Contraception and the Fertility Transition

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Abstract

Dominant paradigms of fertility choice either ignore or assume small, unchanging cost of fertility limitation. Inspired by the historical English experience that is contrary to such assumptions, we modify the Beckerian paradigm to incorporate costly, societal influence on contraception. In the model economy, heterogeneous, generationally-linked households choose between “traditional” and “modern” contraception. The modern has a higher fixed cost (reflecting social opprobrium) but a lower variable cost of averting childbirths. Initially, the rich adopt the modern, and in doing so, unleash a social diffusion process. Eventually everyone switches lowering fertility further and across society. What hastens the switch is the decline in child mortality. The model is broadly consistent with important features of the English transition and has implications for more recent transitions.

KEYWORDS: child mortality, fertility, demographic transition, contraception

JEL CLASSIFICATION: O40, I12, J11

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1 Introduction

The Victorian period represents a major turning point in the population history of England. Three profound and unrelenting demographic transformations originate in this period. First came the mortality transition: life expectancy at birth rose from near 40 by 1830 to 50 by 1900. Much of this progress touched the lives of children: a newborn during 1861-70 had a 27.9% chance of dying before reaching age five, by 1911-20 only 16.5%. The fertility transition followed. Marriages in 1860, when they lasted twenty years or more, produced on average 6.16 births; by 1915, that number had declined to 2.43. Accompanying these radical transitions was the third, a somewhat overlooked revolution in family-size limitation – a contraception revolution, a new paradigm of conscious prevention (via natural or artificial means) of unwanted pregnancies subsequent to reaching parity. There is indirect evidence of such birth control: 32.9% of women born during 1840-69 had two or fewer children (live births) ever, compared to 57.5% of women born during 1870-99 (Hinde, 2003, Table 13.2). While change in nuptiality, the proportion of women marrying, was a proximate determinant of fertility until the eighteenth century, it had little explanatory power for the nineteenth century fertility decline. Similarly, changes in the distribution of age at marriage were too minor to play much of a role.

We propose a theory where these demographic transformations become causally entwined: the mortality transition triggers the contraception revolution, which in turn, makes the fertility transition possible. In our story, one or more factors trigger a reduction in the desired fertility of couples, be it rising adult wages that raise the opportunity cost of childbearing or declining child mortality that creates too many surviving children. All else same, whatever the parity (family-size target) may be, the natural process of procreation, if unrestricted, would likely overshoot it. This isn’t much of a concern if child mortality is high so the number of surviving children is still close to target. But if child mortality declines precipitously, as it did in England, couples have no choice but to adopt some method of birth control so as to stay on target. It is this line of argument that places the contraception revolution firmly at the center of our story and in between the mortality and the (subsequent) fertility transitions.

Placing the contraception revolution center stage does a whole lot more. Economists have long sought to explain the emergence and persistence of the fertility transition. Their search for a single explanation, a single trigger, has led them to doubt the role of the mortality transition and negate the contribution of the contraception transition (Galor, 2012).\footnote{For Becker (1960), rapid industrialization and urbanization circa mid-nineteenth century generated strong wage and income growth which raised the opportunity time-cost of children and triggered the switch from quantity to quality of children. For others, the acceleration in the rate of technological progress during this period increased the role of human capital in production and triggered households to have fewer but higher quality children (see Galor, 2005, 2012, for an overview of this research).} Indeed, the domi-
nant (Beckerian) paradigm of fertility views the contraception revolution as a side show, a mere reflection of changes in the demand for children: in that paradigm, couples perform a simple cost-benefit analysis to compute their fertility target and achieve it at no cost (Becker, 1960). But more importantly, the Beckerian paradigm implicitly assumes couples’ fertility decisions are impervious to social influences of any kind. As we discuss in Section 2, such an assumption is limiting: late nineteenth century England saw significant institutional resistance – from the clergy and the medical profession – to the practice of marital fertility control making a couple's goal of attaining their fertility target costly, challenging, and subject to social opprobrium. Additionally, there is evidence these costs abated over time as information about and access to fertility control became available and the very notion and possibility of fertility regulation became increasingly acceptable. Positioning contraception at the center of the historical fertility transition offer us one possible way to model these social influences on fertility and to study if their inclusion amplifies the potency of an existing trigger, be it wage growth or mortality declines, enough to elevate its status to the explanation.

In our model, the household's Beckerian cost-benefit analysis of fertility is expanded to include social influences on fertility as well as the cost of achieving that fertility via specific contraception strategies. In our setup, the biological process, left unhindered, will produce children in conformity with “natural fertility” whatever may be the true demand for children. If fewer children are desired, this biological process has to be restrained through contraception, with associated costs and benefits. We show that for a given contraception strategy, an exogenous decline in child mortality causes a decline in both childbirths (TFR) and surviving children (NFR). The latter is true only when income crosses a threshold, and once that happens, families start to make investments in the quality of their offspring. This is important for the following reason. A key pillar of historical demographic transitions is that declines in TFR and NFR often followed declines in child mortality. Simple versions of the Becker model, with homothetic preferences, have trouble delivering this link. This has been noted by Doepke (2005) and more recently Galor (2012). In fact Galor (2012) uses it to pass an indictment against such a causal link. Our model is able to deliver this link since costly fertility control naturally gives rise

Existing models within this paradigm could easily be amended to accommodate costly fertility regulation provided the cost is fixed, relatively small, and does not evolve systematically over time or across people. Qualitatively speaking, such an approach would essentially introduce a level effect on fertility relative to a standard Beckerian model (one with zero cost), and hence, would not add much insight.

Becker (1991) writes standard economic theories “appear sufficient to explain major declines in fertility, and simple and sufficiently effective birth control methods have been available to produce these declines” (p.141). We argue such a view is at odds with important ground realities of the English transitions. First, while it is true “simple and sufficiently effective birth control methods” had been known for centuries in England and elsewhere, information about how they worked was not widely available nor always precise. Second, that illegitimate fertility fell in conjunction with marital fertility during England’s transition indicates the means to achieve fertility targets changed (Knodel and van de Walle, 1986).
to non-homothetic preferences over the number of children. It bears mention that our link is conceptually very different from the same obtained via a precautionary-demand-for-children channel – one that arises because of uncertainty regarding the number of child survivors – explored in Kalemli-Ozcan (2008).

