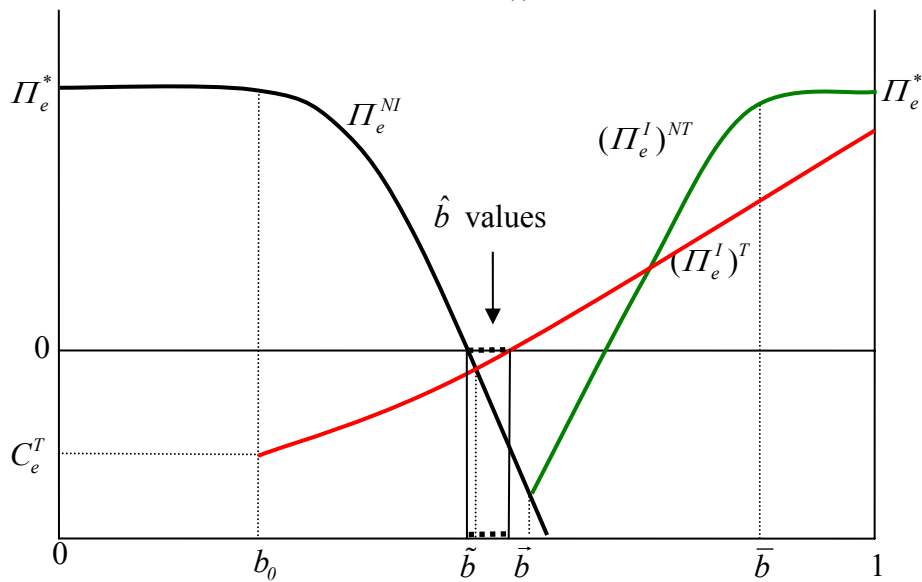
**Corollary 2.** *Entry deterrence becomes more likely, the greater are the entrant's R&D costs (i.e., the larger is  $\beta$ ) and trial costs,  $C_e^T$ , the smaller is the validity parameter,  $\alpha$ , and the incumbent's trial costs,  $C_p^T$ , and the larger are the monopoly profits,  $\Pi_m$ , that can be captured by the incumbent.*

Obviously, the greater are the entrant's R&D costs, the more likely it is that the condition  $\beta \geq 2\beta_0$  will be satisfied. Similarly, the smaller is the validity parameter,  $\alpha$ , and the incumbent's trial costs,  $C_p^T$ , and the larger are the monopoly profits,  $\Pi_m$ , the smaller is the critical value  $\beta_0$  (see Appendix A2). In addition, the smaller is the validity parameter,  $\alpha$ , and the greater are the entrant's R&D and trial costs, the more likely it is that the condition  $C_e^T \geq \frac{512\alpha^2}{6561\beta^3} = \hat{C}_e^T$  will be satisfied.

**Result 7.** When the entry deterrence conditions are satisfied and a  $\bar{b} \in (\bar{b}, 1]$  exists, the patent breadth that deters entry,  $\hat{b}$ , will always be smaller than the maximum patent breadth possible, i.e.,  $\hat{b} \in (b_0, \bar{b})$ .



Panel (i)



Panel (ii)

Figure 5. The entrant's profits under no infringement, infringement and trial and infringement and no trial when entry can be deterred.

Scenario B: Entry cannot be deterred

There are a number of different cases where entry cannot be deterred that lead to different optimal strategies for the incumbent and the entrant. Entry cannot be deterred when at  $\bar{b}$  the entrant's profits under no infringement and under infringement and no trial are positive,

$$\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) > 0; \text{ this is a sufficient condition for the absence of entry deterrence}$$

and is satisfied for  $\beta < 2\beta_0$  (see Appendix A4). Note that the condition  $\Pi_e^{NI}(\bar{b}) = (\Pi_e^I)^{NT}(\bar{b}) > 0$  is not a necessary condition for the absence of entry deterrence as entry cannot be deterred when

$\Pi_e^{NI}(\bar{b}) = (\Pi_e^I)^{NT}(\bar{b}) \leq 0$  and at  $\tilde{b}$   $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b}) > 0$  (i.e., the profit curve  $E(\Pi_e^I)^T$  crosses the profit curve  $\Pi_e^{NI}$  above zero); this case is depicted in Figure 6, panel (iii).

As will become evident in the cases examined below, the optimal strategy for the entrant when entry cannot be deterred (scenario B) depends on the relationship between  $\bar{b}$ ,  $\tilde{b}$  and  $\tilde{\tilde{b}}$ . Two general cases are considered, case I and II, that examine the entrant's profits when  $\tilde{b} \geq \bar{b}$  and  $\tilde{b} < \bar{b}$ , respectively. Under each of these two cases, two sub-cases may emerge depending on whether a  $\tilde{\tilde{b}} \in (\bar{b}, I]$  exists or not. The four cases are illustrated in Figure 6 and the conditions under which they emerge are examined below.

- Case I<sub>A</sub>:  $\tilde{b} \geq \bar{b}$  and a  $\tilde{\tilde{b}} \in (\bar{b}, I]$  does not exist

Under this case (Figure 6, panel (i)),  $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) > 0$ ,

$$\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) \geq E(\Pi_e^I)^T(\bar{b}) \text{ and } (\Pi_e^I)^{NT}(b = I) > E(\Pi_e^I)^T(b = I), \text{ which imply that}$$

this case emerges for  $\beta < 2\beta_0$ ,  $C_e^T \geq \left(\frac{8\alpha^2}{81\beta} + \frac{\beta}{2}\right)\bar{b}^2 - \frac{4}{9}\bar{b} \Rightarrow C_e^T \geq \left(\frac{8\alpha^2}{81\beta} + \frac{\beta}{2}\right)\frac{16}{81\beta_0^2} - \frac{16}{81\beta_0} = \tilde{C}_e^T$  and

$$C_e^T > \frac{8\alpha^2}{81\beta} + \frac{81\beta}{2} \left(\Pi_m - \frac{C_p^T}{1-\alpha}\right)^2 - 4\left(\Pi_m - \frac{C_p^T}{1-\alpha}\right) = \tilde{\tilde{C}}_e^T. \text{ Given these conditions, the entrant's optimal}$$

strategy is to not infringe the patent for patent breadth values  $b \in (b_0, \bar{b})$  and to infringe the patent and not induce a trial for patent breadth values  $b \in [\bar{b}, I]$ ; the strategy of infringing the patent and inducing the incumbent to invoke a trial is never an optimal strategy. This case is most likely to emerge when the entrant's trial costs,  $C_e^T$ , are relatively high, and his R&D costs are relatively low making infringement and trial less attractive to the entrant.

- Case I<sub>B</sub>:  $\tilde{b} \geq \bar{b}$  and a  $\tilde{b} \in (\bar{b}, I]$  exists

Under this case (Figure 6, panel (ii)),  $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) > 0$ ,

$\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) \geq E(\Pi_e^I)^T(\bar{b})$  and  $(\Pi_e^I)^{NT}(b = I) < E(\Pi_e^I)^T(b = I)$  (i.e., a  $\bar{b} \in (\bar{b}, I]$

does not exist). These conditions imply that  $\beta < 2\beta_0$ ,  $C_e^T \geq \tilde{C}_e^T$  and  $C_e^T < \hat{C}_e^T$ . Under this case, the

entrant will find it optimal to not infringe the patent for patent breadth values  $b \in (b_0, \bar{b}]$ , he will

infringe the patent and not induce a trial for patent breadth values  $b \in (\bar{b}, \tilde{b}]$  and he will infringe the

patent and induce a trial for patent breadth values  $b \in (\tilde{b}, I]$ . Thus, under this case, infringement and

trial is an attractive strategy to the entrant only for relatively high patent breadth values, since the

greater is patent breadth the more likely it becomes that infringement will not be found at trial (i.e.,

that the patent will be found invalid).

