Testing for Cointegration in the Presence of Moving Average Errors

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Mindy Mallory and Sergio H. Lence

Abstract

This study explores performance of the Johansen cointegration statistics on data containing negative moving average (NMA) errors. Monte Carlo experiments demonstrate that the asymptotic distributions of the statistics are sensitive to NMA parameters, and that using the standard 5% asymptotic critical values results in severe underestimation of the actual test sizes. We demonstrate that problems associated with NMA errors do not decrease as sample size increases; instead, they become more severe. Further we examine evidence that many U.S. commodity prices are characterized by NMA errors. Pretesting data is recommended before using standard asymptotic critical values for Johansen's cointegration tests.

KEYWORDS: cointegration, Johansen cointegration test, moving average

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1 Introduction

Since the development of cointegration tests by Johansen (1988), Johansen and Juselius (1990) and Johansen (1991), the error correction framework has been used to examine spatial arbitrage and the law of one price for commodities, trade conditions, and purchasing power parity, to name a few prominent examples (Corbae and Ouliaris, 1988; Enders, 1988; Kim, 1990; Goodwin and Schroeder, 1991; Goodwin, 1992; Johansen and Juselius, 1992; Chowdhury, 1993; Kugler and Lenz, 1993; Adamowicz and Luckert, 1997; Bahmani-Oskooee, 1998; Asche et al., 1999; Boyd, Caporale and Smith, 2001; Goodwin and Piggott, 2001; Caporale and Chui, 2002; Sephton, 2003; Hoover et al., 2008).

Because of this, a large literature examining the performance of the Johansen cointegration tests has emerged. There are several studies showing that the Johansen tests often possess poor small-sample properties and have low power against a persistent alternative (Cheung and Lai, 1993; Toda, 1995; Johansen, 2002; Reimers, 1992; Gonzalo and Pitarakis, 1999; Toda and Phillips, 1994; Haug, 1996). For applied researchers, these issues generally are assumed to be of little concern for sufficiently long samples, and there has been little attention to the fact that long samples are not always enough to ensure good performance of the Johansen cointegration tests within certain types of data.

Saikkonen (1992) showed that when a set of cointegrated data has a moving average (MA) error structure, the asymptotic distribution of Johansen's trace and maximum eigenvalue tests are the same as those reported in Johansen (1988) and Osterwald-Lenum (1992)—provided the lag length of the vector error correction model (VECM) tends to infinity with the sample size at a sufficient rate. However, applied studies generally choose lag length by employing one (or more) of the standard lag length selection criterion (e.g., the Akaike Information Criterion or the Bayesian Information Criteria), which may be too small to ensure asymptotic properties of the test statistic. Further, choosing a large lag length has the problem of sacrificing power due to the large number of parameters to be estimated.

Our Monte Carlo evidence suggests that in the presence of MA errors, specifically negative moving average (NMA) errors, the size distortion can be severe even with quite long lag lengths. Further, the power lost by increasing the lag length is noticeably costly. This puts the applied researcher in a quandary: the researcher can either choose a small lag length and a high probability of committing a type I error, or choose a large lag length and a high probability of committing a type II error when a NMA is present in the data generating process (DGP). We make four contributions related to this issue: (1) we explore the nature of this trade off by examining the properties of the Johansen cointegration tests as sample size increases for DGPs containing an NMA in the error term; (2) we offer

to the applied researcher some pre-testing and post-testing techniques, which may be helpful in deciding whether NMA errors are a problem in their specific application; (3) we show how Monte Carlo simulations of the distribution of the test statistic could be used to generate critical values that achieve both appropriate size and high power; and (4) we demonstrate these techniques on a large number of actual price series.

A key component in this type of analysis is determining how much data is required to safely assume asymptotic properties will be obtained. Prior Monte Carlo studies did not draw a connection between the simulation experiments and the type of data they intended to mimic. The simulated DGPs did not correspond to daily, weekly, monthly, quarterly, or annual observations; instead the variance was set within a convenient range. This distinction, however, is critically important since the context of the economic problem determines how much data one needs. For example, a series containing 400 observations of *annual* data is longer than social scientists ever enjoy, but 400 observations of *daily* data contain scarcely more than a year's worth of information. This is a point that is at the essence of the small sample literature regarding observational frequency and power of the tests. While not entirely unanimous, these tests generally conclude that for a given DGP, observing with more frequency has almost no effect on the power of the test (Shiller and Perron, 1985; Hakkio and Rush, 1991; Hooker, 1993; Lahiri and Mamingi, 1995; Otero and Smith, 2000). In our study, we construct simulated data series of varying lengths and varying severity of MA errors that mimic "daily" and "monthly" price data. By this we mean that the random disturbances of our Monte Carlo experiment's DGP have annualized volatilities comparable to those of the actual series of commodity prices.

2 Testing for cointegration using the Johansen tests

Consider an m vector $\underline{X}_t$ of $I(1)$ variables. If they are cointegrated, it means that there exist r ($0 < r < m$) linear combinations of such variables that are stationary. In other words, there are r long-run relationships among the m $\underline{X}_t$ variables. In general, vector $\underline{X}_t$ with cointegrating rank r ($0 \leq r \leq m$) can be represented as the VECM

$$(1) \quad \Delta \underline{X}_t = \Pi \underline{X}_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta \underline{X}_{t-i} + \underline{e}_t,$$

where Π is an $(m \times m)$ matrix with long-run impacts, Γ s are $(m \times m)$ lag parameter matrices, and $\underline{e}_t$ is an m -vector of residuals. If $0 < r < m$, matrix Π can be expressed as $\Pi = \alpha \beta^T$, where α is an $(m \times r)$ matrix comprising the speed-of-adjustment coefficients to the long-run relationships, β is an $(m \times r)$ matrix

containing the r cointegrating vectors, and α and β have rank $r = \text{rank}(I\Gamma)$. This means that one can test for cointegration by testing for the rank of matrix $I\Gamma$. There are r cointegrating relationships among the $\underline{X}_t$ variables if $0 < r < m$, whereas there is no cointegration if $r = 0$. If $r = m$, vector $\underline{X}_t$ is stationary (i.e., $I(0)$).

Johansen (1988), Johansen and Juselius (1990), and Johansen (1991) addressed the full maximum likelihood estimation of (1) for finite k and an m -variate identically and independently normally distributed vector of residuals. They also developed a trace statistic and a maximum eigenvalue statistic to test for cointegration. The trace test statistic for the null hypothesis that there are, at most, r cointegrating vectors is computed as $-T \sum_{i=r+1}^m \ln(1 - \hat{\lambda}_i)$, where T is the number of dates in the sample, $\hat{\lambda}_i$ s are the ordered eigenvalues of $S_{11}^{-1} S_{10} S_{00}^{-1} S_{01}$, $S_{ij} \equiv T^{-1} \sum_t R_{it} R_{jt}^T$, and R_{it} and R_{jt} represent the residuals obtained from regressing $\Delta \underline{X}_t$ and $\underline{X}_{t-1}$, respectively, on $\Delta \underline{X}_{t-1}, \dots, \Delta \underline{X}_{t-k+1}$. The maximum eigenvalue statistic is used to test the null hypothesis that there are r cointegrating vectors against the alternative of $r+1$ cointegrating vectors, and consists of $-T \ln(1 - \hat{\lambda}_{r+1})$.

3 Previous small-sample investigations

Johansen's trace and maximum eigenvalue tests of cointegrating rank are *asymptotic* likelihood ratio tests. Hence, they need not exhibit desirable properties in small samples. It is known that the tests suffer from size distortions and low power in small samples, especially when the error correction model produces residuals that are nearly integrated of order one or $I(1)$. Several Monte Carlo studies have been published outlining the severity of these issues.

Cheung and Lai (1993) determined the finite-sample sizes of the Johansen tests and quantified the finite-sample critical values using response surface analysis. They concluded that the Johansen tests are biased toward rejecting a null of no cointegration too often in finite samples, compared to the asymptotic distribution of the test statistics. Further, they concluded that the bias worsens as the dimension of the system or the length of the lag structure increases. This contrasts with our findings that increasing the lag length reduces the problem of size distortions when NMA errors are present.

Toda (1995) performed an independent study of the finite-sample performance of Johansen's trace tests and determined that the simulated distribution of the asymptotic test statistic under the null was reasonably good, with 100 observations. However, 100 observations were not enough to determine the true cointegrating rank under a stationary alternative, if one or more of the

stationary roots of the process was nearly 1; that is, the trace tests have low power against a persistent alternative. Unlike Cheung and Lai, Toda asserted that this leads to underestimation of the cointegrating rank because of the nature of sequential testing inherent in the Johansen procedure. Further, he found that the trace test's performance is affected by the initial values of the stationary component of the process. Toda concluded that one needs at least 300 observations for Johansen's trace test to perform well uniformly over the range of finite-sample scenarios that he considered. However, his simulated data were not calibrated to a specific frequency, so Toda's recommendation is difficult for the applied researcher to interpret.

Alternative to determining the critical values of the actual finite-sample distribution, small-sample corrections to the test statistics or critical values have been proposed. Johansen (2002) advocated a correction factor that depends on parameters of the error correction model, as well as the sample size. However, the correction is fairly complicated to apply (the components of the correction which depend only on functionals of a random walk are simulated and described in Johansen et al., 2005), and it is not clear that one cannot obtain better estimates of the small-sample critical values from simulating the small-sample distributions directly, since the correction of the statistics developed by Johansen (2002) requires estimating the parameters of the data as well.

Ahn and Reinsel (1990) and Reimers (1992) developed a correction that is a simple function of sample size, system dimension, and lag order. However, as part of their Monte Carlo analysis, Cheung and Lai (1993) concluded that the Ahn-Reinsel method does not yield unbiased estimates of the finite-sample critical values.

Saikkonen and Luukkonen (1997) derive some alternatives to Johansen's trace and maximal eigenvalue statistics. These tests are based on using a finite order VAR to estimate an infinite order or VARMA process, which means that the finite approximating lag length must grow with the sample size. In Monte Carlo simulations designed to compare these tests with their traditional counterparts, Saikkonen and Luukkonen find that the statistics often can achieve large size improvements, but not always:

“However, our results also indicate that one can always find cases where even these tests have problems with their size. In particular, it is possible that the null hypothesis is rejected quite too frequently”

Therefore, there is not a clear picture on how to best proceed when one's DGP is likely to have an MA error structure. The next section describes the Monte Carlo analysis we conducted to investigate the interplay between sample size and lag length, and size and power under these circumstances.

4 A Monte Carlo study

We follow Gonzalo (1994) in choosing the DGP for our study, as it nests the DGPs used by other researchers (e.g., Banerjee, 1986; Haug, 1996). The DGP for the I(1) variables $X_{1,t}$ and $X_{2,t}$ consists of

$$(2) \quad X_{1,t} - b X_{2,t} = \epsilon_t, \quad \epsilon_t = \rho \epsilon_{t-1} + u_t,$$

$$(3) \quad a_1 X_{1,t} + a_2 X_{2,t} = \psi_t, \quad \psi_t = \psi_{t-1} + v_t, \quad v_t = w_t + \theta w_{t-1},$$

$$(4) \quad \begin{bmatrix} u_t \\ w_t \end{bmatrix} \stackrel{iid}{=} N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \sigma_{uw} \\ \sigma_{uw} & \sigma_w^2 \end{bmatrix} \right),$$

where parameters ρ and θ respectively govern the cointegrating relationship between $X_{1,t}$ and $X_{2,t}$, and the MA in the error term. In terms of the VECM representation (1), equations (2) and (3) can be expressed as a system with $\alpha = [a_2/(a_2 + b a_1) (\rho - 1), -a_1/(a_2 + b a_1)]^T$, $\beta = [1, -b]$, $k = 0$, $e_{1t} = (a_2 u_t + b v_t)/(a_2 + b a_1)$, and $e_{2t} = (v_t - a_1 u_t)/(a_2 + b a_1)$.

According to this DGP, variables $X_{1,t}$ and $X_{2,t}$ are cointegrated if $|\rho| < 1$, whereas they are not cointegrated if $\rho = 1$. A positive (negative) MA component in the error terms exists when θ is greater (smaller) than zero. We closely follow Haug (1996) by running simulations for each parameter scenario (ρ, θ) , where $\rho \in \{0.85, 0.90, 0.95, 1\}$ and $\theta \in \{-0.8, 0, 0.8\}$. These parameter values allow us to illustrate the effect of an MA component in the error term on the size distortion and power of the Johansen tests. We run the models for (a_1, a_2, b) such that $a_1 \in \{1, 0\}$, $a_2 \in \{1, 0\}$, $b \in \{0.5, 1, 2\}$, except for the tuples $(a_1, a_2, b) = (1, 0, 0.5)$ and $(0, 1, 2)$ because they result in a non positive definite covariance matrix.