In the dynamic version of our model, we allow for persistent intra- and inter-generational income heterogeneity across households in order to capture cross-sectional differences in fertility behavior before, during and after the English transition (Clark, 2005). We also permit households to choose between two birth control strategies: a traditional method (infanticide, some abstinence, prolonged breast-feeding) and a more efficient, modern one (coitus interruptus, later sheaths and appliances).\(^4\) Associated with each strategy are costs that, for convenience, take the form of utility loss alone. We posit that while the modern strategy comes with a higher fixed cost (social opposition, informational barriers), it has a lower variable cost (more effective) when it comes to averting childbirths. The social diffusion of contraception adoption is specified as a random matching process that brings together adopters and those yet to adopt (Munshi and Myaux, 2006). While the initial adopters of the modern method are the rich – those who are not dissuaded by the associated higher fixed cost – eventually everyone switches to the modern technology so as to economize on the variable cost. What hastens the switch is the decline in child mortality. In addition, adoption of modern contraception unleashes a diffusion process that lowers its fixed cost over time. More people switch, further lowering the TFR and NFR. Our numerical experiments are designed to be suggestive of ways in which our story is in line with the English fertility transition.

While child mortality triggers the English fertility transition in our model, our theory is more general: it can include other catalysts, such as income growth. Income growth can, over time, take the economy from a Malthusian to a post-Malthusian to a modern equilibrium. Along the way, the cross-sectional correlation between household income and fertility goes from being positive to negative. The lower demand for children, from the post-Malthusian phase onwards, means a latent demand for fertility regulation which households are able to act upon when income crosses a threshold and/or child mortality rates fall. In other words, income growth alone can generate contraception and fertility transitions in our model.

The dynamic model incorporates income growth from exogenous productivity improvement. This is done mainly to generate plausible child quality investment in the model in line

\(^4\)Our decision to lump the withdrawal method with modern, appliance-based methods, may appear strange. Data limitations, as discussed in Section 6 below, govern this decision, partly. Perhaps more importantly, Victorian England was characterized by a “general state of profound public ignorance on matters of basic sexual functioning and anatomy...even among trained doctors.” (Garrett et al., 2001) The implication is that while coitus interruptus was known of and even practiced on occasion before 1850, it took a contraception revolution to get people to practice it as part of a deliberate, informed choice to curb childbearing.
with data from mid-nineteenth century. Within the confines of this limited purpose, our quantitative assessment suggests that England’s steep child mortality decline was more instrumental in its ensuing fertility transition than was income growth. A more general setup, one that allows income growth to play a more significant role, could potentially alter this conclusion. Our model, as such, should not be seen competing in quantitative horse races with income growth or other theories of the English transition, such as the precautionary motive (Kalemli-Ozcan, 2008), the gender wage gap (Galor and Weil, 1996) and productivity change (Bar and Leukhina, 2010) to a name a few. Income or human capital growth could have acted as triggers in the English case and in other societies, at other times, particularly where it is unclear whether or not the mortality transition preceded the fertility transition. Our quantitative work also shows that contraception by itself could not have triggered the fertility transition. Nevertheless, it was a vital link between England’s mortality and fertility transitions.

Beyond the relevance of our work for historical, and possibly modern, transitions (see Section 7), this paper makes several theoretical contributions. It shows that costly contraception in the Becker model can generate a link from child mortality to total and net fertility rates without relying on subsistence constraints that may not have applied to all socio-economic groups in nineteenth century England (Galor, 2005; Clark, 2005), or relying on uncertainty regarding child survival as in Kalemli-Ozcan (2008). It also shows how pre-transition economies are characterized by a positive cross-sectional correlation between household income and fertility, a correlation that turns negative during the transition as wealthier households adopt costlier but more efficient methods of contraception.

Our goal is to study how social influences on fertility decisions, social diffusion and the cost of contraception adoption modify our understanding of the Beckerian model. The important lesson learnt is that since the fertility transition is accompanied by rapid social change, understanding social influence in general and contraception choice in particular, can significantly improve our appreciation for mortality decline as a trigger of demographic change.

Some facts about the English transition are outlined in Section 2 below. Section 3 specifies a dynastic model where households care about the quantity and quality of children and face costly fertility regulation. The static and dynamic equilibria are analyzed in Section 4. Section 5 specifies the process of diffusion by which households switch to a more efficient contraception technology. Section 6 parameterizes the model. Based on observed declines in child mortality and indirect evidence on fertility regulation in late nineteenth century England, numerical simulations show that the predicted fertility transition broadly fits the time-series of the English transition. Section 7 discusses the applicability of our model to more recent transitions.
2 The Fertility Transition in England & Wales

In his tome, *The Demographic Transition*, Chesnais (1992) lays out three central propositions that capture major demographic transitions: (i) the chronological sequence of the transition, mortality decline, followed by fertility decline\(^5\), (ii) a general restriction of marriages followed by a limitation of births, and (iii) the context of modernization – the overarching trends in societies evolving from traditional, agrarian to modern, urban forms – and their effect on the onset of fertility decline. The English demographic transition, as we describe below, fits these propositions reasonably well.

2.1 The English mortality and fertility declines

Figure 1 illustrates the transitions in mortality and fertility. The measure of mortality used in the figure is child mortality rate (CMR), the probability of death between ages 0-5, data for which goes back only to 1841. As the upper panel shows, CMR remained high until the 1860s, after which it started to fall steeply.\(^6\) This was part of a broader progress in mortality that was confined to those between the ages of one and forty: mortality in these age groups fell by nearly 50% in the 1890s compared to their levels in the 1830s. Immunization against small pox and other diseases, improvements in sanitation and public health, all played significant roles in this mortality decline. This account of English mortality is largely agreed upon by demographers who use it to place the onset of the child mortality transition somewhere in the mid-to-late nineteenth century.