- Case II<sub>A</sub>:  $\tilde{b} < \bar{b}$  and a  $\tilde{b} \in (\bar{b}, I]$  does not exist

This case can emerge both when at  $\bar{b}$   $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) > 0$  which implies that  $\beta < 2\beta_0$

and when at  $\bar{b}$   $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) \leq 0$  which implies that  $\beta \geq 2\beta_0$  (this latter possibility

is shown in Figure 6, panel (iii)). Under this case, at  $\tilde{b}$   $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b}) > 0$  which implies

that  $C_e^T < \hat{C}_e^T$ . Also, at  $\bar{b}$   $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) < E(\Pi_e^I)^T(\bar{b})$  and

$(\Pi_e^I)^{NT}(b=I) < E(\Pi_e^I)^T(b=I)$  (i.e., a  $\bar{b} \in (\bar{b}, I]$  does not exist) which imply that  $C_e^T < \tilde{C}_e^T$  and  $C_e^T < \tilde{C}_e^T$ , respectively. Under these conditions, the entrant will find it optimal to not infringe the patent for patent breadth values  $b \in (b_0, \bar{b}]$  and infringe the patent inducing the incumbent to invoke a trial for patent breadth values  $b \in (\bar{b}, I]$ ; the strategy of infringing the patent and not inducing the incumbent to invoke a trial is never an optimal strategy. This case is most likely to occur when the entrant's trial costs,  $C_e^T$ , are relatively low and the validity parameter,  $\alpha$ , is relatively high making infringement and trial attractive to the entrant for relatively large patent breadth values. Also, this case is more likely to emerge the lower are the incumbent's trial costs,  $C_p^T$ , since the lower are the incumbent's trial costs, the larger is  $\bar{q}_e$  and the less attractive is infringement and no trial to the entrant.

- Case II<sub>B</sub>:  $\tilde{b} < \bar{b}$  and a  $\tilde{\tilde{b}} \in (\tilde{b}, I]$  exists

This case can emerge both when at  $\bar{b} \lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) > 0$  which implies that  $\beta < 2\beta_0$

and when  $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) \leq 0$  which implies that  $\beta \geq 2\beta_0$  (the former possibility is

shown in Figure 6, panel (iv)). Under this case, at  $\tilde{b} \Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b}) > 0$  which implies that

$C_e^T < \hat{C}_e^T$ . Also, at  $\bar{b} \lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) < E(\Pi_e^I)^T(\bar{b})$ ; in addition

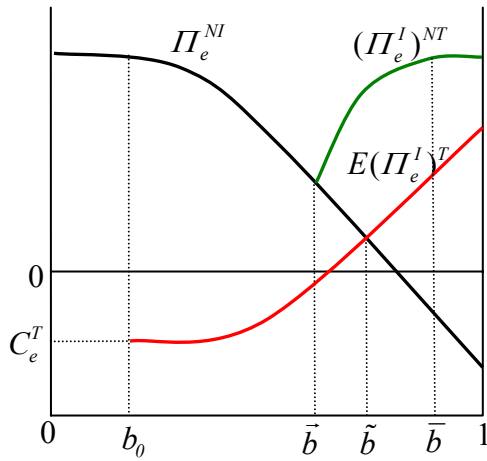
$(\Pi_e^I)^{NT}(b=I) > E(\Pi_e^I)^T(b=I)$  which together imply that  $C_e^T < \tilde{C}_e^T$  and  $C_e^T > \tilde{\tilde{C}}_e^T$ , respectively.

Under these conditions, the entrant's optimal strategy is to not infringe the patent for patent breadth

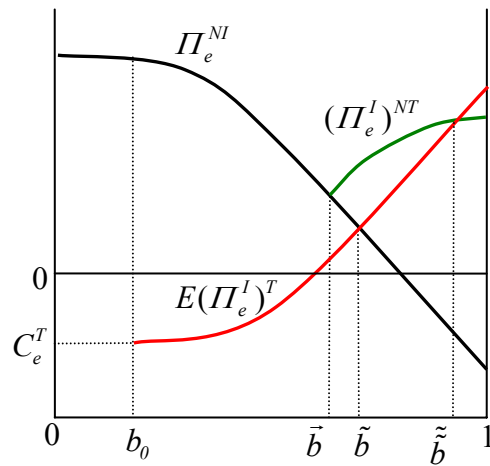
values  $b \in (b_0, \tilde{b})$ , infringe the patent and induce a trial for patent breadth values  $b \in [\tilde{b}, \tilde{\tilde{b}})$  and

infringe the patent and not induce a trial for patent breadth values  $b \in [\tilde{\tilde{b}}, I]$ . This case is most likely

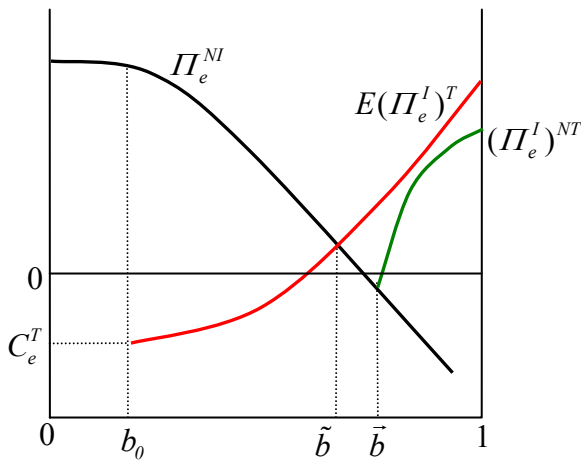
to emerge when the entrant's trial costs,  $C_e^T$ , and R&D costs are relatively low making infringement with trial attractive to the entrant for intermediate patent breadth values.



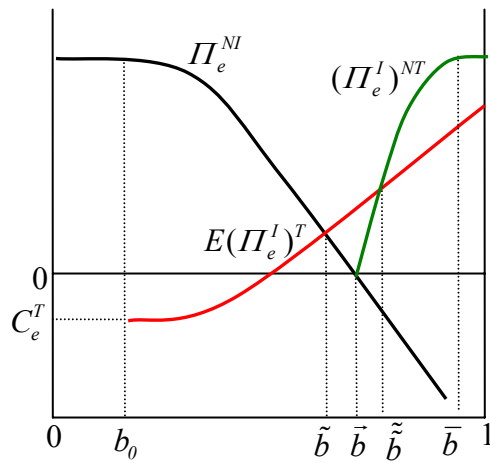
Panel (i): Case I<sub>A</sub>



Panel (ii): Case I<sub>B</sub>



Panel (iii): Case II<sub>A</sub>



Panel (iv): Case II<sub>B</sub>

Figure 6. The entrant's profits under no infringement, under infringement and trial and under infringement and no trial when entry cannot be deterred and  $\tilde{b} \geq \bar{b}$  – panels (i) and (ii) and when  $\tilde{b} < \bar{b}$  – panels (iii) and (iv).

Figure 7 depicts the combination of the entrant's R&D effectiveness,  $\beta$ , and trial cost,  $C_e^T$ , values (for given values of the exogenous parameters) that give rise to entry deterrence (scenario A) and the four cases that may emerge when entry cannot be deterred (scenario B). Note that when  $\beta = 2\beta_0$ , the loci  $\hat{C}_e^T = \tilde{C}_e^T = \frac{64a^2}{6561\beta_0^3} = \bar{C}_e^T$  where  $\bar{C}_e^T = \frac{64a^2}{6561\beta_0^3}$  is the value of the entrant's trial costs that makes his expected profits under infringement and trial equal to zero at  $\bar{b}$ . Also, the locus  $\tilde{\tilde{C}}_e^T$  may be greater, smaller or equal to the locus  $\tilde{C}_e^T$  at  $\beta = 2\beta_0$  depending on the value of the exogenous parameters. Figure 7 depicts the locus  $\tilde{\tilde{C}}_e^T$  crossing the locus  $\tilde{C}_e^T$  so that both cases I<sub>B</sub> and II<sub>B</sub> are feasible.