Values for the covariance matrix of the exogenous shocks u_t and w_t can be selected so as to yield typical volatility levels for price changes. For that purpose, note that for $\rho \in (-1, 1]$ the covariance matrix in (4) is given by¹

$$(5) \quad \sigma_u^2 = 0.5 (1 + \rho) (\sigma_1^2 - 2 b \sigma_{12} + b^2 \sigma_2^2).$$

$$(6) \quad \sigma_w^2 = \frac{1}{1 + \theta^2} (a_1^2 \sigma_1^2 + 2 a_1 a_2 \sigma_{12} + a_2^2 \sigma_2^2),$$

¹See Appendix A for the derivation of (5), (6), and (7).

$$(7) \quad \sigma_{uw} = \frac{1}{1 + \theta(\rho - 1)} [a_1 \sigma_1^2 + (a_2 - b a_1) \sigma_{12} - a_2 b \sigma_2^2],$$

where σ_1^2 and σ_2^2 are the variances of $\Delta X_{1,t}$ and $\Delta X_{2,t}$, respectively, and σ_{12} is the covariance between $\Delta X_{1,t}$ and $\Delta X_{2,t}$. These equations allow one to tailor the Monte Carlo experiment to mimic data of any frequency level.

To set σ_u^2 , σ_w^2 , and σ_{uw} at values consistent with the variance that might be found in real world commodity prices, we estimated the *annualized* volatilities for the first-differenced series listed in Table 1, so as to obtain representative values of σ_1^2 and σ_2^2 that could then be plugged into (5) through (7) to compute the covariance matrix of the exogenous shocks. The mean value of such estimates was 0.25 and the median value was 0.24. The 10% and 90% quantiles were 0.08 and 0.39, respectively. Therefore, the simulated daily (monthly) data series of exogenous shocks u_t and w_t were obtained by fixing $\sigma_1^2 = \sigma_2^2 = 0.00025$ (0.0052).² To avoid unduly high or low correlation between $\Delta X_{1,t}$ and $\Delta X_{2,t}$, their covariance was set at $\sigma_{12} = 0.000125$ (0.0026) to generate the simulated daily (monthly) series, implying a correlation of 0.5.

We performed a Monte Carlo analysis of the Johansen test statistics by simulating 20,000 series for both $\Delta X_{1,t}$ and $\Delta X_{2,t}$ under each parameterization, and for sample lengths varying from one month up to 100 years for daily data, and from 2 years up to 100 years for monthly data. To enhance the estimation of the distributions of the Johansen test statistics, the 20,000 series were obtained by first generating 10,000 series for random variables u_t and w_t , and then computing their antithetic replications (Geweke, 1988). In addition, “common random numbers” were employed to improve accuracy in the comparison across alternative scenarios (i.e., all scenarios are based on the same simulated series of exogenous random variables u_t and w_t).

Each replication was used to fit the bivariate version of the VECM assuming zero-mean residuals, and to compute the Johansen’s trace and maximum eigenvalue test statistics associated with the null hypothesis that $r \leq 0$ for lag specifications of $k = 2, 4$, and 5. Tables 2-5 display the results when the DGP is defined by $(a_1, a_2, b) = (1, 1, 1)$; the remaining DGP scenarios are

²An annualized volatility of $\sigma_{\text{annual}} = 0.25$ is the same as an annualized variance of $\sigma_{\text{annual}}^2 = 0.0625$, which implies daily and monthly variances of $\sigma_{\text{daily}}^2 = 0.00025$ ($= 0.0625/250$, assuming 250 trading days per year) and $\sigma_{\text{monthly}}^2 = 0.0052$ ($= 0.0625/12$), respectively.

presented in Appendix B and demonstrate the robustness of our findings to a variety of DGP scenarios.

Tables 2 and 3 contain the empirical size of the Johansen tests based on the corresponding standard 5% asymptotic critical values for daily and monthly data, respectively, under the different scenarios. The tables also report the actual 5% critical values computed from the simulated distribution of the statistics for each case. According to Table 2, when the MA component of the DGP is zero or positive ($\theta \geq 0$), the small-sample size distortion disappears in daily data with little more than six months of observations, and the statistics have surprisingly small distortions with as little as three months of observations. Interestingly, the max test tends to under-reject the null hypothesis of no cointegration for large samples, especially for positive MA values, as the empirical size corresponding to the standard 5% asymptotic critical value is only 2% for samples longer than 2 years. The likely reason for the under-rejection is that the lag length used for estimation and testing is $k = 2$, whereas the true DGP has a lag length $k = 0$.

In contrast, when data are generated with an NMA in the error term, there are severe over-rejection distortions which increase with sample size. For example, for a lag length of $k = 2$ and samples exceeding 10 years of data, the empirical sizes of the standard 5% asymptotic critical values are 77% and 72% for the trace and max test statistics, respectively. Increasing the lag length to $k = 4$ or $k = 5$ is helpful in reducing the size distortion, but even including 5 lags yields an empirical size of 26% (20%) for the trace (max) test statistics corresponding to the standard 5% asymptotic critical value. And increasing the lag length is costly in terms of precision of the estimate of the mean reverting parameter in the VECM, which is often the primary parameter of interest in these kinds of studies. This finding is related to the work of Saikkonen (1992) and Saukkonen and Luukkonen (1997) regarding estimating the cointegrating rank of vector ARMA processes. They showed that since a stationary ARMA process has a representation as an infinite order AR process, when $T \rightarrow \infty$ and $k \rightarrow \infty$ at the same rate, the asymptotic critical values tabulated in Johansenn (1988) and Osterwald-Lenum (1992) are appropriate.

The columns showing the actual 5% critical values computed from the simulated distributions reveal that for NMA structures, they need not vary monotonically with the size of the sample. Further, they do not seem to achieve convergence even for samples comprising as many as 100 years of daily observations.

In Table 3, we repeat the analysis of the small-sample size distortion for monthly data and we see the same patterns as with daily data. The small-sample size distortion is small when the MA component of the DGP is zero or positive. The empirical size reaches reasonable levels of accuracy with 10 years of data. As

in the daily data simulation, the max test tends to under-reject the null hypothesis of no cointegration.

When the MA term in the DGP is negative, we see the same severity of size distortion as we saw in the daily data for both the trace and the max test. The size distortion is increasing in sample size. The trace and max tests reach an empirical size of 0.75 and 0.70, respectively, with 100 years of monthly observations when $k = 2$ lags are included. As before, increasing the lag length reduces the size distortion, but even with a lag length of $k = 5$, the empirical size of the trace and max tests are 0.26 and 0.20 with 100 years of data, respectively.

As we saw in the daily data, the actual 5% critical values of the simulated distributions of the trace and max statistics are not monotonically increasing in sample size, and the 5% critical values do not seem to have converged even with 100 years of monthly observations.

Comparing Tables 2 and 3 for sample lengths of four or more years, we also observe that frequency of observation does not seem important in the severity of size distortion in very long samples, but it does seem important for shorter ones. For example, with a sample length of 10 years and a lag length of $k = 2$, the empirical size of the trace test when a NMA is present is 0.77 and 0.59 for daily and monthly data, respectively. With 100 years of data and a lag length of $k = 2$, the empirical size of the trace test is 0.77 and 0.75 for daily and monthly data respectively.

Tables 4 and 5 contain the results of a simulation analysis of the power of the test statistics for daily and monthly data, respectively, generated under the alternative hypothesis of cointegration. The reported values are *size-adjusted* powers, so the numbers represent the probability of rejecting the false null hypothesis using the appropriate small-sample critical values shown on the last two columns of Tables 2 (for daily data) and 3 (for monthly data). Given the size distortions reported in Tables 2 and 3, this is an important distinction.³

The trace statistic performs relatively well in terms of power when $\theta \geq 0$. The simulation results demonstrate how the test loses the ability to discern a cointegrating relationship from increasingly persistent alternative hypotheses. When the DGP has $\rho = 0.85$, the power of the test is reasonable for sample lengths as small as six months, and has very high power for samples of two years or longer. The power becomes weaker when the DGP is closer to the null hypothesis of no cointegration with $\rho = 0.90$ and $\rho = 0.95$. In these cases, two years of data are required before the test statistic has reasonable power; however, the power is very high for samples comprising four years of daily observations or more.

³If the asymptotic critical values were used instead, the over-rejection of the null associated with large samples under the NMA scenarios would mean a smaller probability of committing type II errors, which would imply an overstatement of the power of the tests.

In Table 5 we see a similar pattern in the trace and max statistics for monthly data. The trace and max statistics perform better when $\theta \geq 0$ than when $\theta < 0$, but much longer time periods are required to achieve the same level of power for monthly data than for daily. For example, 100 years worth of data are required to achieve a power of 1 in the best case with monthly data, while it takes only two years with daily data. When $\theta < 0$, even 100 years of data may not achieve high power if only two lags ($k = 2$) are used. This, combined with the results of Tables 2 and 3, suggests that data frequency may have substantial impacts on the power of the tests, as well as on the size distortions, due to both finite samples and NMA.

The results of Tables 2 and 3 verify Saikkonen's result that lag length must increase with sample size when an NMA is present, in order to eliminate size distortions. Tables 4 and 5, however, show that this comes at a cost in terms of power, since the number of parameters to be estimated increases significantly with lag length. Testing in this environment may benefit significantly from using critical values from a simulated distribution and a parsimonious specification, achieving both lower size distortion and high power.

5 A visual inspection of the performance of the Johansen statistics

The Monte Carlo results illustrate that cointegration testing when one's data is suspected to have an NMA is not straightforward. In this case, one may benefit from a qualitative examination of how the cointegration tests are performing. As a thought experiment, consider what one would expect to happen to the Johansen test statistics as more observations are added to each time series sample. Additional data points will influence the likelihood function and cause the value of the test statistic to be perturbed by a small amount. If the test statistics converge asymptotically to some distribution, one should expect a decreasing effect of additional entries as the sample size gets large.

In each of Figures 1-3, one 20-year realization of monthly observations for the DGP under each of the following scenarios was simulated: (a) null hypothesis of no cointegration and no NMA in the error term; (b) null hypothesis of no cointegration and an NMA in the error term; and (c) alternative hypothesis of cointegration and an NMA in the error term. It must be kept in mind that Figures 1 through 3 represent a single realization of the DGP and are not necessarily representative of the distributions of the test statistics; however, they still provide a useful tool for analysis because they can be compared to figures we will generate later based on actual data series.

Each point on the solid line denotes the value of the test statistic computed from a sample that starts on the date shown on the horizontal axis corresponding

to that point, and ends on the last date along the horizontal axis. That is, the test statistics depicted by the solid line are based on progressively larger samples as one looks from right to left. In contrast, the test statistics represented by the dotted line are calculated from successively larger samples as one looks from left to right. This is true because each point on the dotted line is a test statistic obtained from a sample starting on the horizontal axis' first date, and ending on the date shown along the horizontal axis corresponding to that point. As a reference, each graph includes the standard 5% asymptotic critical value as a horizontal line.

Figure 1 was created from monthly data generated under the null hypothesis of no cointegration and no NMA in the error term. Recall from the Monte Carlo study that under this scenario the Johansen tests perform well; size distortions quickly disappear and power is quite good even for samples of modest length. The figure is consistent with this as well. The graph shows the cointegration test statistics varying as new data points are added. Each new data point perturbs the statistic, but the line appears to be stationary. Also, recall that the horizontal lines represent the standard 5% asymptotic critical value of the test statistics. Since test statistics are almost always below the critical value, one would correctly fail to reject the null hypothesis except in a few of the subsamples. This is exactly what one would expect to see when the statistics perform as the asymptotics suggest.

Looking at Figure 2, we see a different picture. The DGP underlying this figure involves the null hypothesis of no cointegration, but with an error term featuring an NMA. The plot is characterized by increasing values of the test statistics as sample size increases, and for almost all subsamples one would easily (but incorrectly) reject the null hypothesis of no cointegration.

Figure 3 was constructed using 20 years of monthly data generated under the alternative hypothesis of cointegration, and an NMA in the error term. The NMA causes the values of the cointegration test statistics to increase with sample size in this case as well. The magnitudes of the test statistics under this scenario are generally much larger than under the scenario depicted in Figure 11, but using the standard asymptotic critical values one would easily reject the null hypothesis of no cointegration for both data series generated under the null and under the alternative. This supports the claim that when there is an NMA in the error term, it is not wise to test for cointegration by means of the standard asymptotic critical values for the Johansen statistics. This is due to the fact that when testing using a fixed k , when the sample size is large enough, the test will easily reject under both the null and alternative hypothesis.