The onset of England and Wales’ – indeed much of Europe’s, France being a major outlier – fertility transition has been dated by demographers to somewhere in the 1870s (Figure 1) or a bit later (Chesnais, 1992, Table 4.3). The decline in the total fertility rate that we see in the upper panel of Figure 1 was accompanied by a decline in net fertility (lower panel). Fertility did decline earlier in the nineteenth century, between 1811-20 and 1846, but then remained steady for three decades until the 1870s when it started on its path of precipitous and consistent decline. The principal driver of that earlier decline was nuptiality, changes in the age and incidence of marriage.\(^7\)

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\(^5\)Some authors have noted that France, and possibly Belgium and Germany, were exceptions to this. Chesnais (1992) makes a persuasive case for otherwise.

\(^6\)Hinde (2003, p. 195), using a different yardstick, life expectancy at birth, for which longer time series are available, argues that the English mortality transition went through three phases: there was a “definite, though rather modest” decline during 1780-1830, followed by stagnation during the mid nineteenth century, and finally, a second “decisive” period of decline starting in the 1860s. The expectation of life at birth was roughly 35 years in 1750 and reached 40 years by 1830 (and stayed there till the 1870s); thereafter it rose faster, reaching 60 years by 1930.

\(^7\)Reviewing the literature, Hinde (2003; Ch 13) is definitive on this: “[... ] very little of the massive decline in the average completed fertility between women born in the middle of the nineteenth century and women born at
Consistent with Chesnais's second proposition, the English fertility transition represented a shift in fertility behavior from the extensive margin (nuptiality) to the intensive margin (fertility regulation within marriage), a transition from 'natural' fertility to controlled fertility. Fertility limitation now depended on the number of children a couple already had, reflecting a conscious choice and ability to restrict family size. Average completed family size of ever-married women fell from over 7 for birth cohorts 1826-31 to about 4 for cohorts 1891-96 (Hinde, 2003). 32% of women born in 1840-69 had completed family size of fewer than three children (live births), compared to roughly 56% of women born in 1870-99, suggesting some sort of deliberate family limitation was being pursued. Reher (1999) views this period of history as a break from the past. For the first time, faced with the reality of low child mortality, couples made the decision to “give individual choice priority over social norms and ended up curtailing their fertility...[The] entire context of social, economic and cultural modernization contributed to people's willingness to adopt new strategies when faced with new realities, but intuitively the point of departure would seem to have been incipient mortality transformation.”

What of Chesnais's first proposition? Chesnais makes a prima facie case mortality declines in portions of western Europe preceded fertility declines by at least a century, the implication being the mortality decline had little to do with the fertility decline. However, after accounting for data limitations, wide variability in declines, and exceptions such as France, Chesnais comes to the tempered conclusion that declines in mortality “seem to be necessary” for the fertility transition but “hardly sufficient” (p. 149). Reher (2009) is far more sanguine: “Nearly everywhere mortality decline preceded fertility decline. Child mortality was the first to decline, followed somewhat later by declines in infant mortality”.

For our purpose, it is of some importance that the mortality decline preceded and hence, could have instigated, the subsequent fertility decline. The exact length of the delay is of secondary concern. Our theory below can also generate a fertility transition from income growth alone. Clark (2005) ponders a similar question. He argues that higher social class and gross fertility were negatively associated in late nineteenth century England. Using wills, testators and survivors data, he find a positive association between income and net fertility in pre-transition England and concludes that “this positive association between fertility and income [in pre-industrial England] seemingly becomes negative in the period of the demographic transition”.

the end of that century can be attributed to changes in nuptiality”. This suggests the extensive margin of fertility was not that important which, in turn, justifies the singular focus of this paper on the internal margin. A new and contrarian twist to this issue comes from Garrett et al. (2001): “choosing to marry later may have been as conscious an act as choosing to use a particular method of contraception." And in their view, this explains France and England's "different paths to low fertility" because "postponement of marriage was a 'conscious choice' by individual English couples and thus equivalent to the fertility regulation being practised by their cross-channel neighbours."
2.2 The Costs of Contraception

Knowledge about birth control goes back to several centuries. Hippocratic texts of the late fifth and fourth century BCE contain details about controlling the biology of procreation and, even earlier, the withdrawal method was discussed as a means to avoid conception (McLaren 1991, Kohler, 2001). Despite this, centuries-old knowledge of some methods of contraception was never widely diffused, nor were such forms of contraception widely practiced. Sources often presented medically reliable techniques right next to magical recipes, sowing more confusion than informing. Moreover, while the birth control movement of nineteenth century drew on this centuries-old knowledge, its immediate source were Islamic texts, indicating the lack of accessible European sources (Peel, 1964).

Books extolling the virtues and rational methods of birth control started appearing in England in the early nineteenth century. For a while these had little impact on fertility behavior or public opinion. In his book *The Fruits of Philosophy* published in 1832 in Massachusetts, Charles Knowlton discussed the effectiveness of douching as a new method of birth control and informed readers about the physiology of conception and treatments for fertility and impotence. It too had little immediate impact. It was only in 1876, when the English fertility transition was just unfolding and the book was republished in England that it became enormously controversial and popular. The publishers Annie Besant and Charles Bradlaugh were indicted on grounds of publishing “lewd, filthy, bawdy and obscene” material and sentenced to imprisonment, later overturned on appeal. The attention garnered by the case raised awareness and sales of the book shot up from a few hundred to thousands per year. More publications on birth control followed in the subsequent decades, many promoting relatively easy to use, at-home, methods (Hinde, 2003). Not all such literature was reliable. Many pseudo-scientific handbills and pamphlets were from quacks. McLaren (1991) concludes that “the circulation of the quacks’ literature was in addition (to the expansion in publishing facilities) an indication of the public's continuing desire for some type of instruction on procreation” (p. 80).

There is evidence that in most regions of England, marital fertility was lowered through “stopping” behavior. This was primarily achieved through more frequent usage of abstention and the withdrawal method. The latter was particularly responsible for the sharp reductions in working class fertility achieved during 1880-1910 (McLaren, 1991). Contraceptive devices, on the other hand, were not widely used initially. Condoms and spermicides were not widely available and too expensive until the twentieth century.\(^8\) Secondly, ignorance and unreliable promises made by the penny press made people more skeptical of these devices. Many women

\(^8\)Considerable advancements in reproductive technology happened at the start of the twentieth century. By the 1920s, caps and diaphragms started to become widely available in birth control clinics. Then came spermicidal pessaries, sheaths, and finally, by early 1930s, the latex condom (Fisher and Szreter, 2003).
perceived them to be physically harmful, a perception aggravated by medical advice. Condoms, on the other hand, were associated in the popular mind with prostitution and the prevention of STDs (Seccombe, 1990).