Consider first the situation where entry can be deterred. The combinations of  $\beta$  and  $C_e^T$  values to the right of locus  $\beta = 2\beta_0$  and above the locus  $\hat{C}_e^T$  (i.e.,  $\beta \geq 2\beta_0$  and  $C_e^T \geq \hat{C}_e^T$ ), depicted by the horizontally hatched area in Figure 7, give rise to the entry deterrence outcome; the area to the right of locus  $\beta = 2\beta_0$  and above the locus  $\bar{C}_e^T$  corresponds to the entry deterrence outcome depicted in Figure 5, panel (i) while the area to the right of locus  $\beta = 2\beta_0$ , above the locus  $\hat{C}_e^T$  and below the locus  $\bar{C}_e^T$  corresponds to the entry deterrence outcome depicted in Figure 5, panel (ii). As discussed earlier, entry deterrence is possible if both  $\beta$  and  $C_e^T$  values are relatively large.

Now consider the situation where entry cannot be deterred. The combinations of  $\beta$  and  $C_e^T$  values to the left of locus  $\beta = 2\beta_0$  and above the loci  $\tilde{C}_e^T$  and  $\tilde{\tilde{C}}_e^T$ , depicted by the lightly vertically hatched area in Figure 7, correspond to case I<sub>A</sub> (Figure 6, panel (i)) while the combinations of  $\beta$  and  $C_e^T$  values to the left of locus  $\beta = 2\beta_0$ , above the locus  $\tilde{C}_e^T$  and below the locus  $\tilde{\tilde{C}}_e^T$ , depicted by the heavily vertically hatched area in Figure 7, correspond to case I<sub>B</sub> (Figure 6, panel (ii)). The

combinations of  $\beta$  and  $C_e^T$  values to the left of locus  $\hat{C}_e^T$  and to the right of loci  $\tilde{C}_e^T$  and  $\tilde{\tilde{C}}_e^T$ , depicted by the lightly dotted area in Figure 7, correspond to case II<sub>A</sub> (Figure 6, panel (iii)). Finally, the combinations of  $\beta$  and  $C_e^T$  values above the locus  $\tilde{\tilde{C}}_e^T$  and below the locus  $\tilde{C}_e^T$ , depicted by the heavily dotted area in Figure 7, correspond to case II<sub>B</sub> (Figure 6, panel (iv)).

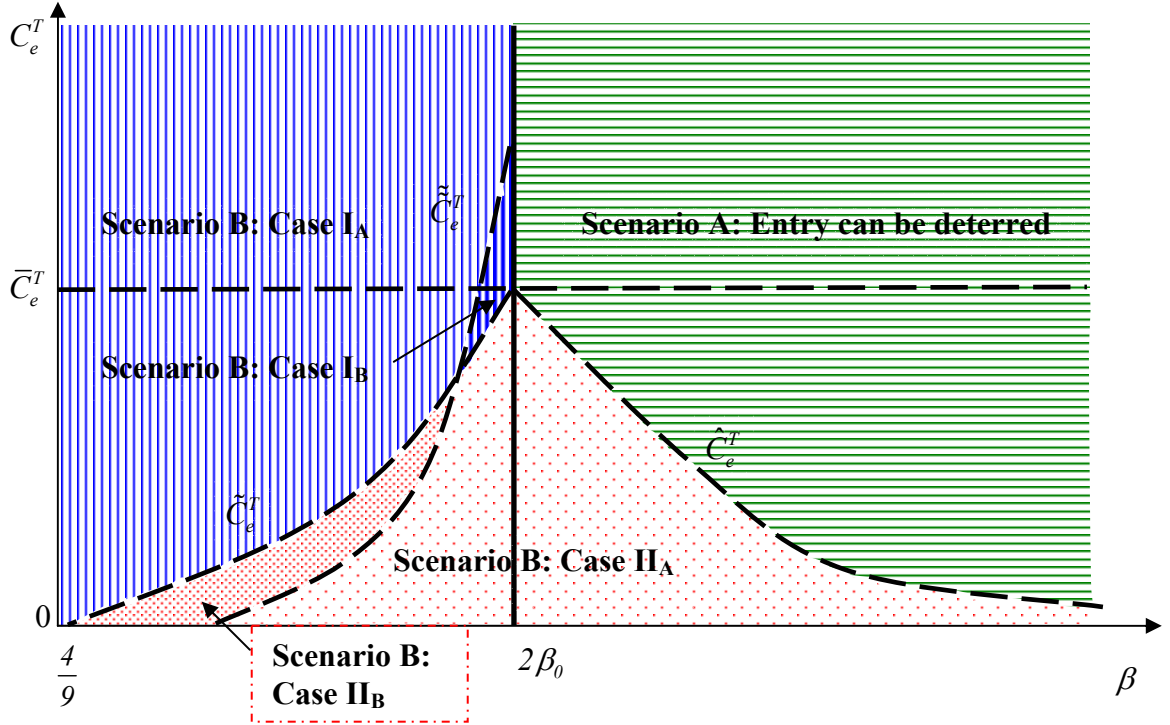


Figure 7. Combinations of  $\beta$  and  $C_e^T$  values for which entry can and cannot be deterred.

### 3.4 Stage 2 – The patent breadth decision

In stage 2 of the game the incumbent chooses the patent breadth  $b$  that maximizes profits, given her knowledge of the entrant's behavior in the third stage of the game. Since the entrant's behavior depends on the values of the parameters  $\Pi_m$ ,  $C_p^T$ ,  $C_e^T$ ,  $\alpha$  and  $\beta$ , the patent breadth chosen by the incumbent also depends on these parameters. Specifically, the following situations are possible, each one corresponding to one of the scenarios and cases outlined above.

Scenario A: Choose patent breadth to deter entry

If there exists a patent breadth  $\hat{b} \in (b_0, I]$  such that  $\Pi_e^{NI}(\hat{b}) \leq 0 \wedge E(\Pi_e^I)^T(\hat{b}) \leq 0 \wedge (\Pi_e^I)^{NT}(\hat{b}) \leq 0$  then the incumbent will choose this patent breadth and deter entry. By deterring entry, the incumbent earns monopoly profits,  $\Pi_m$ . Since these profits are higher than what can be earned under a duopoly, the incumbent always finds it optimal to deter entry.

Scenario B: Entry cannot be deterred

**Result 8.** *When the incumbent wishes to patent her innovation and entry cannot be deterred, she will never find it optimal to choose patent breadth values such that  $b \in (0, b_0]$  and  $b \in [\bar{b}, I]$ .*

As illustrated in Figure 4, if the incumbent chooses patent breadth values  $b \in (0, b_0]$  or  $b \in [\bar{b}, I]$ , the entrant will find it optimal to locate at his most preferred location,  $q_e^*$ , since when  $b \in (0, b_0]$  patent breadth is not binding while when  $b \in [\bar{b}, I]$  the incumbent's optimal strategy when the patent is infringed is to not invoke a trial. Since  $q_e^*$  is where the entrant locates under no patent protection, as long as patenting costs are positive ( $z > 0$ ), the patenting strategy is always dominated by the no patenting strategy for patent breadth values  $b \in (0, b_0]$  or  $b \in [\bar{b}, I]$ . Thus, the relevant patent breadth values that can be chosen by the incumbent when she wishes to patent the innovation and entry cannot be deterred are patent breadth values such that  $b \in (b_0, \bar{b})$ .

- Case I<sub>A</sub>:  $\tilde{b} \geq \bar{b}$  and a  $\tilde{b} \in (\bar{b}, I]$  does not exist

Under this case, it is never optimal for the entrant to infringe the patent and induce the incumbent to invoke a trial. The incumbent has to decide whether to choose a patent breadth  $b \in (b_0, \bar{b})$  that will induce the entrant not to infringe the patent or to choose a patent breadth  $b \in [\bar{b}, \tilde{b})$  that will induce the entrant to infringe the patent without inducing a trial.

- Case I<sub>B</sub>:  $\bar{b} \geq \tilde{b}$  and a  $\tilde{b} \in (\bar{b}, I]$  exists

Under this case, infringement and trial is optimal for the entrant only for relatively large patent

breadth values. In this case, the incumbent has to decide whether to choose a patent breadth

$b \in (b_0, \bar{b}]$  and induce the entrant to not infringe the patent, choose a patent breadth  $b \in (\bar{b}, \tilde{b}]$  and

induce the entrant to infringe the patent and not force a trial or choose a patent breadth  $b \in (\tilde{b}, I]$

and induce the entrant to infringe the patent and force a trial.