6. Characterizing the time series properties of prices for major U.S. commodities

We have illustrated complications in cointegration testing when there is an NMA present in simulated data, but this is only an important question if this problem exists in actual data on which we may like to perform cointegration analysis. In this section we present evidence that these issues appear to be a problem in a large number of U.S. price series for important commodities, including major agricultural goods, energy products, and consumer food prices. Specifically, we show that an NMA error structure appears to be quite common for such series. While it is impossible to determine the true DGP for actual data, we will present empirical evidence from fitting an ARIMA model to each price series. Then we examine the prices using the more qualitative visual approach in the context of the cointegration testing introduced above.

A data dictionary is provided in Table 1, indicating the data source, the beginning and ending of the sample, and the frequency of each data set we examined. Our data series vary in length because we chose to use as much of the data as was available, so that in each case we have the longest data set possible. This is important because we showed earlier that the problems associated with an NMA in the error term are not a small-sample concern, but one that persists and even worsens into the asymptotics. All of the series are quite long, representing 20 to 40 years worth of data. Therefore, to remove the effects of inflation over this long period, each series was deflated by the Consumer Price Index (CPI) maintained by the Bureau of Labor Statistics.⁴

To provide support for the presence of MAs in the DGPs of our data, the ARIMA(p, q) model in equation (8) was fitted to each series enumerated in the data dictionary,

$$(8) \quad \Delta P_t = c + \varepsilon_t + \sum_{i=1}^p \phi_i \Delta P_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j},$$

where $\Delta P_t \equiv P_t - P_{t-1}$, P_t is the price at time t , c is a constant, ε_t is the error at date t , ϕ s are autoregressive (AR) parameters, and θ s are MA parameters. We used the Bayesian information criterion to select the lag lengths p and q ; we specifically noted the sign and tested the significance of the coefficients corresponding to the lagged error terms. Parameter estimates and p-values, as well as the R^2 for each data series are reported in Table 6. Parameters significantly different from zero are denoted by asterisks, and negative point estimates of MA parameters are identified by shaded cells.

⁴CPI data can be found at <http://www.bls.gov/data/>

Over 57% of the series under investigation (20 out of 35) have an NMA component in the error term significantly different from zero at the 5% level. The figure increases to 74% of the series (26 out of 35) when considering all series with negative point estimates on the lagged error terms, regardless of whether they are statistically significant or not. While the list of commodity prices being examined is not exhaustive, these findings indicate that NMA components in the error term are common, and pre-testing for this structure should be part of the routine in testing for cointegration.

6.1 Testing for cointegration in the data

Many subsets of the data analyzed in the previous section may be expected to contain cointegrating relationships based on long-run equilibrium and technological production processes present within industry groups. These include the crack spread (crude oil, gasoline, and diesel); the soybean crush (soybean, soybean oil, and soybean meal); the cattle crush (corn, feeder cattle, and live cattle); wheat, wheat flour, and bran; lean hogs and pork bellies; corn gluten feed and corn gluten meal; whole chicken, breast, and legs; feedgrains (sorghum, corn, barley, and oats); and milk and butter.

We performed Johansen's trace and maximum eigenvalue tests of cointegrating rank on these commodity groups and reported the results in Table 7. The tests clearly indicate that commodity prices in each group have at least one cointegrating vector, as the null of no cointegrating vectors ($r \leq 0$) is rejected for each commodity group at the 1% significance level, except for corn gluten feed and meal where it is rejected at the 5% level of significance. The statistics are generally large so that the null hypothesis of no cointegration is easily rejected. In fact, for lean hogs-pork bellies and the soybean crush, the statistics are so large that in sequential testing for the number of cointegrating relationships, the tests conclude that there are as many cointegrating relationships as there are equations. This is equivalent to a joint test that each price series is stationary. In an empirical study of one of these industries designed to determine if there were an equilibrium relationship among the prices, one would easily conclude that there is indeed an equilibrium relationship if the possibility of an NMA was not considered.

6.2 A visual inspection of the Johansen statistics in actual data for varying sample lengths

It is instructive to draw figures plotting the trace and maximal statistics against sample size, as we did earlier with the simulated data series. To see whether the present data are characterized by similar behavior, the Johansen test statistics

corresponding to the null hypothesis that there are no cointegrating vectors in each commodity group (i.e., $r \leq 0$) were calculated using samples of different sizes, and depicted in Figures 4 through 12, in a similar manner as was done earlier with the simulated data. Recall that in each figure, two lines are plotted. One line begins with the full sample, and then data points that are the furthest back in time are deleted one by one. The other line begins with a short sub-sample from the beginning of the data set, and adds data points one by one until the full sample is created.

It is evident that Figures 4 through 12 do not display the behavior that one would expect from a test statistic sampled from its asymptotic distribution. Despite the large length of the underlying full samples, the values of the Johansen test statistics clearly have not converged, as they seem to be increasing with the number of observations in each sample. This means that the chances of rejecting the null hypothesis of no cointegration increase with the sample size when a fixed k is used, and one will always reject the null hypothesis given enough observations. Using the asymptotic critical values to test for cointegration will result in large size distortions relative to the stated significance levels.

Note that in each of the groups depicted in Figures 4 through 12, at least one commodity price series was estimated to have an NMA component in the error term. The visual patterns of Figures 4 through 12 are consistent with the visual patterns shown in the simulated data that contained NMA errors. This, of course, does not provide a definitive conclusion that the data are characterized by NMA errors exactly as in the simulated data. For example, we did not control for possible structural breaks that may be important in such long series, nor did we control for threshold effects which have been found to characterize some price series similar to the ones studied here.

7 Conclusions and recommendations for the applied researcher

Our results show that when testing for cointegrating rank, it is crucially important to determine whether the data being considered contain an NMA error structure or not. When the error terms do not exhibit an NMA, small-sample size distortions in Johansen test statistics disappear with little more than six months of daily data, and the standard asymptotic critical values perform impressively well with as little as ten years of simulated monthly observations. In contrast, if an NMA error structure is present, our findings indicate that the standard asymptotic critical values for Johansen's test statistics should not be used.

The issue then faced by the applied researcher is how to proceed when NMA errors are found. In the event that NMA errors are suspected, one would adopt a more aggressive policy regarding lag length selection. For example,

instead of relying on the standard lag length selection procedures like the Akaike Information Criterion, Bayesian Information Criterion, or Hannan-Quinn Information Criterion, one could adopt a more aggressive approach in setting longer lag lengths. We saw, however, that this results in significant losses in power as the lag length grows.

The second alternative is to obtain appropriate critical values for testing and inference by performing Monte Carlo simulations. By using simulated critical values for testing, the researcher could maintain a parsimonious model and achieve smaller size distortion without giving up power. Such simulations would be analogous to the ones conducted here, but tailored specifically to the data at hand. For example, if the data involves three commodity prices in the cointegration group, Models (2) through (4) would have to be augmented to incorporate a third (potentially) cointegrated variable. In the past, researchers in possession of long datasets have confidently assumed that asymptotic critical values could be used to test for cointegration. Our results suggest that one could come to incorrect conclusions regarding the cointegrating rank of the data series in question, if proper examination for NMA errors is not undertaken.

Table 1: Data Dictionary.

Price Series	Start Date	End Date	Source ^a	Frequency
Alfalfa meal dehydrated, 17% protein – Kansas City, MO	Oct. 1981	Sep. 2009	ERS Feed grains database	Monthly
Barley No. 2 feed – Portland, OR	Jun. 1975	May 2004	ERS Feed grains database	Monthly
Butter, salted – U.S. City average	Jan. 1980	Dec. 2007	U.S. BLS	Monthly
Chicken, fresh whole – U.S. city average	Jan. 1980	Dec. 2009	U.S. BLS	Monthly
Chicken breast, bone in – U.S. city average	Jan. 1980	Dec. 2002	U.S. BLS	Monthly
Chicken legs, bone in – U.S. city average	Jan. 1980	Dec. 2009	U.S. BLS	Monthly
Corn	3/1/1968	8/1/2009	Barchart “cash”	Monthly
Corn gluten feed 21% – Midwest	Oct. 1981	Sep. 2009	ERS Feed grains database	Monthly
Corn gluten meal 60% – Midwest	Oct. 1981	Sep. 2009	ERS Feed grains database	Monthly
Corn meal, yellow – New York, NY	Sep. 1983	Aug. 2009	ERS Feed grains database	Monthly
Cottonseed meal 41% – Memphis, TN	Oct. 1981	Sep. 2003	ERS Feed grains database	Monthly
Crude oil – Cushing, OK	6/15/1986	7/15/2009	U.S. EIA	Monthly
Diesel, No. 2 – Los Angeles, CA	6/15/1986	7/15/2009	U.S. EIA	Monthly
Eggs, grade A large	Jan. 1980	Dec. 2009	U.S. BLS	Monthly
Feeder cattle	1/1/1970	8/1/2009	Barchart “cash”	Monthly
Gasoline, conventional – NY Harbor	6/15/1986	7/15/2009	U.S. EIA	Monthly
Ground beef – U.S. city average	Jan. 1984	Dec. 2009	U.S. BLS	Monthly
High fructose corn syrup 42% – Midwest	Sep. 1983	Aug. 2009	ERS Feed grains database	Monthly
Lean hogs	3/1/1968	8/1/2009	Barchart “cash”	Monthly
Live cattle	1/1/1970	8/1/2009	Barchart “cash”	Monthly
Meat and bone meal – Central U.S.	Oct. 1981	Sep. 2009	ERS Feed grains database	Monthly

^aERS denotes the Economic Research Service of the U.S. Department of Agriculture (USDA), BLS is the Bureau of Labor Statistics, Barchart denotes spot prices as archived by Barchart.com, EIA is the Energy Information Administration, and NASS is the USDA National Agricultural Statistics Service.

Table 1 (continued): Data Dictionary.

Price Series	Start Date	End Date	Source ^a	Frequency
Milk, price received by farmers, all milk	Jan. 1970	Dec. 2004	NASS, Agricultural Prices	Monthly
Milk, wholesale nonfat dry	Jan. 1970	Dec. 1981	NASS, Agricultural Prices	Monthly
Oats, white heavy – Minneapolis, MN	Jun. 1975	May 1994	ERS Feed grains database	Monthly
Pork bellies	3/1/1968	8/1/2009	Barchart “cash”	Monthly
Potatoes, white – U.S. city average	Jan. 1988	Dec. 2009	U.S. BLS	Monthly
Sorghum, No. 2 yellow – Kansas City, MO	Sep. 1975	Aug. 2009	ERS Feed grains database	Monthly
Soybean	3/1/1968	8/1/2009	Barchart “cash”	Monthly
Soy meal	3/1/1968	8/1/2009	Barchart “cash”	Monthly
Soy oil	3/1/1968	8/1/2009	Barchart “cash”	Monthly
Sugar, white – U.S. city average	Jan. 1980	Dec. 2009	U.S. BLS	Monthly
Urea 42% – Fort Worth, TX	Oct. 1981	Sep. 2009	ERS Feed grains database	Monthly
Wheat	Jun. 1985	Mar 2009	ERS Wheat Yearbook	Monthly
Wheat bran – Kansas City, MO	Oct. 1981	Sep. 2009	ERS Feed grains database	Monthly
Wheat flour	Jun. 1985	Mar 2009	ERS Wheat Yearbook	Monthly

^aERS denotes the Economic Research Service of the U.S. Department of Agriculture (USDA), BLS is the Bureau of Labor Statistics, Barchart denotes spot prices as archived by Barchart.com, EIA is the Energy Information Administration, and NASS is the USDA National Agricultural Statistics Service.