A major hurdle to the democratization of birth control in English society was the deep-seated resistance from the medical profession and, less surprisingly, religious authorities. A former prime minister’s public advocacy of doctors providing guidance on family limitation was widely condemned by the medical press. The medical community labeled family limitation as folk medicine practiced by quacks and midwives. Others went further. Dr Charles H. F. Routh, an influential gynecologist at the Samaritan Hospital for Women and Children, railed against contraception as “a great moral crime” and blamed it for a long list of serious physiological harm and psychological problems including mental decay, cardiac problems, metritis, cancer, sterility and suicidal tendencies. As late as 1901, an editorial in the *British Medical Journal* labelled it as “unnatural and degrading...and oft injurious to both husband and wife” (Soloway, 1982).

Requests for information about contraceptives were regularly rebuffed by public health professionals including nurses and pharmacists and, aggravating matters, physicians failed “to make a distinction between abortifacients and contraceptives” (Seccombe, 1992). Indeed many doctors felt it was their duty to prevent the spread of contraceptive knowledge and devices. Correspondents in Marie Stopes’ *Mother England* mention that doctors had refused to inform them about contraceptive devices, even as they warned patients of the dangers of further pregnancies (Secombe, 1992). McLaren (1991) cites the writing of one doctor: “It was not the doctor's duty to instruct laymen on the details of procreation for such information might be then used to escape the punishment ordained by God for sexual misdemeanours” (p. 80).

Religious authorities, both Protestant (Anglican) and Catholic, reinforced this opposition to any form of contraception until 1930 when the Anglican Church first adopted a more accommodating stance (Szreter, 1996). Religious doctrines of the times proscribed couples from “engaging in any act which would hinder the natural power of procreating life” and rejected appliance-based methods on the grounds they tampered with “nature or God's will” (Fisher, 2006). In many cases, the clergy simply ignored or avoided the topic: “they don't talk about it in church...[yet] everybody that goes to church knows...you're not supposed to use contraceptives” (Fisher, 2006, p. 152).

In an era of insufficient information and weak understanding of the biology of procreation, it is not hard to imagine that medical, religious and political resistance to the idea of birth con-

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9Garrett et. al (2001) describe it succinctly as follows: “Such evidence as is available indicates a generally negative and moralistic attitude towards contraceptives, something that was endorsed as much by a medical profession anxious to avoid slurs on its reputation, as by the Anglican clergy and by working-class leaders.”
trol would have raised the social cost of practicing family limitation in late nineteenth cen-
tury England. They would have been privately costly as well. Certain methods like the rhythm method were unavailable as the precise timing of ovulation was not known. For working classes, contraceptives were expensive items. Female contraceptive devices were especially so, requiring clinical help and being subject to the risk of infection in the absence of indoor plumbing. And methods such as withdrawal and abstinence that had been practiced only sporadically earlier, would have been psychologically costly because of their perceived health and moral failings (Seccombe, 1990).

It is in order to discuss a contrarian view of all this due to Garrett et al. (2001). Their argument relies on two planks: a) that there was no “emergence of novel ideas or attitudes concerning family size....[no] dramatic social change or cultural innovation in the widespread use of appliance methods of birth control”, and b) “available...survey evidence all indicates that ‘traditional', non-appliance techniques of birth control, involving attempted abstinence, coitus interruptus and abortion, were the main methods of birth control.” In our view, one shared by decades of demography research including the European Fertility Project’s, is that what emerged was the very idea that fertility was indeed controllable which meant, for the first time, ‘conscious choice’ could be applied to fertility. At the same time as this new notion is surfacing, the demand for some kind of fertility control is starting to rise as mortality starts to fall appreciably and incomes continue to rise. This urgency for birth control stokes the need to get more information on the process of procreation leading to a renewed attention on coitus interruptus. All this was impossible to ignore because the issues were getting discussed publicly, in meetings, newspapers, pamphlets. The outrage, we describe above, ensued precisely because a social change was perceived to be underway: it was a reaction to this change. It implies the social change was considered unacceptable possibly because it was new, not “traditional”.

2.3 Diffusion

The English fertility decline broadly followed socio-economic lines with the fertility differential across social and occupational categories widening in the early days of the transition. The decline started among the upper and middle classes where women marrying in the 1880s were having 3.8 children compared to 6.3 for those who had married in the 1860s (Woods, 2000, Ta

10“The year 1876 should not, we would thus argue, be taken as a chronological marker either of the emergence of novel ideas or attitudes concerning family size, or of the advent of new methods of birth control. Indeed, Szreter has seriously questioned the extent to which there is any plausible evidence that there was any dramatic social change or cultural innovation in the widespread use of appliance methods of birth control before the inter-war decades of the next century. He has pointed out that careful scrutiny of the available quantitative, qualitative and survey evidence all indicates that ‘traditional', non-appliance techniques of birth control, involving attempted abstinence, coitus interruptus and abortion, were the main methods of birth control used before the Great War.”
ble 4.1, age at marriage 20-24). Even though fertility fell in successive marriage cohorts for all socio-economic groups, the faster decline among urban property classes opened up a fertility differential. By the late nineteenth century, working-class fertility started falling noticeably, first among Yorkshire and Lancashire's textile workers many of whom were married women, then among other working-classes well into the twentieth century (Seccombe, 1990; Haines, 1992).