**Lemma 2.** *The incumbent's profits under infringement and no trial are greater than her profits under infringement and trial for all patent breadth values  $b \in (b_0, \bar{b})$ ; i.e., the infringement and trial strategy is always dominated by the infringement and no trial strategy.*

The incumbent will choose infringement and no trial over infringement and trial when

$$(\Pi_p^I)^{NT} \geq E(\Pi_p^I)^T \text{ where } (\Pi_p^I)^{NT} = \frac{\bar{q}_e}{9} = \Pi_m - \frac{C_p}{1-ab} \text{ and}$$

$$E(\Pi_p^I)^T = (1-ab)\Pi_m + ab\frac{(q_e^I)^T}{9} - C_p^T = (1-ab)\Pi_m + a^2b^2\frac{q_e^*}{9} - C_p^T. \text{ It is straightforward to show}$$

that  $(\Pi_p^I)^{NT} \geq E(\Pi_p^I)^T$  implies that  $9(\Pi_m - \frac{C_p^T}{1-ab}) \geq abq_e^* \Rightarrow \bar{q}_e \geq (q_e^I)^T$ . The condition  $\bar{q}_e \geq (q_e^I)^T$

holds for all patent breadth values that can be chosen by the incumbent when entry cannot be

deterred, i.e.,  $b \in (b_0, \bar{b})$  (see Result 8), since  $(q_e^I)^T = abq_e^* < q_e^* \forall a \in (0, 1) \wedge b \in (0, 1)$  and  $q_e^* < \bar{q}_e$

$\forall b \in (b_0, \bar{b})$ ; see Figure 4. Thus, when deciding between infringement and no trial and infringement

and trial, the incumbent's optimal strategy is to always choose infringement and no trial.

**Proposition 3.** *When entry cannot be deterred and it is either (a) never optimal for the entrant to infringe the patent and face a trial (case I<sub>A</sub>) or (b) infringement and trial is optimal for the entrant only for relatively large patent breadth values (case I<sub>B</sub>), the incumbent maximizes her profits by*

claiming the patent breadth  $\bar{b}$  that makes her indifferent between not having her patent infringed and not defending her patent by invoking a trial under infringement.

**Proof:** Given the results in lemma 2, the incumbent's decision under cases I<sub>A</sub> and I<sub>B</sub> is reduced to deciding whether to induce no infringement or infringement and no trial. Since the incumbent's profits are increasing in the entrant's quality choice  $q_e$ , both under no infringement and under

infringement and no trial i.e.,  $\Pi_p^{NI} = (\Pi_p^I)^{NT} = \pi_p^* = \frac{q_e}{9}$ , the incumbent maximizes her profits by

forcing the entrant to locate the furthest away possible in the quality product space. When the

incumbent chooses the patent breadth  $\bar{b}$ , the entrant is indifferent between not infringing the patent and infringing the patent and not facing a trial, i.e.,  $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b})$ ; for any  $b < \bar{b}$  the

entrant does not infringe the patent by choosing  $q_e^{NI} = b + e$  while for any  $b > \bar{b}$  the entrant

infringes the patent and does not induce a trial by choosing  $(q_e^I)^{NT} = \bar{q}_e$ . When the patent breadth  $\bar{b}$

is chosen, the incumbent is indifferent between not having the patent infringed and not defending

the patent by invoking a trial under infringement, i.e.,  $\lim_{e \rightarrow 0} \Pi_p^{NI}(\bar{b} + e) = (\Pi_p^I)^{NT}(\bar{b}) = \frac{\bar{b}}{9}$ . Since any

patent breadth that is greater than  $\bar{b}$  (e.g.,  $\tilde{b}$ ), will lead to the entrant locating closer to the

incumbent (note that  $\bar{q}_e = \bar{b}$  at  $\bar{b}$  while  $\bar{q}_e < b \forall b \in (\bar{b}, I]$ ), the choice of the patent breadth  $\bar{b}$

forces the entrant to locate the furthest away possible in the quality product space, maximizing product differentiation and, thus, the incumbent's profits.

It should be stressed that, even though the incumbent does not find it optimal to defend her patent, the patent is nevertheless valuable. Indeed, without the patent, the entrant would not locate at  $\bar{b}$  (or  $\bar{b} + e$ ), but rather at  $q_e^* < \bar{b}$ . The presence of the patent means that the option for the incumbent to take the entrant to court to defend the patent exists. It is the entrant's desire to avoid

this option (since doing so increases his profits) that results in him locating further from the incumbent than would be the case in the absence of patent protection. Thus, undefended patents need not signal exploitation or “robbery” by the entrant, but rather represent the outcome of a game in which lack of patent defense emerges as an optimal strategy.

**Corollary 3.** *When entry cannot be deterred and it is either (a) never optimal for the entrant to infringe the patent and face a trial (case  $I_A$ ) or (b) infringement and trial is optimal for the entrant only for relatively large patent breadth values (case  $I_B$ ), the incumbent maximizes her profits by claiming a relatively narrow rather than broad patent protection as a narrow patent breadth leads to greater product differentiation.*

This result shares a similarity with a result in Yiannaka and Fulton (2006). In their model, where litigation is not endogenous and a trial always occurs upon infringement, if the incumbent finds it optimal to induce non-infringement she chooses a patent breadth that induces the entrant to locate at the edge of patent breadth. In our model the infringement and no trial outcome implies the choice of a patent breadth that similarly induces the entrant to locate by the edge of patent breadth.

The incumbent’s profits under case  $I_A$  are depicted in Figure 8, panel (i) while her profits under case  $I_B$  when the optimal patent breadth under infringement and trial is given by  $b^I = \tilde{b} + e$  are depicted in Figure 8, panel (ii) (for the derivation of the optimal patent breadth values under infringement and trial see Appendix A<sub>5</sub>).

- Case II<sub>A</sub>:  $\tilde{b} < \vec{b}$  and a  $\tilde{b} \in (\vec{b}, I]$  does not exist

Under this case, it is never optimal for the entrant to infringe the patent without inducing a trial. The incumbent has to decide whether to choose a patent breadth  $b \in (b_0, \tilde{b}]$  and induce the entrant to not infringe the patent or to choose a patent breadth  $b \in (\tilde{b}, I]$  and induce the entrant to infringe the

patent and induce a trial. This case has been examined by Yiannaka and Fulton (2006) who find that the incumbent will induce non infringement by claiming  $b^{NI} = \tilde{b}$  or induce infringement and trial by claiming either  $b^I = \tilde{b} + e$  or  $b^I = I$ . The optimal strategy for the incumbent depends in a complex way on the values of the parameters  $\Pi_m$ ,  $C_p^T$ ,  $C_e^T$ ,  $\alpha$  and  $\beta$ . In general, the greater are the incumbent's monopoly profits, the greater is the incumbent's incentive to induce infringement and trial since the only opportunity the incumbent has to realize monopoly profits (when entry cannot be deterred) is when her patent is infringed and its validity is upheld during the infringement trial. At the same time, the greater are the monopoly profits, the more likely it becomes that the incumbent will find it optimal to induce infringement and trial by claiming a relatively small patent breadth  $-\tilde{b} + e$  rather than  $b = I$  (see the Appendix A5). The larger are the incumbent's trial costs and the validity parameter and the smaller are the entrant's R&D costs the more likely it is that the incumbent will find it optimal to induce non infringement by claiming  $b^{NI} = \tilde{b}$ . Case II<sub>A</sub> when the optimal patent breadth under infringement and trial is given by  $b^I = \tilde{b} + e$  is depicted in Figure 8, panel (iii). As shown in this figure, when the incumbent's expected profits under infringement and trial are given by the curve AB, the incumbent's optimal strategy is to claim patent breadth  $b^I = \tilde{b} + e$  and induce infringement and trial while if her expected profits are given by CD, the incumbent's optimal strategy is to claim  $b^{NI} = \tilde{b}$  and induce non infringement.