Table 2: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 1)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	1 month	0.18	0.12	24.38	20.24
	3 months	0.08	0.04	19.58	16.15
	6 months	0.06	0.03	18.49	15.35
	2 years	0.05	0.03	18.22	15.34
	4 years	0.05	0.03	18.27	15.17
	10 years	0.05	0.03	18.44	15.33
	20 years	0.05	0.03	18.16	15.00
	100 years	0.05	0.03	18.22	15.11
$\theta = 0.8, k = 2$	1 month	0.20	0.14	25.07	20.89
	3 months	0.07	0.04	19.33	16.02
	6 months	0.06	0.03	18.63	15.44
	2 years	0.05	0.03	18.11	14.95
	4 years	0.04	0.02	17.72	14.69
	10 years	0.04	0.02	17.74	14.88
	20 years	0.04	0.02	17.83	14.84
	100 years	0.04	0.02	17.68	14.62

^a θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

^bThe 5% asymptotic critical values are 18.17 and 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table 2: (continued) Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 1)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = -0.8, k = 2$	1 month	0.22	0.15	25.13	20.92
	3 months	0.39	0.30	28.37	25.06
	6 months	0.60	0.52	35.48	32.03
	2 years	0.73	0.67	53.01	49.91
	4 years	0.75	0.70	58.54	55.68
	10 years	0.77	0.71	63.71	61.04
	20 years	0.77	0.72	66.14	62.99
	100 years	0.77	0.72	68.08	65.29
$\theta = -0.8, k = 4$	1 month	0.48	0.39	37.50	32.03
	3 months	0.18	0.12	23.40	19.95
	6 months	0.25	0.18	25.46	22.09
	2 years	0.35	0.28	30.89	27.65
	4 years	0.37	0.30	31.95	29.25
	10 years	0.38	0.32	33.75	30.41
	20 years	0.39	0.32	34.08	31.11
	100 years	0.38	0.32	34.23	31.39
$\theta = -0.8, k = 5$	1 month	0.81	0.76	70.92	63.00
	3 months	0.17	0.10	23.25	19.49
	6 months	0.18	0.11	23.40	20.03
	2 years	0.24	0.18	26.22	23.20
	4 years	0.25	0.19	27.53	24.40
	10 years	0.26	0.20	28.16	25.05
	20 years	0.26	0.20	28.34	25.01
	100 years	0.27	0.21	28.64	25.70

^a θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

^bThe 5% asymptotic critical values are 18.17 and 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table 3: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 1)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	2 years	0.15	0.09	22.84	19.03
	4 years	0.08	0.04	19.80	16.45
	10 years	0.06	0.03	18.68	15.68
	20 years	0.06	0.03	18.60	15.19
	100 years	0.05	0.03	18.21	15.06
$\theta = 0.8, k = 2$	2 years	0.16	0.11	23.67	20.00
	4 years	0.08	0.05	19.91	16.78
	10 years	0.06	0.03	18.76	15.53
	20 years	0.05	0.03	18.04	15.09
	100 years	0.04	0.02	17.73	14.74
$\theta = -0.8, k = 2$	2 years	0.22	0.13	24.72	20.73
	4 years	0.31	0.23	26.65	22.86
	10 years	0.59	0.51	34.69	31.52
	20 years	0.68	0.62	43.32	40.35
	100 years	0.75	0.70	59.85	57.09
$\theta = -0.8, k = 4$	2 years	0.35	0.26	31.36	26.27
	4 years	0.18	0.11	23.65	19.68
	10 years	0.23	0.16	24.92	21.68
	20 years	0.30	0.23	27.66	24.63
	100 years	0.37	0.30	32.70	29.50
$\theta = -0.8, k = 5$	2 years	0.56	0.47	42.32	36.50
	4 years	0.18	0.12	24.13	19.92
	10 years	0.17	0.12	23.26	19.99
	20 years	0.22	0.16	25.09	21.74
	100 years	0.26	0.20	27.87	24.60

^a θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

^b Standard 5% asymptotic critical values are 18.17 for the trace test and 16.87 for the max test (Osterwald-Lenum, 1992).

Table 4. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 1)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	1 month	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	3 months	0.10	0.09	0.09	0.10	0.08	0.04	0.08	0.04
	6 months	0.29	0.28	0.27	0.28	0.11	0.03	0.14	0.05
	2 years	1	1	1	1	0.46	0.07	0.89	0.80
	4 years	1	1	1	1	1	0.97	1	1
	10 years	1	1	1	1	1	1	1	1
	20 years	1	1	1	1	1	1	1	1
	100 years	1	1	1	1	1	1	1	1
$\rho = 0.90$	1 month	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05
	3 months	0.07	0.07	0.07	0.07	0.07	0.04	0.07	0.05
	6 months	0.16	0.14	0.15	0.15	0.09	0.04	0.10	0.04
	2 years	0.99	0.99	0.99	1	0.22	0.03	0.57	0.32
	4 years	1	1	1	1	0.71	0.25	1	1
	10 years	1	1	1	1	1	1	1	1
	20 years	1	1	1	1	1	1	1	1
	100 years	1	1	1	1	1	1	1	1
$\rho = 0.95$	1 month	0.05	0.05	0.06	0.06	0.05	0.05	0.05	0.05
	3 months	0.05	0.05	0.06	0.06	0.06	0.04	0.05	0.05
	6 months	0.09	0.08	0.08	0.08	0.05	0.04	0.06	0.04
	2 years	0.55	0.54	0.54	0.60	0.09	0.03	0.18	0.05
	4 years	1	1	1	1	0.17	0.02	0.61	0.32
	10 years	1	1	1	1	0.90	0.62	1	1
	20 years	1	1	1	1	1	1	1	1
	100 years	1	1	1	1	1	1	1	1

^a ρ is the persistence parameter of the DGP (2)-(4).

^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table 5. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 1)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	2 years	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.13
	4 years	0.07	0.07	0.07	0.07	0.07	0.04	0.06	0.05
	10 years	0.26	0.23	0.24	0.25	0.13	0.03	0.15	0.05
	20 years	0.79	0.84	0.80	0.84	0.20	0.03	0.35	0.14
	100 years	1	1	1	1	1	1	1	1
$\rho = 0.90$	2 years	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.13
	4 years	0.07	0.06	0.07	0.06	0.06	0.04	0.05	0.05
	10 years	0.14	0.11	0.14	0.14	0.08	0.03	0.10	0.04
	20 years	0.46	0.49	0.48	0.51	0.12	0.03	0.20	0.06
	100 years	1	1	1	1	0.90	0.60	1	1
$\rho = 0.95$	2 years	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.14
	4 years	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.05
	10 years	0.08	0.06	0.08	0.08	0.06	0.04	0.06	0.04
	20 years	0.16	0.14	0.17	0.16	0.06	0.03	0.09	0.04
	100 years	1	1	1	1	0.24	0.03	0.78	0.59

^a ρ is the persistence parameter of the DGP (2)-(4).^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table 6: Estimated ARIMA Models.^a

Model: $\Delta P_t = c + \varepsilon_t + \sum_{i=1}^p \phi_i \Delta P_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j}$									
Commodity	ϕ_1	ϕ_2	ϕ_3	ϕ_4	θ_1	θ_2	θ_3	θ_4	R^2
Alfalfa meal	1.73*** (160.34)	-0.99*** (-92.25)			-1.70*** (-71.71)	0.96*** (40.51)			0.087
Barley	0.91*** (6.17)	-0.63*** (-7.56)	0.13** (2.35)		-0.69*** (-4.66)	0.37*** (4.08)			0.145
Butter	1.32*** (60.02)	-0.95*** (-43.31)			-1.30*** (-129.25)	0.99*** (99.11)			0.050
Chicken, whole	0.23 (0.94)				-0.23 (-0.09)				0.043
Chicken breast	-0.72 (-1.49)				0.67 (1.31)				0.003
Chicken Legs	1.60*** (18.90)	-0.87*** (-11.14)			-1.58*** (-16.00)	0.81*** (8.09)			0.058
Corn	-0.13 (-0.27)				0.10 (0.22)	0.17*** (3.42)	-0.10 (-0.95)	-0.11 (-1.62)	0.048
Corn gluten feed	1.04*** (6.14)	0.06 (0.19)	-0.45** (-2.06)	-0.14** (-2.29)	-1.02*** (-6.15)	-0.21 (-0.74)	0.68*** (4.13)		0.106
Corn gluten meal	0.87*** (13.54)	-0.07 (-0.98)	0.004 (0.058)	0.10 (1.65)	-0.97*** (-31.40)				0.062
Corn meal	-0.35 (-1.51)				0.54*** (2.57)				0.037
Cottonseed meal	-0.23 (-0.51)	-0.18 (-1.03)	-0.06 (-0.55)	-0.13* (-1.91)	-0.15 (-0.33)				0.143
Crude oil	1.27*** (15.88)	-0.23** (-2.23)	-0.18*** (-2.69)		-0.90*** (-16.06)				0.244

*** (**, *) denotes significantly different from zero at the 1% (5%, 10%) level, based on the two-sided *t*-test.

^a *t*-ratios are shown within parentheses below the point estimates of the respective parameters.

Table 6: (continued) Estimated ARIMA Models.^a

Commodity	ϕ_1	ϕ_2	ϕ_3	ϕ_4	θ_1	θ_2	θ_3	θ_4	R^2
Diesel	1.10 ^{***} (13.55)	-0.1 (-1.11)	-0.12 [*] (-1.91)		-0.93 ^{***} (-16.52)				0.073
Eggs	1.62 ^{***} (5.59)	-0.81 (-1.60)	-0.11 (-0.39)		-1.79 ^{***} (-6.14)	1.11 ^{**} (2.22)	-0.08 (-0.27)		0.123
Feeder cattle	-0.90 ^{***} (-55.08)	-0.98 ^{***} (-60.26)			0.87 ^{***} (55.42)	0.98 ^{***} (63.59)			0.040
Gasoline	1.90 ^{***} (31.53)	-1.53 ^{***} (-11.99)	0.58 ^{***} (4.46)	-0.25 ^{***} (-3.94)	-1.73 ^{***} (64.41)	1.00 ^{***} (64.41)			0.198
Ground beef	-0.02 (-0.07)				-0.17 (-0.58)				0.033
High fructose corn syrup	1.67 ^{***} (28.04)	-1.81 ^{***} (-15.98)	1.50 ^{***} (13.91)	-0.87 ^{***} (-16.17)	-1.55 ^{***} (-24.84)	1.56 ^{***} (15.01)	-1.39 ^{***} (-14.47)	0.82 ^{***} (15.02)	0.266
Lean hogs	0.47 ^{**} (2.21)	-0.02 (-0.46)	-0.12 ^{**} (-2.22)		-0.52 ^{**} (-2.43)				0.026
Live cattle	0.11 (0.52)				-0.04 (-0.21)	-0.06 (-1.34)	-0.20 ^{***} (-4.32)	-0.19 ^{***} (-2.98)	0.085
Meat and bone meal	-0.99 ^{***} (-9.18)	-0.34 ^{***} (-4.46)	-0.30 ^{***} (-3.38)	-0.13 ^{**} (-2.33)	0.90 ^{***} (9.31)				0.077
Milk, farmgate	2.59 ^{***} (23.35)	-3.11 ^{***} (-11.76)	1.94 ^{***} (7.82)	-0.62 ^{***} (-6.87)	-2.12 ^{***} (-19.78)	2.27 ^{***} (11.17)	-1.43 ^{***} (-8.90)	0.57 ^{***} (8.99)	0.520
Milk, wholesale	-0.07 (-0.15)	0.11 (0.44)	-0.21 ^{**} (-1.94)		0.56 (1.07)				0.214
Oats	-0.04 (-0.15)				0.29 (1.18)				0.058
Pork bellies	0.80 ^{***} (15.86)	0.15 ^{***} (2.63)	-0.09 [*] (-1.91)		-0.96 ^{***} (-40.57)				0.046

*** (**, *) denotes significantly different from zero at the 1% (5%, 10%) level, based on the two-sided *t*-test.

^a *t*-ratios are shown within parentheses below the point estimates of the respective parameters.

Table 6 (continued): Estimated ARIMA Models.^a

Commodity	ϕ_1	ϕ_2	ϕ_3	ϕ_4	θ_1	θ_2	θ_3	θ_4	R^2
Potatoes	1.38*** (24.31)	-0.49*** (-8.92)			-0.96*** (-36.63)				0.256
Sorghum	0.37 (0.21)				0.23 (1.24)				0.063
Soybean	0.85*** (15.94)	0.09 (1.59)	-0.12** (-2.46)		-0.94*** (-30.63)				0.043
Soy meal	0.13 (0.58)	0.06 (1.43)	-0.20*** (-4.43)		-0.12 (-0.52)				0.041
Soy oil	0.11 (0.35)	0.23** (2.31)	-0.17*** (-2.97)		-0.36 (-1.12)				0.137
Sugar	0.97*** (15.49)	-0.95*** (-13.83)	0.51*** (8.66)	-0.01 (-0.19)	-0.43*** (-11.33)	0.89*** (26.50)			0.541
Urea	-0.69*** (-9.99)	0.83*** (8.00)	0.34*** (4.11)	-0.39 (-6.83)	0.95*** (16.80)	-0.64*** (-6.55)	-0.85*** (-15.26)		0.222
Wheat	0.00 (0.003)				0.31 (1.86)*				0.088
Wheat bran	1.06*** (4.64)	0.17 (0.42)	-0.67*** (-2.95)		-1.28*** (-5.72)	-0.03 (-0.08)	0.88*** (3.22)	-0.27*** (-4.34)	0.160
Wheat flour	-1.00*** (-156)	1.08*** (19.61)			0.09 (1.63)				0.05

*** (**, *) denotes significantly different from zero at the 1% (5%, 10%) level, based on the two-sided *t*-test.

^a *t*-ratios are shown within parentheses below the point estimates of the respective parameters.

Table 7: Johansen Cointegration Test Results for Commodity Groups.