Many demographers see this pattern of differential fertility transition as indicative of social diffusion, as “the new reproductive habits began to spread (from the upper- and middle-classes) amongst the less privileged social groups” (Woods, 2000, p. 114, quoting from the work of J. A. Banks). This view of social diffusion was reinforced by the European Fertility Project's findings that fertility decline often followed religious and linguistic lines across economically disparate regions of the continent. Szreter's (1992) reinterpretation of the English data modifies Banks' hypothesis. The process of diffusion and absorption of new reproductive norms, Szreter argues, could not have been as simple as copying the family limitation practices of the upper- and middle-classes by lower socio-economic groups. If this were the dominant process of diffusion, one would have expected female domestic servants hired in upper- and middle-class families to have been major agents of change. The evidence, however, suggests women in domestic service were socially isolated and least likely to acquire knowledge of sexual matters. Instead, any diffusion of behavior would have worked through job expectations: “as the servant-employing class itself found it increasingly necessary to restrict its own family size...it may well have adjusted downwards, either consciously or not, its tolerance for large families among its resident servants” (Szreter, 1992, p. 479).\(^{11}\)

Family limitation behavior of the upper- and middle-classes may have, however, indirectly affected the behavior of other classes. As we pointed out earlier, social activism and the arrival of new literature on birth control increasingly placed family limitation in the popular consciousness. That a couple could choose to restrict family size, an idea that would have alien just a few decades earlier, gained popularity and feasible by the late nineteenth century. Diffusion, in this case, took the form of social influence.

Where social learning, the diffusion of knowledge about birth control, played a role was within some socio-economic groups. In her review of Marie Stopes' correspondence with working-class women soliciting birth control information, Seccombe (1990, 1992) offers convincing evidence of this. Working-class women acquired knowledge of contraceptives through word of mouth. The main hurdles were not so much a stigma against discussing these as ignorance about contraception and a taboo against the use of artificial means to restrict fertility within

\(^{11}\)This motivates our formalization of contraception adoption as an arms-length process of diffusion from early to late adopters.
marriage. Many of Stopes’ correspondents were fearful of the side effects of contraception in general and “preventatives” in particular while others were unsure where to find contraceptives since they were not yet sold over the counter in local shops.

3 The Model

Demographers classify theories of the fertility transition in two categories, “innovation-based” and “adjustment-based”(see Hinde, 2003, Ch. 13). In the former view, for example in the findings of the European Fertility Project, a sharp distinction is drawn between pre-transition communities where most people found the very idea of family limitation inconceivable and subsequent post-transition communities where the question, “how many children should we have?” was routinely asked and answered (Woods, 2000, p. 169). In this view, the transition is precipitated by a social innovation, the widespread recognition of the notion that family limitation does lie in the realm of conscious action. Proponents of this view argue fertility behavior changed as a consequence of this social innovation and, in turn, fostered increased adoption of contraception over time and space.

Adjustment-based theories, on the other hand, highlight how socio-economic changes such as increased urbanization and industrialization lowered the demand for children and, in response, families welcomed birth control into their fertility calculus. Adjustment-based theories – what economists call the demand-side view – assume, however, a household’s fertility objective is achieved at little or no cost. They also assume family limitation was always part of the realm of conscious action: if prevailing socio-economic conditions justified high fertility, family limitation, even when available, would not have been adopted. The evidence from Victorian England we have presented in Section 2.2 suggests that actively regulating marital fertility would have been financially and psychologically costly in the face of social opposition, scarce information and non-availability of reliable external devices. Hence, in our view, any plausible account of the historical fertility transition needs to recognize these restrictions (see the conclusion for a discussion of how this relates to modern fertility transitions).

The theory we propose is an amalgam of the innovation and adjustment hypotheses. Diffusion of the notion of fertility control alone cannot explain the onset of the English transition. That fertility started falling across all socio-economic groups from the 1870s – groups with very different birth-control adoption rates – suggests an underlying common cause that necessitated a need for smaller families. In our model, declining child mortality across all income groups is

12 It is important to recognize that evidence of a contraception revolution does not imply a contraceptive revolution. Indeed contraceptives – external devices and medications – were not widely used during much of the English transition.
that common cause, one which lowers the demand for children and motivates the need for an
adjustment in fertility objectives. Initially-available birth control methods are not as effective
in achieving those objectives and adoption of more effective methods involves hefty switching
costs. Richer households are the first to respond: they reduce family size by switching to more
efficient forms of birth control, despite the high cost. These costs fall over time as information
about modern methods becomes widely available and social opposition to their use in marital
fertility regulation abates. As child mortality and fertility limitation costs keep falling, effective
fertility control becomes a no-brainer for more and more households. The fertility transition,
ultimately, sweeps through all rungs of the socio-economic ladder as economy-wide fertility
reaches near-replacement levels.

We start by developing the basic model for a given birth control technology, with social dif-
fusion shut down (until we reach Section 5). The starting point of our model is the eve of the
fertility transition. In the model economy, the social innovation discussed above has already
happened, that is, family limitation is firmly in the fertility consciousness of agents and tra-
ditional forms of birth control have been adopted. Child mortality, the common-cause, the
adjustment, is about to sharply decline.

3.1 Basics

Consider a single-good economy consisting of an infinite sequence of three period-lived over-
lapping generations of agents. The three stages of life are childhood, adulthood and old age. At
each date $t = 1, 2, ..., \infty$, a continuum of agents (children) with mass $n_t$ is born to existing adults.
An exogenous fraction, $\phi_t$, of them survive to adulthood.\textsuperscript{13} In childhood, agents are passive;
they simply acquire human capital, which is paid for by their parents. As young adults, they be-
come active decision-makers: they work, determine their family size and strategies to achieve
that, spend time and resources raising children, invest in their schooling, consume, and save
for old age. Everyone retires in old age and consumes all wealth before dying.

Adults are endowed with one unit of time which they allocate between work and raising chil-
dren. Adults differ in their labor productivity. Specifically, they are heterogeneous on a single
innate, unobserved dimension which, for convenience, we call ability. Ability $\epsilon$ is drawn identi-
cally and independently across adults (and across generations) from a cumulative distribution
$G[\bar{\epsilon}, \tilde{\epsilon}]$ with positive support ($\bar{\epsilon} > \tilde{\epsilon} > 0$) and an unconditional mean of unity. An adult’s ability
draw is realized upon joining the labor market. If a child’s parent invests $x_{t-1}$ units of goods in
her education, similar to Moav (2005), her human capital as an adult (at the beginning of $t$), $q_t$,

\textsuperscript{13}Henceforth, $\phi$ will be dubbed the child survival rate and $1 - \phi$ the child mortality rate. We implicitly assume $\phi$ is
revealed early in childhood, before parental investments are made. Anyone who survives to adulthood is assumed
to survive till the end of old age.
is given by

\[ q_t = Q(x_{t-1}) \equiv a(1 + x_{t-1})^\sigma, \quad a > 0, \quad \sigma \in (0, 1). \]  

(1)

This human capital, combined with her ability, produces labor market income. For a child born at \( t - 1 \) whose ability \( \varepsilon_i \) is realized as an adult at \( t \), “full income” in period \( t \) is given by \( v_{it} \equiv \varepsilon_i q_t w_t \), where \( w_t \) is the competitive wage rate per efficiency unit of labor.