- Case II<sub>B</sub>:  $\tilde{b} < \bar{b}$  and a  $\tilde{b} \in (\bar{b}, I]$  exists

Under this case, the incumbent has to decide whether to choose a patent breadth  $b \in (b_0, \tilde{b}]$  and induce the entrant to not infringe the patent, choose a patent breadth  $b \in (\tilde{b}, \bar{b})$  and induce the entrant to infringe the patent and face a trial or choose a patent breadth  $b \in [\tilde{b}, \bar{b})$  and induce the entrant to infringe the patent and not face a trial.

**Lemma 3.** *Under case II<sub>B</sub>, the incumbent's profits under no infringement are greater than her profits under infringement and no trial; i.e., the infringement and no trial strategy is always dominated by the no infringement strategy.*

If the incumbent were to choose to induce non infringement the optimal strategy would be to choose the patent breadth  $\tilde{b}$  since this is the patent breadth that forces the entrant to locate the furthest away possible in the quality space without infringing the patent. If the incumbent were to choose to induce infringement and no trial then the optimal strategy would be to choose patent breadth  $\tilde{\tilde{b}}$  since this is the patent breadth that induces the entrant to locate the furthest away possible under infringement and no trial (for any  $b > \tilde{\tilde{b}}$  the entrant locates closer to the incumbent). Since the incumbent's profits under no infringement and under infringement and no trial are both increasing in the quality chosen by the entrant,  $q_e$ , the incumbent is better off choosing  $\tilde{b}$  rather than  $\tilde{\tilde{b}}$ , i.e.,

$$\Pi_p^{NI}(\tilde{b}) > (\Pi_p^I)^{NT}(\tilde{\tilde{b}}).$$

**Proposition 4.** *When entry cannot be deterred and the entrant finds it optimal to infringe the patent and face a trial for intermediate patent breadth values and infringe the patent and not face a trial for relatively large patent breadth values (i.e., case II<sub>B</sub>), the optimal strategy for the incumbent is to choose a patent breadth value that induces the entrant to not infringe the patent, i.e.,  $b^{NI} = \tilde{b}$ .*

**Proof:** The result in proposition 4 follows directly from lemma 2 and 3.

Case II<sub>B</sub> when the optimal patent breadth under infringement and trial is given by  $b^I = \tilde{b} + e$  is depicted in Figure 8, panel (iv).

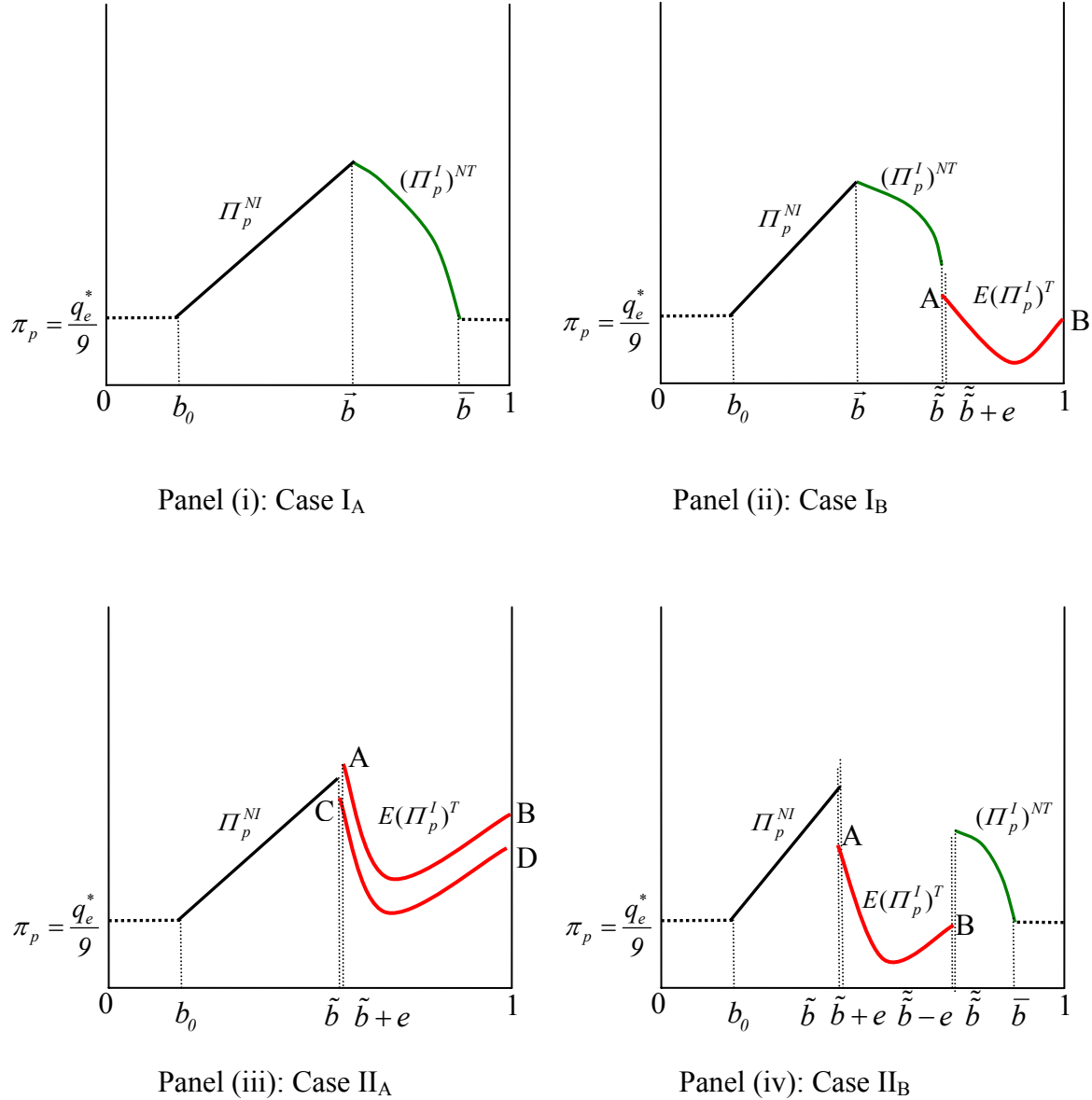


Figure 8. The incumbent's profits when entry cannot be deterred (cases I<sub>A</sub>, I<sub>B</sub>, II<sub>A</sub> and II<sub>B</sub>).

### 3.5 Stage 1 – The patenting decision

In stage 1 of the game the incumbent decides whether to patent her innovation or not given her knowledge of the entrant's response to her patent breadth and trial decisions. The incumbent will choose to patent her innovation when the profits earned under patenting are greater than the profits earned under no patent protection,  $\Pi_p^P \geq \Pi_p^{NP}$ . As described in the preceding sections entry cannot

be deterred if a patent has not been obtained; the entrant will enter the market choosing his most preferred quality  $q_e^*$  and the incumbent will earn profits  $\Pi_p^{NP} = \frac{q_e^*}{9} = \frac{4}{81\beta}$ . As expected, the incumbent's profits under no patent protection are increasing in the entrant's R&D effectiveness; the lower are the entrant's R&D costs, the further away the entrant locates from the incumbent and the less intense is price competition at the last stage of the game. The profits realized by the incumbent under patent protection depend on the value of the parameters  $\Pi_m, C_p^T, C_e^T, \alpha$  and  $\beta$  which determine whether she can use patent breadth to deter entry (Scenario A) or not (Scenario B) and, when entry cannot be deterred, which of the four cases examined in the previous section will emerge (cases I<sub>A</sub>, I<sub>B</sub>, II<sub>A</sub> and II<sub>B</sub>).

As shown in proposition 1, the incumbent will always find it optimal not to patent the innovation when  $q_e^* \geq \bar{b}$  (i.e.,  $\beta < \beta_0$ ), regardless of the level of patenting costs. In this case, had the incumbent chosen to patent, the entrant would always choose  $q_e^*$  and the incumbent would not find it optimal to invoke a trial. Thus, if under patenting the incumbent can never enforce her patent rights when the patent is infringed she always chooses not to patent. When, however, the entrant's location choice affects whether the incumbent will find it optimal to invoke a trial under infringement (i.e., when  $q_e^* < \bar{b}$ ), the optimal strategy for the incumbent depends on the values of the parameters  $\Pi_m, C_p^T, C_e^T, \alpha$  and  $\beta$  and on the incumbent's patenting costs,  $z > 0$ .