Commodity Group	H_0^a	Trace Test			Maximum Eigenvalue Test		
		Test Statistic	Critical Value ^b		Test Statistic	Critical Value ^b	
			5%	1%		5%	1%
Chicken: whole, breast, and legs	$r \leq 0$	56.49	29.68	35.65	48.57	20.97	25.52
	$r \leq 1$	7.91	15.41	20.04	5.22	14.07	18.63
	$r \leq 2$	2.70	3.76	6.65	2.70	3.76	6.65
Corn gluten feed and corn gluten meal	$r \leq 0$	19.69	15.41	20.04	16.76	14.07	18.63
	$r \leq 1$	2.93	3.76	6.65	2.93	3.76	6.65
Crack spread	$r \leq 0$	109.79	29.68	35.65	64.93	20.97	25.52
	$r \leq 1$	44.87	15.41	20.04	39.09	14.07	18.63
	$r \leq 2$	5.77	3.76	6.65	5.77	3.76	6.65
Corn, sorghum, barley, and oats	$r \leq 0$	80.61	47.21	54.46	40.14	27.07	32.24
	$r \leq 1$	40.47	29.68	35.65	23.32	20.97	25.52
	$r \leq 2$	17.15	15.41	20.04	10.73	14.07	18.63
	$r \leq 3$	6.41	3.76	6.65	6.42	3.76	6.65
Lean hogs and pork bellies	$r \leq 0$	42.46	15.41	20.04	27.11	14.07	18.63
	$r \leq 1$	15.35	3.76	6.65	15.35	3.76	6.65
Live cattle, feeder cattle, and corn	$r \leq 0$	92.15	29.68	35.65	71.37	20.97	25.52
	$r \leq 1$	20.78	15.41	20.04	15.74	14.07	18.63
	$r \leq 2$	5.05	3.76	6.65	5.05	3.76	6.65
Milk and butter	$r \leq 0$	45.20	15.41	20.04	43.85	14.07	18.63
	$r \leq 1$	1.35	3.76	6.65	1.35	3.76	6.65
Soybean, soybean oil, and soybean meal	$r \leq 0$	84.35	29.68	35.65	52.04	20.97	25.52
	$r \leq 1$	32.31	15.41	20.04	24.19	14.07	18.63
	$r \leq 2$	8.11	3.76	6.65	8.11	3.76	6.65
Wheat, flour, and bran	$r \leq 0$	100.43	29.68	35.65	63.18	20.97	25.52
	$r \leq 1$	37.25	15.41	20.04	34.45	14.07	18.63
	$r \leq 2$	2.78	3.76	6.65	2.77	3.76	6.65

^aFor the trace test, $H_0 = r \leq z$ means a null hypothesis that there are z or fewer cointegrating vectors, against the alternative that there are more than z cointegrating vectors. For the maximum eigenvalue test, $H_0 = r \leq z$ means a null hypothesis that there are z cointegrating vectors, against the alternative that there are $z+1$ cointegrating vectors.

^bStandard asymptotic critical values for the trace and maximum eigenvalue tests are from Osterwald-Lenum (1992).

Note: The estimation assumes no constant term in the VECM (2).

Figure 1: Johansen's Trace and Max Statistics for Varying Sample Lengths, Based on Twenty Years of Simulated Monthly Data Under the Null Hypothesis of No Cointegration – No NMA in the Error Term.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 2: Johansen's Trace and Max Statistics for Varying Sample Lengths, Based on Twenty Years of Simulated Monthly Data Under the Null Hypothesis of No Cointegration – NMA in the Error Term.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 3: Johansen's Trace and Max Statistics for Varying Sample Lengths, Based on Twenty Years of Simulated Monthly Data Under the Alternative Hypothesis of Cointegration – NMA in the Error Term.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 4: Johansen's Trace and Max Statistics for Varying Sample Lengths of Crack Spread Price Data.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 5: Johansen's Trace and Max Statistics for Varying Sample Lengths of Wheat, Flour, and Bran Price Data.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 6: Johansen's Trace and Max Statistics for Varying Sample Lengths of Soybean Crush Price Data.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 7: Johansen's Trace and Max Statistics for Varying Sample Lengths of Cattle Crush Price Data.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 8: Johansen's Trace and Max Statistics for Varying Sample Lengths of Lean Hogs and Pork Bellies Price Data.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 9: Johansen's Trace and Max Statistics for Varying Sample Lengths of Milk and Butter Price Data.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 10: Johansen's Trace and Max Statistics for Varying Sample Lengths of Corn Gluten Feed and Corn Gluten Meal Price Data.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 11: Johansen's Trace and Max Statistics for Varying Sample Lengths of Chicken Prices.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Figure 12: Johansen's Trace and Max Statistics for Varying Sample Lengths of Sorghum, Corn, Barley, and Oats.

Note: The horizontal lines represent the respective standard 5% asymptotic critical value of the trace and max tests.

Feed products included are alfalfa meal, corn gluten feed, corn gluten meal, urea, and wheat bran

Appendix A: Derivation of the VECM Model and of Expressions (5), (6), and (7)

First-differencing (2) and (3) yield (A1) and (A2), respectively:

$$(A1) \quad \Delta X_{1,t} - b \Delta X_{2,t} = \Delta \epsilon_t,$$

$$(A2) \quad a_1 \Delta X_{1,t} + a_2 \Delta X_{2,t} = \Delta \psi_t.$$

Solving equations (A1) and (A2) for the two unknowns $\Delta X_{1,t}$ and $\Delta X_{2,t}$ in terms of the error differences $\Delta \epsilon_t$ and $\Delta \psi_t$ yields (A3) and (A4), respectively:

$$(A3) \quad \Delta X_{1,t} = a_2 / (a_2 + b a_1) \Delta \epsilon_t + b / (a_2 + b a_1) \Delta \psi_t,$$

$$(A4) \quad \Delta X_{2,t} = -a_1 / (a_2 + b a_1) \Delta \epsilon_t + 1 / (a_2 + b a_1) \Delta \psi_t.$$

By substituting $[u_t + (\rho - 1) \epsilon_{t-1}]$ and v_t for $\Delta \epsilon_t$ and $\Delta \psi_t$, respectively, and noting that $\epsilon_{t-1} = X_{1,t-1} - b X_{2,t-1}$, expressions (A4) and (A5) can be written as the VECM (A5)-(A6):

$$(A5) \quad \Delta X_{1,t} = a_2 / (a_2 + b a_1) (\rho - 1) (X_{1,t-1} - b X_{2,t-1}) + (b v_t + a_2 u_t) / (a_2 + b a_1),$$

$$(A6) \quad \Delta X_{2,t} = -a_1 / (a_2 + b a_1) (\rho - 1) (X_{1,t-1} - b X_{2,t-1}) + (v_t - a_1 u_t) / (a_2 + b a_1).$$

Alternatively, since $\Delta \epsilon_t = u_t + (\rho - 1) (u_{t-1} + \rho u_{t-2} + \rho^2 u_{t-3} + \dots)$ and $\Delta \psi_t = w_t + \theta w_{t-1}$, equations (A3) and (A4) can also be expressed as (A7) and (A8), respectively:

$$(A7) \quad \Delta X_{1,t} = \{b (w_t + \theta w_{t-1}) + a_2 [u_t + (\rho - 1) (u_{t-1} + \rho u_{t-2} + \rho^2 u_{t-3} + \dots)]\} / (a_2 + b a_1),$$

$$(A8) \quad \Delta X_{2,t} = \{(w_t + \theta w_{t-1}) - a_1 [u_t + (\rho - 1) (u_{t-1} + \rho u_{t-2} + \rho^2 u_{t-3} + \dots)]\} / (a_2 + b a_1).$$

From (A7) and (A8), it follows that the variances of $\Delta X_{1,t}$ and $\Delta X_{2,t}$ may be written as (A9) and (A10), respectively, and their covariance as (A11):

$$(A9) \quad \sigma_1^2 = \frac{1}{(a_2 + b a_1)^2} \{b^2 (1 + \theta^2) \sigma_w^2 + 2 a_2 b [1 + \theta (\rho - 1)] \sigma_{uw} + a_2^2 \frac{2}{(1 + \rho)} \sigma_u^2\},$$

$$(A10) \sigma_2^2 = \frac{1}{(a_2 + ba_1)^2} \{ (1 + \theta^2) \sigma_w^2 - 2 a_1 [1 + \theta(\rho - 1)] \sigma_{uw} + a_1^2 \frac{2}{(1 + \rho)} \sigma_u^2 \},$$

$$(A11) \sigma_{12} = \frac{1}{(a_2 + ba_1)^2} \{ b(1 + \theta^2) \sigma_w^2 + (a_2 - b a_1) [1 + \theta(\rho - 1)] \sigma_{uw} - a_1 a_2 \frac{2}{(1 + \rho)} \sigma_u^2 \}.$$

Expressions (5), (6), and (7) are obtained by solving (A9) through (A11) for σ_u^2 , σ_w^2 , and σ_{uw} in terms of σ_1^2 , σ_2^2 , and σ_{12} .

Appendix B: Results from Additional DGP Parameter Scenarios

Table B.1: Empirical Size of Asymptotic Critical Values and Finite-Sample Critical Values, Johansen Cointegration Tests for Daily Data from a Bivariate System, $(a_1, a_2, b) = (1, 1, 0.5)$.

	Sample Length	Empirical Size Based on Standard		Actual	
		5% Asymptotic Critical Value ^b		5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	1 month	0.12	0.07	21.67	21.67
	3 months	0.07	0.03	19.09	15.93
	6 months	0.05	0.03	18.33	15.32
	2 years	0.06	0.03	18.51	15.24
	4 years	0.05	0.03	18.18	15.12
	10 years	0.05	0.03	18.07	15.10
	20 years	0.05	0.03	18.14	15.10
	100 years	0.05	0.03	18.23	15.06
$\theta = 0.8, k = 2$	1 month	0.15	0.10	22.91	22.91
	3 months	0.08	0.05	20.00	16.69
	6 months	0.07	0.04	19.24	16.18
	2 years	0.06	0.04	18.83	15.84
	4 years	0.06	0.04	19.06	15.92
	10 years	0.06	0.03	18.52	15.61
	20 years	0.05	0.03	18.40	15.35
	100 years	0.06	0.03	18.67	15.66
$\theta = -0.8, k = 2$	1 month	0.28	0.19	25.99	25.99
	3 months	0.63	0.55	35.24	31.97
	6 months	0.76	0.70	46.38	43.49
	2 years	0.82	0.78	64.43	61.53
	4 years	0.83	0.79	72.47	69.24
	10 years	0.84	0.80	80.47	77.91
	20 years	0.84	0.80	83.97	81.59
	100 years	0.84	0.81	84.52	81.54
$\theta = -0.8, k = 4$	1 month	0.26	0.17	26.96	26.96
	3 months	0.25	0.17	25.44	21.78
	6 months	0.34	0.26	28.62	25.49
	2 years	0.42	0.35	33.68	30.88
	4 years	0.43	0.37	36.59	33.29
	10 years	0.45	0.39	37.41	34.53
	20 years	0.45	0.39	37.85	35.14
	100 years	0.46	0.39	38.60	35.55
$\theta = -0.8, k = 5$	1 month	0.33	0.23	30.37	30.37
	3 months	0.19	0.12	23.58	20.04
	6 months	0.23	0.17	25.20	21.89
	2 years	0.28	0.22	27.63	24.63
	4 years	0.30	0.24	29.28	26.49
	10 years	0.31	0.25	30.76	27.29
	20 years	0.32	0.26	30.54	27.47
	100 years	0.32	0.26	30.70	27.77

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).