The combined savings of adult-workers gets converted into the future aggregate capital stock, \( K \), which depreciates fully upon use and earns a return \( \rho_{t+1} \) between \( t \) and \( t + 1 \). The single good is produced from physical and human capital using the technology \( F(K_t, H_t) = A_t K_t^\mu H_t^{1-\mu}, \ A_t > 0, \ \mu \in (0, 1) \) where \( A_t = (1 + g) A_{t-1} \) and \( g \geq 0 \) is the fixed rate of exogenous TFP growth. If \( g > 0 \), TFP grows over time causing \( w \) to grow, which in turn, causes \( v_{it} \) to trend. Until further notice we set \( g = 0 \) and \( A_t = A \ \forall \ t \).

### 3.2 Contraception technology

As noted before, demand-side models in the Beckerian tradition assume households’ desired fertility can be costlessly implemented and without regard to social influences. Departing from this tradition, we model family limitation as a conscious albeit costly attempt to control fertility (Michael and Willis, 1976; Easterlin, 1975), one susceptible to social pulls. In our setup, the household’s Beckerian trade-offs are adjusted further to respect the cost of achieving its fertility target. Our formulation is related to Easterlin (1975) and Easterlin et al.’s (1980) “synthesis” model of fertility wherein a household faces costly fertility regulation and a child production function (“supply” of births). Unlike us, Easterlin treats the availability and use of contraception methods as exogenous, and does not consider the transition from ‘natural’ to controlled fertility as arising from the mortality decline and ensuing social change.

The details are as follows. Left unfettered, the biological process of procreation is assumed to produce \( \eta \) children in any household. Here \( \eta \) corresponds to natural fecundity which depends on biological factors and possibly social factors such as nuptiality and age at marriage. This is taken to be exogenous as our focus is on the intensive margin of fertility regulation. If a household wishes to attain a fertility level lower than \( \eta \), the biological process has to be subjected to active birth control through the adoption of a contraception strategy.\(^{14}\)

We distinguish between two contraception strategies: a readily-available, traditional strat-
strategy and a more efficient, modern one. It is convenient to think of the latter as "appliance-based," caps, condoms, diaphragms, pessaries and the like. But recall from above that the English transition initially relied on methods such as withdrawal that had been known for much longer. What was different was a new attitude towards fertility regulation. Even though the withdrawal method is easy to comprehend with our modern attitudes, in a society where it had been practiced sparingly, the biology of procreation ill understood, and physicians routinely blurred the distinction between abortion and contraception, it is not hard to imagine that it would have been significantly costly to adopt. As such we club together this newly adopted withdrawal method with newly available appliance-based methods and label them modern. The traditional method of birth control should then be interpreted as interventions such as longer duration breast-feeding, occasional abstinence (particularly by the rich who could afford separate bedrooms for the spouses) and possibly age at marriage. Associated with each strategy are costs that, for convenience, take the form of utility loss alone. While some strategies are costly in terms of diminished sexual satisfaction or lack of spontaneity, others bring about real or imagined decreases in physical health, require sacrifice of religious principles, or involve an embarrassing search for reliable information about their implementation and availability.

Let \( j \in \{1, 2\} \) denote contraception strategies where the first strategy (technology) is identified with the traditional method, the second with the modern method. Let \( \alpha_j \geq 0 \) be a fixed cost while \( e_j \) denotes how faithfully the adult household employs strategy \( j \). The utility cost of adopting contraceptive strategy \( j \in \{1, 2\} \) is \( \Gamma^j = \Gamma(e_j) = \alpha_j + e_j^2 \) where the second term represents a convex ‘variable cost’. Higher values of \( e \) enable the household to more effectively attain its fertility target but at higher variable cost. In Section 5, we specify how the fixed cost \( \alpha_j \) depends on social influence. For now, it is enough to note that \( \alpha_2 > \alpha_1 \) in a society on the cusp of fertility regulation.

Contraception strategy \( j \) is associated with the child production function

\[
h^j(e) = \eta \left(1 - \frac{e}{\sqrt{\lambda_j}} \right), \quad 0 \leq e \leq \sqrt{\lambda_j},
\]

where we have dropped the subscript on \( e \). If contraception is not practiced by the household, no births are prevented and \( \eta \) children are necessarily born: \( h^j(0) = \eta \). Since \( \frac{\partial h^j(e)}{\partial e} < 0 \), more intensive implementation of \( j \) via higher \( e \) lowers the number of children relative to \( \eta \). Also note that \( \frac{\partial h^j(e)}{\partial e} \), the marginal efficiency of contraception method \( j \), depends on \( \lambda_j \). A higher value of \( \lambda_j \), ceteris paribus, makes technology \( j \) less effective at averting births. Hence-

---

\(^{15}\)The Lewis-Faning survey found only 1.4% of the non-appliance method users referred to the monetary cost of appliance methods as being the chief deterrent to their use. (Fisher, 2006; p. 143)
forth, we refer to $\lambda$ as the ineffectiveness of a contraception strategy and assume $\lambda_1 > \lambda_2$. For a given $e$, more births are averted with technology 2 (the modern one) than with technology 1 (the traditional one) as illustrated in Figure 2a.