Table 1 presents the effect of the exogenous parameters on the incumbent's decision to patent when entry can be deterred (Scenario A) and when entry cannot be deterred (Scenario B, cases I<sub>A</sub>, I<sub>B</sub>, II<sub>A</sub> and II<sub>B</sub>).

Table 1. Effect of exogenous parameters on the incumbent's decision to patent

Scenarios	Patenting profitable when	Impact on the incumbent's decision to patent				
		$\Pi_m$	$\beta$	$\alpha$	$C_p^T$	$C_e^T$
Scenario A: $b = \hat{b}$	$z \leq \Pi_m - \frac{4}{81\beta}$	(+)	(+)			
Scenario B: case I <sub>A</sub> , case I <sub>B</sub> : $b = \bar{b}$	$z \leq \frac{\bar{b}}{9} - \frac{4}{81\beta}$	(+)	(+)	(-)	(-)	
Scenario B: case II <sub>A</sub> , case II <sub>B</sub> : $b = \tilde{b}$	$z \leq \frac{\tilde{b}}{9} - \frac{4}{81\beta}$			(-)		(+)
Scenario B: case II <sub>A</sub> : $b = \tilde{b} + e$	$z \leq (1 - \alpha(\tilde{b} + e))\Pi_m + \frac{4\alpha^2(\tilde{b} + 1)^2}{81\beta} - C_p^T$	(+)			(-)	(+)
Scenario B: case II <sub>A</sub> : $b = 1$	$z \leq (1 - \alpha)\Pi_m + \frac{4(\alpha^2 - 1)}{81\beta} - C_p^T$	(+)	(+)		(-)	

Note: blank cells imply that the impact cannot be determined without knowledge of the magnitude of the parameters

Under patent protection when entry can be deterred (Scenario A), the incumbent's profits are given by  $\Pi_p^P = \Pi_m - z$ . The decision to patent or not in this case depends on the monopoly profits that can be captured by the incumbent, the magnitude of the patenting costs and the entrant's R&D effectiveness. As shown in Table 1, as long as patenting costs are such that  $z \leq \Pi_m - \frac{4}{81\beta}$ , patenting is more profitable than no patenting for the incumbent,  $\Pi_p^P \geq \Pi_p^{NP}$ . The greater are the monopoly profits and the entrant's R&D costs (the greater is  $\beta$  and thus the closer to the incumbent the entrant finds it optimal to locate), the more likely it is that patenting will result in greater profits than no patenting for the incumbent.

When entry cannot be deterred under patent protection (Scenario B) and under cases I<sub>A</sub> and I<sub>B</sub>, the incumbent's optimal patent breadth strategy is to choose patent breadth  $b = \bar{b}$  and the

incumbent's profits are given by  $\Pi_p^P = \frac{q_e = \tilde{b}}{9} - z = \frac{1 + 9a\Pi_m - \sqrt{1 + 36aC_p^T - 18a\Pi_m + 81a^2\Pi_m^2}}{18a} - z$ .

The greater are the monopoly profits and the entrant's R&D costs and the smaller is the validity parameter and the incumbent's trial costs, the more likely it is that patenting will be more profitable than no patenting for the incumbent.

Under case  $\Pi_A$  the incumbent will either induce the entrant to not infringe the patent by claiming patent breadth  $b = \tilde{b}$  or to infringe the patent by claiming  $b = \tilde{b} + e$  or  $b = 1$ . When the incumbent's optimal patent breadth strategy is to choose  $b = \tilde{b}$  and induce non infringement her profits under patent protection are given by

$\Pi_p^P = \frac{q_e = \tilde{b}}{9} - z = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2} - z$ . This expression also gives the

incumbent's profits under case  $\Pi_B$  where the incumbent always finds it optimal to induce non infringement by choosing  $b = \tilde{b}$ . Under this case, the smaller is the validity parameter,  $\alpha$ , and the greater are the entrant's trial costs, the more likely it is that patenting will be more profitable than no patenting.

When the incumbent's optimal strategy under patenting is to induce infringement and trial by claiming  $b = \tilde{b} + e$  then her profits under patent protection are given by

$\Pi_p^P = E(\Pi_p^I)^T - z = (1 - \alpha(\tilde{b} + e))\Pi_m + \frac{4\alpha^2(\tilde{b} + e)^2}{81\beta} - C_p^T - z$ . Under this case, the greater are the

monopoly profits and the smaller are the incumbent's trial costs, the more likely it is that patenting will be more profitable than no patenting for the incumbent. Finally, when the incumbent's optimal strategy under patenting is to induce infringement and trial by claiming  $b = 1$  her profits under

patent protection are given by  $\Pi_p^P = E(\Pi_p^I)^T - z = (1 - \alpha)\Pi_m + \frac{4\alpha^2}{81\beta} - C_p^T - z$ . In this case, the greater

are the monopoly profits and the entrant's R&D costs and the smaller are the incumbent's trial costs the more likely it becomes that patenting will be profitable for the incumbent.

#### **4. Concluding remarks**

A game theoretic model was developed to examine how an innovator's decision to seek patent protection and her optimal patent breadth decision affect and are affected by her ability to enforce her patent rights. The innovator in our model seeks patent protection for a product innovation under potential entry by a firm producing a better quality product. The innovator must decide whether to patent her innovation or not; if she chooses to patent her product, she must decide how broad to make the patent. The entrant observes the patent breadth granted to the innovator's product under patenting and decides whether to enter and, if entry occurs, where to locate in the quality product space. Finally, the innovator observes the entrant's quality choice and in the case of infringement decides on whether to invoke a trial or not. A key feature of the model is that the entrant may be able, by his choice of product quality, to affect the innovator's trial decision when the patent is infringed.

Patenting is only optimal if the existence of the patent causes the entrant to locate further away from the incumbent in product space than would be the case in the absence of a patent. The incumbent is unable to use a patent to influence the entrant's location choice when her trial costs are large, when the monopoly profits that can be captured are relatively small and when the entrant's R&D costs are relatively low. Thus, even when reverse engineering of the incumbent's innovation is possible and costless, the patenting strategy is always dominated by the no patenting strategy when patenting costs are positive.

When these conditions do not obtain, the incumbent is able to use patent breadth to induce the entrant to locate further away in the quality product space than he would have located under no patent protection. When such relocation is possible, patenting may become an optimal strategy, with

the patenting choice dependent on the relative magnitude of the patenting costs vis-à-vis the extra profits that are obtained as a result of inducing the entrant to alter his location choice.

An important result of the paper is that a patent can be effective at altering the entrant's location choice even if the incumbent innovator does not defend the patent when it is violated. This result occurs because the incumbent's decision to defend a patent by invoking a trial is one that the entrant can influence by his choice of location. Under a specific set of conditions – most notably when the entrant's R&D costs are relatively low and his trial costs are relatively high – the incumbent selects a patent breadth that results in the entrant choosing a location that maximizes her profit, even though the choice of this location also means that the incumbent will not find it optimal to defend the patent by invoking a trial. Thus, situations where the patentee does not actively defend violated patents may in fact be optimal – the entrant is not getting away with robbery, but instead has been induced to select this location by the incumbent. Such a strategy is optimal because a relatively narrow rather than broad patent breadth achieves greater product differentiation and thus greater profits for the incumbent.

The above results hold under our model assumptions of complete and perfect information, single entry, a deterministic R&D process and possible and costless reverse engineering of the innovator's product. Relaxing the above assumptions is the focus of future research.

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## Appendix

### A<sub>1</sub>. The existence of $\bar{b} \in (0, 1]$

The solution of the condition  $b = \bar{q}_e$  in terms of  $b$  yields the following two roots:

$$b_1 = \frac{1 + 9\alpha\Pi_m + \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha} \quad \text{and} \quad b_2 = \frac{1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}.$$

The root  $b_1$  is rejected as a possible solution since  $b_1 > 1 \forall \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}), \Pi_m > 0, C_p^T \geq 0, \alpha \in (0, 1)$ .