^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.2: Empirical Size of Asymptotic Critical Values and Finite-Sample Critical Values, Johansen Cointegration Tests for Monthly Data in a Bivariate System, $(a_1, a_2, b) = (1, 1, 0.5)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	2 years	0.15	0.10	23.02	19.22
	4 years	0.08	0.05	20.06	16.64
	10 years	0.06	0.03	18.76	15.52
	20 years	0.06	0.03	18.54	15.22
	100 years	0.05	0.02	18.11	14.91
$\theta = 0.8, k = 2$	2 years	0.19	0.13	24.38	20.69
	4 years	0.10	0.06	20.94	17.51
	10 years	0.08	0.04	19.67	16.26
	20 years	0.07	0.04	19.10	15.98
	100 years	0.06	0.04	18.83	15.74
$\theta = -0.8, k = 2$	2 years	0.27	0.17	25.71	21.57
	4 years	0.42	0.32	29.03	25.21
	10 years	0.71	0.64	39.40	36.12
	20 years	0.79	0.74	50.65	47.85
	100 years	0.84	0.80	74.97	72.35
$\theta = -0.8, k = 4$	2 years	0.37	0.27	31.47	26.59
	4 years	0.21	0.13	24.34	20.44
	10 years	0.29	0.21	26.80	23.31
	20 years	0.37	0.30	30.20	26.94
	100 years	0.44	0.37	36.22	32.90
$\theta = -0.8, k = 5$	2 years	0.56	0.47	41.78	35.62
	4 years	0.20	0.13	24.41	20.43
	10 years	0.21	0.14	24.48	20.94
	20 years	0.24	0.18	25.97	22.72
	100 years	0.30	0.24	29.53	26.41

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.3. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 0.5)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	1 month	0.06	0.06	0.13	0.11	0.04	0.03	0.04	0.04
	3 months	0.25	0.23	0.61	0.62	0.04	0.01	0.07	0.03
	6 months	0.80	0.83	0.99	1.00	0.04	0.01	0.14	0.04
	2 years	1.00	1.00	1.00	1.00	0.11	0.00	0.88	0.82
	4 years	1.00	1.00	1.00	1.00	0.82	0.61	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.90$	1 month	0.06	0.05	0.10	0.09	0.04	0.03	0.04	0.05
	3 months	0.15	0.13	0.36	0.34	0.03	0.01	0.05	0.02
	6 months	0.50	0.49	0.87	0.88	0.03	0.01	0.07	0.02
	2 years	1.00	1.00	1.00	1.00	0.04	0.00	0.46	0.24
	4 years	1.00	1.00	1.00	1.00	0.21	0.02	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.95$	1 month	0.05	0.05	0.07	0.06	0.05	0.04	0.04	0.05
	3 months	0.07	0.06	0.15	0.14	0.03	0.02	0.04	0.03
	6 months	0.18	0.15	0.39	0.37	0.02	0.01	0.04	0.01
	2 years	0.80	0.85	0.99	0.99	0.01	0.00	0.07	0.01
	4 years	1.00	1.00	1.00	1.00	0.02	0.00	0.42	0.23
	10 years	1.00	1.00	1.00	1.00	0.35	0.06	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

^a ρ is the persistence parameter of the DGP (2)-(4).

^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.4. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 0.5)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	2 years	0.06	0.05	0.11	0.09	0.05	0.04	0.05	0.05
	4 years	0.10	0.09	0.24	0.22	0.03	0.02	0.05	0.03
	10 years	0.43	0.44	0.85	0.87	0.04	0.01	0.08	0.03
	20 years	0.96	0.98	1.00	1.00	0.05	0.00	0.22	0.08
	100 years	1.00	1.00	1.00	1.00	0.98	0.93	1.00	1.00
$\rho = 0.90$	2 years	0.05	0.05	0.09	0.07	0.05	0.04	0.05	0.04
	4 years	0.08	0.07	0.16	0.14	0.03	0.02	0.04	0.03
	10 years	0.23	0.22	0.55	0.56	0.03	0.01	0.06	0.02
	20 years	0.74	0.78	0.98	0.99	0.03	0.00	0.09	0.02
	100 years	1.00	1.00	1.00	1.00	0.37	0.08	1.00	1.00
$\rho = 0.95$	2 years	0.05	0.05	0.07	0.07	0.05	0.05	0.04	0.04
	4 years	0.06	0.05	0.09	0.09	0.04	0.03	0.04	0.04
	10 years	0.10	0.09	0.22	0.20	0.03	0.02	0.04	0.02
	20 years	0.26	0.25	0.59	0.59	0.02	0.01	0.04	0.01
	100 years	1.00	1.00	1.00	1.00	0.03	0.00	0.73	0.61

^a ρ is the persistence parameter of the DGP (2)-(4).^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.5: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 2)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	1 month	0.11	0.06	21.21	21.21
	3 months	0.06	0.04	18.90	15.83
	6 months	0.06	0.03	18.60	15.34
	2 years	0.05	0.02	17.97	15.01
	4 years	0.05	0.03	18.33	15.10
	10 years	0.05	0.03	18.22	15.14
	20 years	0.05	0.03	18.19	15.13
	100 years	0.05	0.03	18.12	14.94
$\theta = 0.8, k = 2$	1 month	0.15	0.10	23.16	23.16
	3 months	0.08	0.05	20.08	16.65
	6 months	0.07	0.04	19.29	16.31
	2 years	0.06	0.04	18.94	15.93
	4 years	0.06	0.04	19.04	15.88
	10 years	0.06	0.04	18.86	15.83
	20 years	0.06	0.03	18.62	15.66
	100 years	0.06	0.03	18.72	15.69
$\theta = -0.8, k = 2$	1 month	0.30	0.20	26.33	26.33
	3 months	0.63	0.55	35.28	31.99
	6 months	0.76	0.70	45.74	42.86
	2 years	0.82	0.77	64.02	61.22
	4 years	0.83	0.79	73.08	70.55
	10 years	0.85	0.81	80.33	77.75
	20 years	0.84	0.80	82.68	80.15
	100 years	0.84	0.80	84.82	81.45
$\theta = -0.8, k = 4$	1 month	0.26	0.17	26.59	26.59
	3 months	0.24	0.17	25.62	21.92
	6 months	0.34	0.26	28.65	25.34
	2 years	0.41	0.34	33.64	30.37
	4 years	0.44	0.38	35.76	33.04
	10 years	0.45	0.39	37.88	35.07
	20 years	0.46	0.40	38.22	35.37
	100 years	0.45	0.39	38.65	35.55
$\theta = -0.8, k = 5$	1 month	0.34	0.24	29.71	29.71
	3 months	0.18	0.12	23.16	19.75
	6 months	0.22	0.16	24.96	21.60
	2 years	0.28	0.21	27.64	24.53
	4 years	0.30	0.24	29.32	26.19
	10 years	0.30	0.25	30.53	27.52
	20 years	0.32	0.26	30.88	27.88
	100 years	0.31	0.26	31.27	28.23

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).

^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.6: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 2)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	2 years	0.15	0.09	23.06	19.27
	4 years	0.08	0.05	20.04	16.86
	10 years	0.06	0.03	18.61	15.38
	20 years	0.05	0.03	18.33	15.31
	100 years	0.05	0.03	18.12	14.99
$\theta = 0.8, k = 2$	2 years	0.19	0.13	24.80	21.04
	4 years	0.11	0.06	21.00	17.71
	10 years	0.08	0.04	19.67	16.33
	20 years	0.06	0.03	18.86	15.71
	100 years	0.06	0.03	18.87	15.70
$\theta = -0.8, k = 2$	2 years	0.27	0.17	26.03	21.74
	4 years	0.42	0.32	28.98	25.29
	10 years	0.71	0.64	39.38	36.14
	20 years	0.78	0.73	51.24	48.44
	100 years	0.83	0.79	74.66	71.85
$\theta = -0.8, k = 4$	2 years	0.37	0.27	31.66	26.84
	4 years	0.20	0.13	24.31	20.38
	10 years	0.29	0.21	26.54	23.08
	20 years	0.36	0.29	30.31	27.00
	100 years	0.43	0.37	36.08	33.17
$\theta = -0.8, k = 5$	2 years	0.56	0.47	41.28	35.57
	4 years	0.19	0.12	24.22	20.24
	10 years	0.20	0.13	24.29	20.82
	20 years	0.25	0.18	25.93	22.69
	100 years	0.30	0.24	29.63	26.36

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.7. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 2)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	1 month	0.08	0.06	0.12	0.10	0.04	0.03	0.04	0.04
	3 months	0.27	0.24	0.62	0.63	0.04	0.01	0.06	0.03
	6 months	0.79	0.83	0.99	1.00	0.05	0.01	0.13	0.04
	2 years	1.00	1.00	1.00	1.00	0.12	0.01	0.88	0.83
	4 years	1.00	1.00	1.00	1.00	0.81	0.54	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.90$	1 month	0.07	0.06	0.10	0.08	0.04	0.04	0.05	0.05
	3 months	0.15	0.13	0.35	0.34	0.03	0.02	0.04	0.02
	6 months	0.48	0.50	0.86	0.88	0.03	0.01	0.07	0.02
	2 years	1.00	1.00	1.00	1.00	0.04	0.00	0.46	0.27
	4 years	1.00	1.00	1.00	1.00	0.20	0.01	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.95$	1 month	0.06	0.06	0.07	0.06	0.04	0.04	0.05	0.05
	3 months	0.08	0.07	0.14	0.13	0.03	0.03	0.03	0.03
	6 months	0.16	0.15	0.39	0.35	0.02	0.01	0.04	0.01
	2 years	0.83	0.86	0.99	0.99	0.02	0.00	0.08	0.01
	4 years	1.00	1.00	1.00	1.00	0.02	0.00	0.47	0.25
	10 years	1.00	1.00	1.00	1.00	0.37	0.07	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

^a ρ is the persistence parameter of the DGP (2)-(4).

^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.8. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (1, 1, 2)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	2 years	0.05	0.05	0.10	0.09	0.04	0.04	0.04	0.04
	4 years	0.10	0.08	0.23	0.21	0.04	0.02	0.05	0.04
	10 years	0.45	0.45	0.85	0.87	0.04	0.01	0.09	0.03
	20 years	0.96	0.98	1.00	1.00	0.05	0.00	0.22	0.08
	100 years	1.00	1.00	1.00	1.00	0.98	0.93	1.00	1.00
$\rho = 0.90$	2 years	0.05	0.05	0.08	0.08	0.04	0.04	0.04	0.04
	4 years	0.07	0.06	0.15	0.13	0.04	0.03	0.04	0.04
	10 years	0.24	0.23	0.55	0.55	0.03	0.01	0.06	0.02
	20 years	0.74	0.77	0.98	0.99	0.03	0.00	0.09	0.02
	100 years	1.00	1.00	1.00	1.00	0.39	0.08	1.00	1.00
$\rho = 0.95$	2 years	0.05	0.05	0.06	0.06	0.05	0.05	0.04	0.04
	4 years	0.05	0.05	0.09	0.08	0.03	0.03	0.04	0.04
	10 years	0.10	0.09	0.21	0.19	0.03	0.02	0.04	0.02
	20 years	0.27	0.25	0.61	0.61	0.02	0.01	0.04	0.01
	100 years	1.00	1.00	1.00	1.00	0.03	0.00	0.74	0.60

^a ρ is the persistence parameter of the DGP (2)-(4).^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.9: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (0, 1, 1)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	1 month	0.11	0.07	21.42	21.42
	3 months	0.07	0.04	18.97	15.76
	6 months	0.05	0.03	18.37	15.42
	2 years	0.05	0.02	18.10	15.01
	4 years	0.05	0.03	18.18	15.21
	10 years	0.05	0.03	18.15	14.99
	20 years	0.05	0.03	18.20	15.04
	100 years	0.05	0.03	18.21	15.08
$\theta = 0.8, k = 2$	1 month	0.14	0.09	22.69	22.69
	3 months	0.07	0.04	19.55	16.44
	6 months	0.06	0.03	18.80	15.66
	2 years	0.06	0.03	18.70	15.43
	4 years	0.05	0.03	18.51	15.57
	10 years	0.05	0.03	18.45	15.32
	20 years	0.05	0.03	18.09	15.14
	100 years	0.06	0.03	18.55	15.37
$\theta = -0.8, k = 2$	1 month	0.27	0.17	25.55	25.55
	3 months	0.59	0.51	33.83	30.70
	6 months	0.72	0.66	43.62	40.30
	2 years	0.79	0.74	59.26	56.47
	4 years	0.80	0.75	66.47	64.12
	10 years	0.81	0.75	73.47	70.55
	20 years	0.81	0.77	76.56	73.74
	100 years	0.81	0.77	78.71	75.78
$\theta = -0.8, k = 4$	1 month	0.25	0.16	26.40	26.40
	3 months	0.24	0.16	24.99	21.78
	6 months	0.31	0.23	27.96	24.58
	2 years	0.39	0.32	32.45	29.46
	4 years	0.41	0.35	34.47	31.37
	10 years	0.43	0.36	35.66	32.96
	20 years	0.42	0.36	36.51	33.33
	100 years	0.43	0.36	36.87	33.88
$\theta = -0.8, k = 5$	1 month	0.34	0.25	29.79	29.79
	3 months	0.18	0.11	23.26	19.65
	6 months	0.22	0.15	25.20	21.60
	2 years	0.27	0.20	27.68	24.50
	4 years	0.28	0.22	28.66	25.71
	10 years	0.29	0.23	29.62	26.43
	20 years	0.30	0.24	30.29	27.05
	100 years	0.29	0.23	29.29	26.31

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).