After inverting $h(e_j)$ and substituting into $\Gamma^j$, rewrite the utility cost of contraception method $j$ as

$$\Gamma^j(n) = \alpha_j + \lambda_j \left(\frac{\eta - n}{\eta}\right)^2, \quad \alpha_j \geq 0, \ 0 \leq n < \eta$$

(2)
a decreasing and convex function of childbirth. For a given technology, as the need for lower $n$ rises, the variable cost of having to avert $\eta - n$ births also rises. Also, from (2), we have

$$\Gamma^2(n) = \alpha_2 + \lambda_2 \left(\frac{\eta - n}{\eta}\right)^2, \ \Gamma^1(n) = \alpha_1 + \lambda_1 \left(\frac{\eta - n}{\eta}\right)^2; \quad 0 \leq \lambda_2 < \lambda_1; \ \alpha_1 < \alpha_2$$

implying technology 2 is more effective (has lower variable cost) at averting births than technology 1 but comes with a higher initial fixed cost. For example, in Figure 2b, $\Gamma^2(n)$ and $\Gamma^1(n)$ are shown to cross; the former is higher at $\eta$ and the latter is higher at some $n'$. These differences underlie some of the results we obtain later. While the initial adopters of the modern, appliance-based technology are the rich – those who are not fazed by the associated higher fixed cost, $\alpha_2$ – eventually everyone switches to the modern technology so as to economize on the variable cost.

With a slight abuse of notation, henceforth, the utility cost of restricting fertility using contraception strategy $j$ will be denoted $\Gamma^j(n)$, an increasing and convex function of the number of births avoided through contraception.

### 3.3 Preferences

Adult households view children as consumption goods and derive direct utility from the number of surviving children, $\phi_t n_t$. Following Galor and Weil (2000), Greenwood and Seshadri (2002), de la Croix and Doepke (2003) and many others, parents are assumed to be imperfectly altruistic, receiving a warm-glow from the average human capital (earning potential), $\phi_t n_t q_{t+1}$, of their surviving children. Parents make their investment subsequent to child survival and care only about the expected human capital of the child, that is, they ignore uncertainty with respect to child ability. In addition, they care about their own consumption in adulthood and old-age, denoted by $c_{1t}$ and $c_{2t+1}$ respectively. An adult agent-household $i$ at date $t$ employing contraceptive method $j$ derives utility $V_{it}^j$ where

$$V_{it}^j \equiv u\left(c_{1it}\right) + \beta u\left(c_{2it+1}\right) + H\left(\phi_t n_t^j q_{t+1}^j\right) - \pi \Gamma^j(n_t^j)$$

(3)
where $u$, $y$ and $H$ are strictly increasing and strictly concave functions. We take these functions to be logarithmic and reformulate (3) to

$$V_{jt} = \ln c_{1t}^i + \ln c_{2t+1}^i + \gamma \ln(\phi_t n_t^i) + \gamma(1 - \theta) \ln q_{t+1}^i - \pi \Gamma^i(n_t^i)$$

(4)

where $\beta > 0$, $\gamma \in (0, 1)$, $\theta \in (0, 1)$ and $\pi \in [0, 1]$ is an indicator function. When $\pi = 0$, only the well-known Beckerian calculus is operative and achieving desired fertility is costless. When $\pi = 1$, the household’s fertility is the outcome of a birth-control technology. It can, with certainty, have any number $(n < \eta)$ of children it desires but now, besides the Beckerian quantity-quality concerns, it must take into account the cost of achieving its desired fertility.\(^{16}\)

Three points bear emphasis here. First, one must not think of the Beckerian calculus as, in some sense, preceding the contraception calculus. In the model, the contraception calculus is omnipresent; households do not face a choice between adoption and non-adoption of contraception. In Section 5 below, they will entertain the possibility of switching from traditional forms of birth control to modern ones. Second, rejecting the contraception calculus unfailingly implies $n = \eta$. Of course it is possible that families choose $n = \eta$, in effect behaving “as if” they had not adopted contraception. We rule this out via Assumption 1(d) below. Third, our formulation of contraception costs ostensibly appears similar to a quadratic norm over fertility, that is the cost of deviating one’s fertility choice from the social norm, captured by some average $\bar{n}$, is $(n - \bar{n})^2$. But typically such formulations rely on births not survivors. For us the relevant deviation is $\eta - n$, births averted from a state of natural fertility. An earlier work, Bhattacharya and Chakraborty (2012), implies that in order for social norms and child mortality to matter for net fertility, it is not sufficient to have non-homothetic preferences, social norms have to be defined with respect to childbirths instead of surviving children. The present paper may be viewed as building on that idea but differs fundamentally in that social influence matters only for the cost of modern contraception, not family size norm.

3.4 The household’s problem

For now abstract from ability heterogeneity and the choice between alternative contraception strategies and, as such, suppress the $i$ and $j$ superscripts. Assume everyone has already adopted the natural method of contraception. Each adult household allocates its time endowment between market activity – which brings in income $v_t$ for each unit of labor time – and child rearing. Each surviving child requires parental nurture and care of $\delta > 0$ units of goods, and possibly an additional $x$ units of goods of the parents’ choosing in schooling invest-

\(^{16}\)In this setup, with continuous fertility, birth control implies stopping not spacing behavior: households use birth control to stop future births once desired family size is reached, not to space births better.
ment. Time-cost and goods-cost of child rearing are distinct and not equivalent. Let $s_t$ denote saving and $R_{t+1}$ be the gross one-period return on period-$t$ saving. A typical adult-household, taking $v_t$ as pre-determined and $\phi_t$ and $R_{t+1}$ as parametric, solves the following program at date $t$:

$$\max_{\{s_t, n_t, x_t\}} V_t \equiv \ln c_{1t} + \beta \ln c_{2t+1} + \gamma \theta \ln(\phi_t n_t) + \gamma (1 - \theta) \ln q_{t+1} - \pi \Gamma(n_t)$$

subject to the budget constraints

$$c_{1t} + s_t = (1 - \tau \phi_t n_t) v_t - \delta \phi_t n_t - \phi_t n_t x_t,$$

$$c_{2t+1} = R_{t+1} s_t,$$

equations (1) and (2) and the additional constraints

$$c_{1t} \geq 0, c_{2t+1} \geq 0, s_t \geq 0, n_t \geq 0, x_t \geq 0.$$

Note that costs of child rearing and direct utility from children depend on the number of surviving children, hence on $\phi$. In contrast, the cost of contraception is independent of $\phi$ since it depends on the number of childbirths.