The root  $b_2$  is accepted as a possible solution as  $b_2 \in (0, 1)$  for

$$\Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}), \Pi_m > 0, C_p^T \geq 0, \alpha \in (0, 1). \text{ Given the above } b_2 = \bar{b}.$$

### A<sub>2</sub>. Conditions for no patent protection $q_e^* > \bar{b}$

Given that  $q_e^* = \frac{4}{9\beta}$  and  $\bar{b} = \frac{1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}$  the condition  $q_e^* > \bar{b}$  can be

written as  $\beta < \frac{8\alpha}{9(1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})} = \beta_0$ ; thus,  $\bar{b}$  can be written as  $\bar{b} = \frac{4}{9\beta_0}$ .

- The Effect of  $\alpha$ ,  $C_p^T$  and  $\Pi_m$  on  $\beta_0$ .

$$\frac{\partial \beta_0}{\partial \alpha} = \frac{-8\alpha(9\Pi_m - \frac{162\alpha\Pi_m^2 - 18\Pi_m + 36C_p^T}{2\sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}) + 8(1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})}{9(1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})^2} \geq 0$$

$$\forall \alpha \in (0, 1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}).$$

$$\frac{\partial \beta_0}{\partial C_p^T} = \frac{16\alpha^2}{(\sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})(-1 - 9\alpha\Pi_m + \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})^2} \geq 0$$

$$\forall \alpha \in (0, 1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}).$$

$$\frac{\partial \beta_0}{\partial \Pi_m} = \frac{8\alpha \left( \frac{162\alpha^2 \Pi_m - 18\alpha}{2\sqrt{1+36\alpha C_p^T - 18\alpha \Pi_m + 81\alpha^2 \Pi_m^2}} - 9\alpha \right)}{9(\sqrt{1+36\alpha C_p^T - 18\alpha \Pi_m + 81\alpha^2 \Pi_m^2} - 1 - 9\alpha \Pi_m)^2} \leq 0 \quad \forall \alpha \in (0,1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}).$$

$$\frac{\partial^2 \beta_0}{\partial \Pi_m^2} = -\frac{16\alpha \left( \frac{162\alpha^2 \Pi_m - 18\alpha}{2\sqrt{1+36\alpha C_p^T - 18\alpha \Pi_m + 81\alpha^2 \Pi_m^2}} - 9\alpha \right)^2}{9(\sqrt{1+36\alpha C_p^T - 18\alpha \Pi_m + 81\alpha^2 \Pi_m^2} - 1 - 9\alpha \Pi_m)^3} +$$

$$\frac{8\alpha \left( \frac{(162\alpha^2 \Pi_m - 18\alpha)^2}{4(1+36\alpha C_p^T - 18\alpha \Pi_m + 81\alpha^2 \Pi_m^2)^{3/2}} + \frac{18\alpha^2}{\sqrt{1+36\alpha C_p^T - 18\alpha \Pi_m + 81\alpha^2 \Pi_m^2}} \right)}{9(\sqrt{1+36\alpha C_p^T - 18\alpha \Pi_m + 81\alpha^2 \Pi_m^2} - 1 - 9\alpha \Pi_m)^2} \geq 0$$

$$\forall \alpha \in (0,1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}).$$

The above imply that, the greater are the validity parameter,  $\alpha$ , and the incumbent's trial costs,  $C_p^T$ , and the smaller are the monopoly profits,  $\Pi_m$ , the greater is the critical value  $\beta_0$  and the more likely it is that patenting will not be an optimal strategy for the incumbent.

### A<sub>3</sub>. Conditions for the existence of $\tilde{b}$ and $\bar{b}$

If a patent breadth  $\tilde{b}$  that makes the entrant indifferent between infringing and not infringing the patent exists, it should satisfy the condition  $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b})$  and it should take values  $\tilde{b} \in (b_0, \bar{b}]$  when  $\bar{b} \in (\bar{b}, 1]$  exists and  $\tilde{b} \in (b_0, 1]$  when  $\bar{b} \in (\bar{b}, 1]$  does not exist. The solution of

$$\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b}) \Rightarrow \left( \frac{8\alpha^2}{81\beta} + \frac{\beta}{2} \right) \tilde{b}^2 - \frac{4}{9} \tilde{b} - C_e = 0 \text{ in terms of } \tilde{b} \text{ yields the following two roots:}$$

$$\tilde{b}_{1,2} = \frac{9(4\beta \pm \sqrt{2\sqrt{\beta}\sqrt{16C_e^T \alpha^2 + 8\beta + 81C_e^T \beta^2}})}{16\alpha^2 + 81\beta^2}. \text{The root}$$

$$\tilde{b}_1 = \frac{9(4\beta - \sqrt{2\sqrt{\beta}\sqrt{16C_e^T \alpha^2 + 8\beta + 81C_e^T \beta^2}})}{16\alpha^2 + 81\beta^2} \leq 0 \quad \forall \beta > \frac{4}{9}, \alpha \in (0,1) \wedge C_e^T \geq 0 \text{ and it is thus rejected.}$$

$$\text{The root } \tilde{b}_1 = \frac{9(4\beta + \sqrt{2\sqrt{\beta}\sqrt{16C_e^T \alpha^2 + 8\beta + 81C_e^T \beta^2}})}{16\alpha^2 + 81\beta^2} \geq 0 \text{ for } \beta > \frac{4}{9}, \alpha \in (0,1) \wedge C_e^T \geq 0 \text{ and it is}$$

accepted as a possible solution. If  $\tilde{b} = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2}$  exists it should also satisfy

the condition  $b_0 < \tilde{b} \leq 1$  if  $\bar{b} \in (\bar{b}, 1]$  does not exist or the condition  $b_0 < \tilde{b} < \bar{b}$  if  $\bar{b} \in (\bar{b}, 1]$  exists. The

condition  $\tilde{b} > b_0$  is satisfied since  $\tilde{b} - b_0 = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2} - \frac{4}{9\beta} > 0 \quad \forall \beta > \frac{4}{9}$ ,

$\alpha \in (0, 1) \wedge C_e^T \geq 0$ . When  $\bar{b} \in (\bar{b}, 1]$  does not exist, the condition  $\tilde{b} \leq 1$  is satisfied for certain combinations

of  $\beta$ ,  $\alpha$  and  $C_e^T$  values. To determine the combinations of  $\beta$ ,  $\alpha$  and  $C_e^T$  values which satisfy the

condition  $\tilde{b} \leq 1$ , the pairs of  $\beta$ ,  $\alpha$  and  $C_e^T$  values that satisfy the above constraint as an equality ( $\tilde{b} = 1$ ) are

determined first. The solution of  $\tilde{b} = 1$  with respect to  $C_e^T$  yields  $C_e^T = \frac{16\alpha^2 - 72\beta + 81\beta^2}{162\beta}$ . The area to the

right of the locus  $\tilde{b} = 1$  represents all combinations of  $\beta$  and  $C_e^T$  values, for a given  $\alpha$  value, for which

$\tilde{b} < 1$ . When  $\bar{b} \in (\bar{b}, 1]$  exists, the condition  $\tilde{b} < \bar{b}$  is satisfied when

$$C_e^T < \frac{(4 + 81\beta C_p^T - 81\beta \Pi_m)(-288\alpha\beta + (16\alpha^2 + 81\beta^2)(81\beta C_p^T + 4) - 81\beta(4\alpha - 9\beta)^2 \Pi_m)}{162\alpha^2 \beta(4 - 81\beta \Pi_m)^2}.$$

If a  $\tilde{b}$  that makes the entrant indifferent between infringing the patent and inducing a trial and infringing the patent and not inducing a trial exists, it should solve  $E(\Pi_e^I)^T(\tilde{b}) = (\Pi_e^I)^{NT}(\tilde{b})$  and it should take values  $\tilde{b} \in (\bar{b}, \bar{b})$  when  $\bar{b} \in (\bar{b}, 1]$  exists and  $\tilde{b} \in (\bar{b}, 1]$  when  $\bar{b} \in (\bar{b}, 1]$  does not exist. The solution of  $E(\Pi_e^I)^T(\tilde{b}) = (\Pi_e^I)^{NT}(\tilde{b})$  in terms of  $\tilde{b}$  yields four roots one of which satisfies all parameter constraints.