^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.10: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (0, 1, 1)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	2 years	0.15	0.09	22.88	19.30
	4 years	0.08	0.04	19.98	16.54
	10 years	0.06	0.03	18.72	15.58
	20 years	0.05	0.03	18.33	15.21
	100 years	0.05	0.02	18.00	14.84
$\theta = 0.8, k = 2$	2 years	0.17	0.12	24.09	20.42
	4 years	0.10	0.05	20.65	17.21
	10 years	0.07	0.04	19.20	15.95
	20 years	0.06	0.03	18.50	15.37
	100 years	0.05	0.03	18.34	15.34
$\theta = -0.8, k = 2$	2 years	0.25	0.16	25.42	21.37
	4 years	0.38	0.28	28.05	24.16
	10 years	0.66	0.58	37.49	34.31
	20 years	0.75	0.69	47.42	44.48
	100 years	0.80	0.76	69.05	65.98
$\theta = -0.8, k = 4$	2 years	0.35	0.26	31.03	26.09
	4 years	0.20	0.13	24.16	20.39
	10 years	0.26	0.20	26.07	22.82
	20 years	0.34	0.27	29.16	26.01
	100 years	0.42	0.35	34.95	31.89
$\theta = -0.8, k = 5$	2 years	0.55	0.46	41.92	35.76
	4 years	0.19	0.12	24.20	20.25
	10 years	0.19	0.13	23.96	20.56
	20 years	0.23	0.17	25.59	22.29
	100 years	0.28	0.22	28.69	25.78

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.11. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (0, 1, 1)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	1 month	0.06	0.06	0.10	0.09	0.05	0.04	0.05	0.05
	3 months	0.21	0.19	0.43	0.42	0.06	0.02	0.08	0.03
	6 months	0.70	0.72	0.93	0.95	0.07	0.01	0.17	0.05
	2 years	1.00	1.00	1.00	1.00	0.23	0.01	0.87	0.79
	4 years	1.00	1.00	1.00	1.00	0.94	0.79	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.90$	1 month	0.06	0.05	0.08	0.07	0.04	0.04	0.05	0.04
	3 months	0.13	0.11	0.25	0.22	0.05	0.02	0.06	0.03
	6 months	0.41	0.39	0.67	0.69	0.05	0.01	0.09	0.03
	2 years	1.00	1.00	1.00	1.00	0.09	0.01	0.48	0.25
	4 years	1.00	1.00	1.00	1.00	0.39	0.05	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.95$	1 month	0.05	0.05	0.07	0.06	0.05	0.05	0.05	0.05
	3 months	0.08	0.06	0.12	0.11	0.04	0.03	0.05	0.03
	6 months	0.15	0.12	0.27	0.25	0.04	0.02	0.05	0.03
	2 years	0.73	0.76	0.91	0.94	0.03	0.01	0.11	0.02
	4 years	1.00	1.00	1.00	1.00	0.06	0.01	0.49	0.27
	10 years	1.00	1.00	1.00	1.00	0.61	0.22	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

^a ρ is the persistence parameter of the DGP (2)-(4).

^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.12. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Monthly Data in a Bivariate System, for $(a_1, a_2, b) = (0, 1, 1)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	2 years	0.06	0.05	0.10	0.08	0.04	0.04	0.05	0.04
	4 years	0.08	0.07	0.17	0.16	0.05	0.03	0.05	0.03
	10 years	0.35	0.35	0.65	0.67	0.07	0.02	0.10	0.03
	20 years	0.92	0.95	1.00	1.00	0.10	0.01	0.27	0.10
	100 years	1.00	1.00	1.00	1.00	0.99	0.98	1.00	1.00
$\rho = 0.90$	2 years	0.06	0.05	0.07	0.06	0.05	0.04	0.05	0.05
	4 years	0.07	0.07	0.12	0.11	0.04	0.03	0.05	0.04
	10 years	0.20	0.18	0.38	0.37	0.05	0.02	0.07	0.03
	20 years	0.63	0.66	0.89	0.91	0.06	0.01	0.13	0.04
	100 years	1.00	1.00	1.00	1.00	0.61	0.21	1.00	1.00
$\rho = 0.95$	2 years	0.06	0.05	0.06	0.06	0.05	0.04	0.05	0.05
	4 years	0.06	0.06	0.08	0.08	0.04	0.04	0.04	0.04
	10 years	0.09	0.08	0.16	0.14	0.04	0.02	0.04	0.03
	20 years	0.23	0.20	0.41	0.41	0.03	0.02	0.06	0.02
	100 years	1.00	1.00	1.00	1.00	0.07	0.01	0.72	0.54

^a ρ is the persistence parameter of the DGP (2)-(4).^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.13: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (0, 1, 0.5)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	1 month	0.12	0.07	21.81	21.81
	3 months	0.06	0.03	18.68	15.57
	6 months	0.05	0.03	18.40	15.21
	2 years	0.05	0.03	18.28	15.19
	4 years	0.05	0.02	18.04	15.06
	10 years	0.05	0.02	17.94	14.64
	20 years	0.05	0.03	18.22	15.22
	100 years	0.05	0.02	18.13	15.14
$\theta = 0.8, k = 2$	1 month	0.12	0.07	21.79	21.79
	3 months	0.06	0.04	19.16	15.84
	6 months	0.05	0.03	18.44	15.21
	2 years	0.04	0.02	17.78	15.01
	4 years	0.04	0.02	17.92	14.90
	10 years	0.04	0.02	17.65	14.80
	20 years	0.04	0.02	17.54	14.81
	100 years	0.04	0.03	17.87	14.97
$\theta = -0.8, k = 2$	1 month	0.23	0.14	24.63	24.63
	3 months	0.51	0.42	31.69	28.49
	6 months	0.66	0.59	40.17	37.20
	2 years	0.74	0.69	51.93	49.23
	4 years	0.75	0.70	59.38	56.47
	10 years	0.76	0.71	63.93	60.96
	20 years	0.76	0.71	66.71	63.93
	100 years	0.75	0.71	67.69	64.73
$\theta = -0.8, k = 4$	1 month	0.24	0.16	26.43	26.43
	3 months	0.22	0.15	24.41	20.97
	6 months	0.27	0.20	26.81	23.66
	2 years	0.35	0.28	30.68	27.69
	4 years	0.37	0.31	33.10	29.74
	10 years	0.38	0.32	33.59	31.05
	20 years	0.40	0.33	34.53	31.43
	100 years	0.39	0.33	34.38	31.73
$\theta = -0.8, k = 5$	1 month	0.33	0.23	29.48	29.48
	3 months	0.17	0.10	23.23	19.47
	6 months	0.20	0.14	24.29	20.80
	2 years	0.24	0.18	26.34	23.19
	4 years	0.25	0.20	27.44	24.34
	10 years	0.27	0.21	28.09	24.88
	20 years	0.26	0.20	28.50	25.30
	100 years	0.26	0.21	28.42	25.45

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).

^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.14: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (0, 1, 0.5)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	2 years	0.15	0.10	23.06	19.16
	4 years	0.08	0.05	19.92	16.69
	10 years	0.06	0.03	18.95	15.63
	20 years	0.05	0.03	18.39	15.41
	100 years	0.05	0.02	17.98	14.83
$\theta = 0.8, k = 2$	2 years	0.17	0.11	23.79	20.02
	4 years	0.09	0.05	20.16	16.80
	10 years	0.05	0.03	18.44	15.48
	20 years	0.05	0.03	18.37	15.31
	100 years	0.04	0.02	17.62	14.66
$\theta = -0.8, k = 2$	2 years	0.21	0.14	24.73	20.68
	4 years	0.32	0.23	26.70	22.94
	10 years	0.58	0.50	35.04	31.82
	20 years	0.68	0.62	43.56	40.51
	100 years	0.76	0.71	61.82	58.87
$\theta = -0.8, k = 4$	2 years	0.35	0.25	30.66	25.96
	4 years	0.18	0.11	23.42	19.69
	10 years	0.24	0.17	25.31	21.85
	20 years	0.30	0.23	27.88	24.87
	100 years	0.39	0.32	33.21	30.51
$\theta = -0.8, k = 5$	2 years	0.57	0.47	41.90	35.64
	4 years	0.19	0.12	23.91	20.08
	10 years	0.17	0.12	23.62	20.04
	20 years	0.21	0.15	24.65	21.38
	100 years	0.25	0.20	27.66	24.41

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.15. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (0, 1, 0.5)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	1 month	0.05	0.05	0.06	0.06	0.06	0.05	0.05	0.04
	3 months	0.17	0.15	0.16	0.17	0.10	0.04	0.08	0.04
	6 months	0.55	0.57	0.56	0.62	0.15	0.03	0.22	0.07
	2 years	1.00	1.00	1.00	1.00	0.51	0.08	0.90	0.80
	4 years	1.00	1.00	1.00	1.00	0.99	0.95	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.90$	1 month	0.05	0.05	0.06	0.06	0.05	0.05	0.05	0.04
	3 months	0.12	0.10	0.11	0.10	0.07	0.03	0.07	0.04
	6 months	0.30	0.29	0.30	0.32	0.09	0.03	0.13	0.04
	2 years	0.99	0.99	0.99	1.00	0.23	0.03	0.57	0.32
	4 years	1.00	1.00	1.00	1.00	0.67	0.22	1.00	0.99
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.95$	1 month	0.05	0.05	0.06	0.06	0.05	0.05	0.04	0.05
	3 months	0.07	0.06	0.07	0.06	0.06	0.04	0.06	0.04
	6 months	0.12	0.10	0.12	0.12	0.06	0.03	0.08	0.04
	2 years	0.56	0.57	0.61	0.63	0.09	0.03	0.18	0.05
	4 years	0.99	1.00	1.00	1.00	0.16	0.03	0.56	0.29
	10 years	1.00	1.00	1.00	1.00	0.90	0.62	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

^a ρ is the persistence parameter of the DGP (2)-(4).

^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.16. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (0, 1, 0.5)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	2 years	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.05
	4 years	0.07	0.06	0.08	0.08	0.07	0.04	0.06	0.05
	10 years	0.25	0.24	0.30	0.30	0.11	0.03	0.13	0.05
	20 years	0.81	0.84	0.82	0.87	0.19	0.03	0.34	0.13
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.90$	2 years	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05
	4 years	0.06	0.06	0.07	0.06	0.06	0.04	0.06	0.04
	10 years	0.14	0.13	0.16	0.15	0.08	0.03	0.09	0.04
	20 years	0.48	0.48	0.50	0.53	0.12	0.03	0.19	0.06
	100 years	1.00	1.00	1.00	1.00	0.85	0.50	1.00	1.00
$\rho = 0.95$	2 years	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	4 years	0.06	0.05	0.06	0.06	0.05	0.05	0.05	0.04
	10 years	0.08	0.07	0.08	0.08	0.06	0.04	0.06	0.04
	20 years	0.17	0.13	0.17	0.16	0.06	0.03	0.09	0.03
	100 years	1.00	1.00	1.00	1.00	0.19	0.02	0.76	0.52

^a ρ is the persistence parameter of the DGP (2)-(4).^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.17: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (1, 0, 1)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	1 month	0.12	0.07	21.50	21.50
	3 months	0.06	0.03	18.93	15.74
	6 months	0.05	0.03	18.39	15.28
	2 years	0.05	0.03	18.30	15.25
	4 years	0.05	0.02	18.16	15.05
	10 years	0.05	0.03	18.15	14.95
	20 years	0.04	0.02	17.92	14.99
	100 years	0.05	0.03	18.18	15.00
$\theta = 0.8, k = 2$	1 month	0.13	0.08	22.55	22.55
	3 months	0.07	0.04	19.25	16.13
	6 months	0.06	0.03	18.95	15.87
	2 years	0.05	0.03	18.30	15.19
	4 years	0.06	0.03	18.56	15.44
	10 years	0.06	0.03	18.77	15.39
	20 years	0.05	0.03	18.48	15.39
	100 years	0.05	0.03	18.36	15.17
$\theta = -0.8, k = 2$	1 month	0.26	0.17	25.60	25.60
	3 months	0.59	0.50	33.87	30.58
	6 months	0.72	0.66	43.44	40.37
	2 years	0.78	0.74	58.86	56.03
	4 years	0.80	0.75	67.04	64.21
	10 years	0.81	0.76	73.34	70.27
	20 years	0.82	0.78	76.04	73.37
	100 years	0.82	0.78	78.81	75.95
$\theta = -0.8, k = 4$	1 month	0.25	0.17	26.30	26.30
	3 months	0.23	0.16	24.91	21.44
	6 months	0.31	0.23	27.87	24.55
	2 years	0.38	0.32	32.23	29.19
	4 years	0.41	0.34	34.29	31.34
	10 years	0.42	0.36	36.14	33.21
	20 years	0.43	0.37	37.07	33.75
	100 years	0.43	0.37	37.24	34.38
$\theta = -0.8, k = 5$	1 month	0.34	0.25	30.12	30.12
	3 months	0.18	0.11	23.35	19.61
	6 months	0.22	0.15	25.12	21.69
	2 years	0.27	0.20	27.21	24.21
	4 years	0.28	0.22	28.41	25.45
	10 years	0.29	0.23	29.36	26.34
	20 years	0.30	0.24	29.84	26.94
	100 years	0.29	0.23	30.20	27.05

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).