#### A<sub>4</sub>. Conditions under which a $\hat{b}$ that deters entry exists

First we determine the necessary conditions for entry deterrence by noting that when at  $\bar{b}$

$\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) > 0$  there is no patent breadth value that, if chosen by the incumbent, can deter

the entrant from entering the market. This is so because  $\frac{\partial \Pi_e^{NI}}{\partial b} < 0 \quad \forall b \in (b_0, \bar{b}]$  and  $\frac{\partial (\Pi_e^I)^{NT}}{\partial b} > 0 \quad \forall b \in [\bar{b}, \bar{b})$

while at  $\bar{b} \lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b})$ . Thus, entry deterrence requires that at  $\bar{b}$

$\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) \leq 0$ . It is straightforward to show that  $\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) = 0$  when

$\beta = 2\beta_0$  where  $\beta_0$  is defined in Appendix A<sub>2</sub>. Since both  $\Pi_e^{NI}$  and  $(\Pi_e^I)^{NT}$  are decreasing in  $\beta$ ,

$\lim_{e \rightarrow 0} \Pi_e^{NI}(\bar{b} + e) = (\Pi_e^I)^{NT}(\bar{b}) \leq 0$  for  $\beta \geq 2\beta_0$ . Thus, the condition  $\beta \geq 2\beta_0$  is a necessary condition for entry

deterrence. Given the above, the patent breadth  $\hat{b} = \bar{b}$  will deter entry when for  $\beta = 2\beta_0$ ,  $E(\Pi_e^I)^T(\bar{b}) \leq 0$

(the assumption is made that when the entrant is indifferent he will not enter). Substituting for  $b = \bar{b}$  and

$\beta = 2\beta_0$  into the entrant's profit function under entry, infringement and trial, given by equation (13), and

noting that  $\bar{b} = \frac{4}{9\beta_0}$  we get  $E(\Pi_e^I)^T(\bar{b}) = \frac{64\alpha^2}{6561\beta_0^3} - C_e^T$ . Thus,  $E(\Pi_e^I)^T(\bar{b}) = \frac{64\alpha^2}{6561\beta_0^3} - C_e^T \leq 0$  when

$C_e^T \geq \frac{64\alpha^2}{6561\beta_0^3} = \bar{C}_e^T$ . Thus, entry can be deterred when  $\beta \geq 2\beta_0$  and  $C_e^T \geq \frac{64\alpha^2}{6561\beta_0^3} = \bar{C}_e^T$ ; these conditions

give rise to the entry deterrence outcome depicted in Figure 5, panel (i). Entry can also be deterred when for

$\beta = 2\beta_0$ ,  $E(\Pi_e^I)^T(\bar{b}) > 0$ , a  $\tilde{b} \in (b_0, 1]$  exists and at  $\tilde{b}$ ,  $E(\Pi_e^I)^T(\tilde{b}) \leq 0$ . Substitution of  $\tilde{b}$  into the entrant's

profit function under entry, infringement and trial, given by equation (13), yields

$E(\Pi_e^I)^T(\tilde{b}) = \frac{512\alpha^2}{6561\beta^3} - C_e^T$  and thus,  $E(\Pi_e^I)^T(\tilde{b}) = \frac{512\alpha^2}{6561\beta^3} - C_e^T \leq 0$  when  $C_e^T \geq \frac{512\alpha^2}{6561\beta^3} = \hat{C}_e^T$ . Thus, in

this case, entry will be deterred when  $\beta \geq 2\beta_0$ ,  $C_e^T < \frac{64\alpha^2}{6561\beta_0^3} = \bar{C}_e^T$  and  $C_e^T \geq \frac{512\alpha^2}{6561\beta^3} = \hat{C}_e^T$ ; these

conditions give rise to the entry deterrence outcome depicted in Figure 5, panel (ii). Note that when

$\beta = 2\beta_0$ ,  $\hat{C}_e^T = \frac{64\alpha^2}{6561\beta_0^3} = \bar{C}_e^T$  and that  $\hat{C}_e^T < \bar{C}_e^T$  for  $\beta > 2\beta_0$ . Thus, the condition  $C_e^T \geq \frac{512\alpha^2}{6561\beta^3} = \hat{C}_e^T$  is a

sufficient condition for entry deterrence. Given the above, the conditions  $\beta \geq 2\beta_0$  and  $C_e^T \geq \frac{512\alpha^2}{6561\beta^3} = \hat{C}_e^T$

are the necessary and sufficient conditions, respectively, for entry deterrence (see Figure 7).

### A5. Optimal patent breadth under infringement and trial

When the incumbent wishes to induce infringement and trial she chooses the patent breadth that maximizes her expected profits given by equation (4) subject to the constraint  $\tilde{b} + e \leq b \leq 1$  as shown in equation (A5.1):

$$(A5.1) \quad \begin{aligned} \max_b E(\Pi_p^I)^T &= \mu \Pi_m + (1 - \mu) \pi_p - C_p^T = (1 - ab) \Pi_m + ab \frac{q_e}{\theta} - C_p^T \\ \text{s.t. } \tilde{b} + e &\leq b \leq 1 \quad \text{where } e \rightarrow 0 \end{aligned}$$

The patent breadth that solves equation (A5.1),  $\tilde{b} = \frac{81\beta}{8\alpha} \Pi_m$  does not maximize the incumbent's expected profits as the second order conditions for a maximum are violated. Thus, the optimal patent breadth under infringement and trial is one of the corner values,  $\tilde{b} + e$  or  $b = 1$ . When determining the patent breadth that maximizes her profits under infringement and trial, the incumbent faces the following tradeoff; by claiming a lower patent breadth (i.e.,  $\tilde{b} + e$ ) the incumbent reduces the risk of having the patent revoked during trial but the entrant locates closer to her than if a larger patent breadth was chosen (i.e.,  $b = 1$ ) (recall that under infringement and trial the entrant's quality choice is proportional to the breadth of the patent, i.e.,  $(q_e^I)^T = \alpha b q_e^*$ ). Which of the above two values maximizes the incumbent's expected profits under infringement and trial depends on the values of the exogenous parameters. Specifically, if  $\tilde{b} \leq \tilde{b} + e$ , then the optimal patent breadth under entry and infringement is given by  $b = 1$ , while if  $\tilde{b} > 1$ , then  $b = \tilde{b} + e$ . Since  $\tilde{b}$  is increasing in  $\Pi_m$ , the larger are the incumbent's monopoly profits, the more likely it is that the incumbent will choose  $b = \tilde{b} + e$  as the patent breadth with which to induce infringement. If  $\tilde{b} + e < \tilde{b} < 1$ , then the choice of the optimal patent breadth that induces infringement depends on where the incumbent's expected profits are greater. With  $b = \tilde{b} + e$ , the incumbent's expected profits are:

$$(A5.2) \quad \lim_{e \rightarrow 0} E(\Pi_p^I)_{b=\tilde{b}+e} = (1 - \alpha \tilde{b}) \Pi_m + \frac{4\alpha^2}{81\beta} \tilde{b}^2 - C_p^T$$

while with  $b = 1$ , the expected profits are:

$$(A5.3) \quad \lim_{e \rightarrow 0} E(\Pi_p^I)_{b=1} = (1 - \alpha) \Pi_m + \frac{4\alpha^2}{81\beta} - C_p^T$$

Assuming the incumbent induces infringement, she chooses  $b = \tilde{b} + e$  when

$E(\Pi_p^I)_{b=\tilde{b}+e} \geq E(\Pi_p^I)_{b=l}$ . This condition is satisfied when  $\Pi_m \geq \frac{4\alpha}{81\beta}(\tilde{b} + 1)$ . Thus, the greater are the

monopoly profits and the smaller is the validity parameter and the entrant's R&D effectiveness (i.e., the greater is  $\beta$ ), the more likely it is that the incumbent will find it optimal to induce infringement by choosing

the smaller patent breadth value (i.e.,  $b = \tilde{b} + e$ ).