^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.18: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (1, 0, 1)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	2 years	0.15	0.09	22.80	19.08
	4 years	0.08	0.05	19.65	16.67
	10 years	0.06	0.03	18.71	15.71
	20 years	0.06	0.03	18.57	15.48
	100 years	0.05	0.02	17.97	14.83
$\theta = 0.8, k = 2$	2 years	0.18	0.12	24.45	20.24
	4 years	0.09	0.06	20.72	17.34
	10 years	0.07	0.04	19.13	15.92
	20 years	0.06	0.03	18.77	15.52
	100 years	0.06	0.03	18.47	15.45
$\theta = -0.8, k = 2$	2 years	0.18	0.12	24.45	20.24
	4 years	0.09	0.06	20.72	17.34
	10 years	0.07	0.04	19.13	15.92
	20 years	0.06	0.03	18.77	15.52
	100 years	0.06	0.03	18.47	15.45
$\theta = -0.8, k = 4$	2 years	0.36	0.27	31.00	26.36
	4 years	0.20	0.12	24.11	20.33
	10 years	0.26	0.19	25.78	22.50
	20 years	0.34	0.27	28.97	25.74
	100 years	0.41	0.34	34.95	31.70
$\theta = -0.8, k = 5$	2 years	0.55	0.46	42.04	35.74
	4 years	0.19	0.12	24.06	20.12
	10 years	0.19	0.12	23.69	20.19
	20 years	0.23	0.17	25.22	22.25
	100 years	0.28	0.22	28.68	25.69

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.19. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (1, 0, 1)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	1 month	0.06	0.05	0.10	0.09	0.05	0.04	0.05	0.04
	3 months	0.22	0.20	0.44	0.43	0.06	0.02	0.08	0.03
	6 months	0.70	0.73	0.92	0.95	0.07	0.01	0.17	0.05
	2 years	1.00	1.00	1.00	1.00	0.24	0.02	0.89	0.81
	4 years	1.00	1.00	1.00	1.00	0.93	0.78	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.90$	1 month	0.05	0.05	0.08	0.07	0.04	0.04	0.05	0.05
	3 months	0.13	0.11	0.27	0.24	0.05	0.02	0.06	0.03
	6 months	0.40	0.40	0.67	0.68	0.05	0.01	0.10	0.03
	2 years	1.00	1.00	1.00	1.00	0.09	0.01	0.49	0.27
	4 years	1.00	1.00	1.00	1.00	0.38	0.05	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.95$	1 month	0.06	0.06	0.07	0.06	0.05	0.05	0.05	0.05
	3 months	0.08	0.06	0.12	0.11	0.04	0.03	0.04	0.03
	6 months	0.15	0.12	0.27	0.25	0.04	0.02	0.05	0.02
	2 years	0.72	0.75	0.93	0.95	0.03	0.01	0.11	0.02
	4 years	1.00	1.00	1.00	1.00	0.06	0.01	0.49	0.26
	10 years	1.00	1.00	1.00	1.00	0.62	0.23	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

^a ρ is the persistence parameter of the DGP (2)-(4).

^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.20. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (1, 0, 1)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	2 years	0.06	0.05	0.08	0.08	0.05	0.05	0.04	0.04
	4 years	0.09	0.07	0.17	0.16	0.05	0.03	0.05	0.04
	10 years	0.37	0.34	0.64	0.66	0.06	0.02	0.10	0.04
	20 years	0.91	0.94	0.99	1.00	0.10	0.01	0.27	0.10
	100 years	1.00	1.00	1.00	1.00	0.99	0.98	1.00	1.00
$\rho = 0.90$	2 years	0.06	0.05	0.07	0.07	0.05	0.05	0.05	0.04
	4 years	0.07	0.06	0.11	0.10	0.04	0.03	0.05	0.04
	10 years	0.19	0.16	0.39	0.38	0.05	0.02	0.07	0.03
	20 years	0.62	0.65	0.88	0.91	0.05	0.01	0.14	0.04
	100 years	1.00	1.00	1.00	1.00	0.60	0.20	1.00	1.00
$\rho = 0.95$	2 years	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05
	4 years	0.06	0.05	0.08	0.07	0.05	0.04	0.04	0.04
	10 years	0.08	0.07	0.16	0.14	0.03	0.02	0.05	0.03
	20 years	0.21	0.18	0.41	0.41	0.03	0.02	0.06	0.02
	100 years	1.00	1.00	1.00	1.00	0.08	0.01	0.72	0.56

^a ρ is the persistence parameter of the DGP (2)-(4).^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.21: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (1, 0, 2)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	1 month	0.12	0.07	21.60	21.60
	3 months	0.06	0.03	18.95	15.75
	6 months	0.06	0.03	18.59	15.26
	2 years	0.05	0.03	18.15	15.26
	4 years	0.05	0.02	18.02	14.71
	10 years	0.05	0.03	18.12	15.06
	20 years	0.05	0.03	18.21	15.13
	100 years	0.05	0.03	17.99	14.98
$\theta = 0.8, k = 2$	1 month	0.13	0.08	22.06	22.06
	3 months	0.06	0.04	18.97	15.80
	6 months	0.05	0.03	18.22	15.16
	2 years	0.04	0.02	17.79	14.85
	4 years	0.05	0.02	18.00	14.98
	10 years	0.05	0.03	17.81	14.95
	20 years	0.04	0.02	17.72	14.75
	100 years	0.04	0.02	17.79	14.88
$\theta = -0.8, k = 2$	1 month	0.22	0.14	24.54	24.54
	3 months	0.51	0.42	32.00	28.68
	6 months	0.65	0.58	39.70	36.76
	2 years	0.73	0.68	52.59	49.85
	4 years	0.75	0.70	60.17	57.43
	10 years	0.77	0.71	63.86	61.26
	20 years	0.76	0.71	67.81	64.69
	100 years	0.76	0.72	68.90	65.90
$\theta = -0.8, k = 4$	1 month	0.24	0.16	26.04	26.04
	3 months	0.21	0.14	24.52	20.97
	6 months	0.28	0.21	26.84	23.51
	2 years	0.35	0.29	31.20	28.15
	4 years	0.37	0.31	32.21	29.41
	10 years	0.38	0.32	33.95	31.03
	20 years	0.40	0.33	34.40	31.68
	100 years	0.39	0.33	34.62	31.68
$\theta = -0.8, k = 5$	1 month	0.33	0.24	29.81	29.81
	3 months	0.17	0.10	23.15	19.49
	6 months	0.20	0.14	24.35	20.89
	2 years	0.24	0.18	26.29	23.33
	4 years	0.25	0.19	27.29	24.27
	10 years	0.26	0.20	28.65	25.05
	20 years	0.27	0.21	28.49	25.51
	100 years	0.28	0.22	28.85	25.69

^a θ is the MA parameter of the DGP (2)-(4), k is the number of lags in the VECM (2).

^bAsymptotic critical values are 18.17 & 16.87 for the trace and max tests (Osterwald-Lenum, 1992).

Table B.22: Empirical Size of Standard Asymptotic Critical Values and Actual Finite-Sample Critical Values of Johansen Cointegration Tests for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (1, 0, 2)$.

	Sample Length	Empirical Size Based on Standard 5% Asymptotic Critical Value ^b		Actual 5% Critical Value	
		Trace Test	Max Test	Trace Test	Max Test
$\theta = 0, k = 2^a$	2 years	0.14	0.09	22.76	19.03
	4 years	0.09	0.05	19.91	16.65
	10 years	0.06	0.03	18.80	15.61
	20 years	0.05	0.02	18.37	15.16
	100 years	0.06	0.03	18.48	15.23
$\theta = 0.8, k = 2$	2 years	0.16	0.10	23.43	19.85
	4 years	0.09	0.05	20.05	16.97
	10 years	0.06	0.03	18.64	15.54
	20 years	0.05	0.03	18.19	15.09
	100 years	0.05	0.03	18.06	15.10
$\theta = -0.8, k = 2$	2 years	0.22	0.14	24.89	20.92
	4 years	0.33	0.23	26.45	22.90
	10 years	0.59	0.51	35.17	31.89
	20 years	0.69	0.62	44.10	40.95
	100 years	0.75	0.71	60.45	57.80
$\theta = -0.8, k = 4$	2 years	0.34	0.25	30.23	25.67
	4 years	0.19	0.11	23.49	19.92
	10 years	0.24	0.17	25.23	21.78
	20 years	0.31	0.24	28.10	24.75
	100 years	0.37	0.31	33.02	29.85
$\theta = -0.8, k = 5$	2 years	0.56	0.47	41.98	35.68
	4 years	0.18	0.12	23.94	20.02
	10 years	0.18	0.12	23.29	19.83
	20 years	0.21	0.15	25.00	21.69
	100 years	0.25	0.20	27.48	24.39

^a θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

^b Standard 5% asymptotic critical values are 18.17 for the trace test and 16.87 for the max test (Osterwald-Lenum, 1992).

Table B.23. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Daily Data from a Bivariate System, for $(a_1, a_2, b) = (1, 0, 2)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	1 month	0.05	0.05	0.06	0.06	0.06	0.05	0.05	0.05
	3 months	0.16	0.14	0.17	0.16	0.09	0.03	0.09	0.04
	6 months	0.54	0.58	0.59	0.63	0.15	0.03	0.22	0.07
	2 years	1.00	1.00	1.00	1.00	0.48	0.08	0.89	0.78
	4 years	1.00	1.00	1.00	1.00	0.99	0.94	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.90$	1 month	0.06	0.05	0.06	0.06	0.06	0.05	0.05	0.05
	3 months	0.10	0.09	0.11	0.10	0.07	0.03	0.07	0.04
	6 months	0.29	0.29	0.32	0.33	0.10	0.03	0.14	0.05
	2 years	0.99	1.00	0.99	1.00	0.22	0.02	0.55	0.29
	4 years	1.00	1.00	1.00	1.00	0.65	0.19	1.00	1.00
	10 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.95$	1 month	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	3 months	0.07	0.06	0.07	0.07	0.05	0.04	0.05	0.04
	6 months	0.11	0.10	0.12	0.11	0.06	0.03	0.07	0.03
	2 years	0.56	0.56	0.61	0.65	0.08	0.02	0.16	0.04
	4 years	0.99	1.00	1.00	1.00	0.15	0.02	0.59	0.31
	10 years	1.00	1.00	1.00	1.00	0.90	0.60	1.00	1.00
	20 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

^a ρ is the persistence parameter of the DGP (2)-(4).

^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

Table B.24. Size-Adjusted Finite-Sample Power of Johansen Cointegration Tests under the Alternative of Cointegration for Monthly Data from a Bivariate System, for $(a_1, a_2, b) = (1, 0, 2)$.

	Sample Length	$\theta = 0, k = 2^b$		$\theta = 0.8, k = 2^b$		$\theta = -0.8, k = 2^b$		$\theta = -0.8, k = 4^b$	
		Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test	Trace Test	Max Test
$\rho = 0.85^a$	2 years	0.05	0.05	0.06	0.06	0.06	0.05	0.06	0.05
	4 years	0.07	0.06	0.08	0.07	0.07	0.04	0.06	0.05
	10 years	0.26	0.25	0.27	0.28	0.11	0.03	0.14	0.05
	20 years	0.80	0.85	0.83	0.88	0.19	0.03	0.33	0.13
	100 years	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\rho = 0.90$	2 years	0.06	0.05	0.06	0.05	0.05	0.04	0.05	0.05
	4 years	0.06	0.05	0.07	0.06	0.06	0.04	0.05	0.04
	10 years	0.15	0.13	0.16	0.15	0.08	0.03	0.09	0.04
	20 years	0.48	0.50	0.50	0.55	0.10	0.02	0.17	0.05
	100 years	1.00	1.00	1.00	1.00	0.88	0.56	1.00	1.00
$\rho = 0.95$	2 years	0.05	0.05	0.06	0.05	0.05	0.05	0.06	0.05
	4 years	0.05	0.05	0.06	0.05	0.05	0.04	0.05	0.04
	10 years	0.08	0.06	0.08	0.08	0.05	0.04	0.06	0.04
	20 years	0.17	0.15	0.18	0.18	0.06	0.03	0.08	0.04
	100 years	1.00	1.00	1.00	1.00	0.22	0.03	0.77	0.57

^a ρ is the persistence parameter of the DGP (2)-(4).^b θ is the MA parameter of the DGP (2)-(4), and k is the number of lags in the VECM (2) used to perform the tests.

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