Logarithmic preferences imply that non-negativity constraints for consumption (hence, saving) and fertility will not bind in equilibrium. The household may, however, choose to have $n_t \leq \eta$ children ($n = \eta$) or not invest in child quality ($x = 0$). Denoting the Langrange multipliers associated with $n_t \sum \eta$ and $x_t \prod 0$, by $\lambda$ and $\mu$ respectively, the necessary first order conditions for optima are:

$$\begin{align*}
\frac{1}{c_{1t}} &= \frac{\beta R_{t+1}}{c_{2t+1}} \\
\frac{\gamma \theta}{n_t} + \beta \ln(\eta - n_t) &= \frac{\tau \phi_t v_t + \phi_t (\delta + x_t)}{(1 - \tau \phi_t n_t) v_t - \delta \phi_t n_t - \phi_t n_t x_t - s_t}, \zeta_n \geq 0, \zeta_n(n - n_t) = 0 \\
\frac{\gamma(1 - \theta)\sigma}{1 + x_t} + \zeta_x &= \frac{\phi_t n_t}{(1 - \tau \phi_t n_t) v_t - \delta \phi_t n_t - \phi_t n_t x_t - s_t}, \zeta_x \geq 0, \zeta_x x_t = 0
\end{align*}$$

for $s_t$, $n_t$ and $x_t$ respectively. We introduce a few parameter restrictions.

**Assumption 1** For all $t \geq 1$, assume that

(a) $v_t > (1 - \delta) / \tau \equiv v_{\text{min}}$

(b) $\theta > \max \{\sigma / (1 + \sigma), 1/2\} = 1/2$

(c) $\delta < 1$

(d) $\phi_t \in (\phi_{\text{min}}, 1]$ where $\phi_{\text{min}} \equiv \gamma (\theta - \sigma (1 - \theta)) / [\eta \tau [\psi + \gamma (\theta - \sigma (1 - \theta))] ],$

$\psi \equiv [1 + \beta + \gamma \sigma (1 - \theta)].$
Part (a) ensures that income is high enough to afford children. Below we show it also guarantees a tradeoff between $c_{1t}$ and $n_t$. Since the budget constraint can turn non-convex in the presence of both child quality and quantity, part (b) is sufficient to ensure second order conditions hold (see appendix A for details). Such a parameter restriction is, by now, standard in quantity-quality models of fertility (for example, Jones et al., 2008). Part (d) implies the child survival rate corresponding to natural fecundity ($n = \eta$) is $\phi_{\text{min}}$. By restricting $\phi_t \in (\phi_{\text{min}}, 1]$, we ensure $n$ stays below $\eta$.\footnote{For interior $x$, it follows from (10) that $\gamma \sigma(1 - \theta)c_{1t} = \phi n_t(1 + x_t)$ implying $n = 0$, the other corner, is not a possible optimum.}

The importance of part (c) is clarified later.

### 3.5 Some Intuition

Given the somewhat non-standard household preference, it is useful to build up some intuition on how it works. Our focus is solely on two items, the income-expansion path and the connection between the survival probability ($\phi$) and surviving fertility ($z \equiv \phi n$). For convenience, we drop the $t, i$ and $j$ subscripts here. Also, for the present, we ignore the saving decision. Start with $V = u(c) + H(z)$, where $u$ and $H$ are strictly concave, with the corresponding budget constraint $c = (1 - \tau z) v - \delta z$. This is a very basic setup with no contraception and no quality-quantity trade-off – in short, a Malthusian, no-contraception regime (Galor, 2012, Section 3).

As in all models of fertility choice, the price of children $p_z = \delta + \tau v$ is income-dependent as children require both sacrifice of consumption goods and foregone labor income. Recall that homothetic tastes can be represented by any utility function that has the mathematical property of being homogeneous; also, for homothetic preferences, the ratio of demands for the two goods is independent of income. Suppose, for example, $u$ and $H$ are each of the CES form, i.e., $u(d) = H(d) = d^{1-\omega_1}/(1 - \omega_1)$. Then optimality implies that $z/c = (\delta + \tau v)^{-1/\omega_1}$ – the relative demand for children is monotonically falling in household full income as children are relatively more expensive for richer households.

More generally, household optimality requires that $u'(c(z))(\tau v + \delta) = H'(z)$. Notice that $\phi$ is not an independent argument anywhere in this equation and, hence, any $z$ that solves this equation is independent of $\phi$. In other words, an increase in $\phi$ raises the marginal cost and lowers the marginal benefit of surviving children proportionately. It follows $\phi(\partial n/\partial \phi) + n = 0$ so the number of childbirths, $n$, is proportionately declining in $\phi$. A natural question would be, is the independence of $z$ from $\phi$ a consequence of the assumed homogeneity of $u$ and $H$? The answer is no. To break the homogeneity suppose that $u(d) = d^{1-\omega_1}/(1 - \omega_1)$ and $H(d) = d^{1-\omega_2}/(1 - \omega_2)$. It is easy to check, even here, the optimal $z$ is independent of $\phi$.

Galor (2012) argues a necessary condition for declines in mortality to lower surviving fertility is a stochastic $z$. Faced with uncertain child survival, parents may hold a “buffer stock”
of children. We take an alternative route by adding a convex cost to achieving target fertility through contraception, \( \Gamma(n) \). Suppose \( V = u(c) + H(z) - \Gamma(n) \). The first order condition is \( \phi H'(z) - \Gamma'(n) = \phi (\delta + \tau v) u'(c(z)) \) where the marginal benefit of increasing fertility is now higher because it requires households to less faithfully follow contraception. More importantly, \( \phi \) is now an independent argument in this equation. While parents care only about the number of surviving children, the cost of having them is incurred over the number of childbirths.

Finally, note that the quadratic form for \( \Gamma(n) \) renders the lifetime utility function (5) non-homogenous and non-homothetic. The presence of \( \Gamma(n) \) breaks the homogeneity of \( V \) even though the homogeneity of \( u \) and \( H \) are retained. This will play a role in shaping the income-expansion path, helping generate the non-monotonicity discussed in Section 5 below.

## 4 Fertility Regimes

For the household's decision problem specified above, we first characterize properties of the fertility equilibrium at any given date \( t \) where \( x_t > 0 \), then proceed to analyze settings in which parents do not invest in child quality. A regime with \( x_t > 0 \) is labeled *Modern*, while that without child quality investment \( (x_t = 0) \) is termed *Malthusian*.\(^{18}\)

### 4.1 Modern regime

Such a regime is characterized by \( x_t > 0 \) and \( n_t < \eta \). It applies to a point in time at which the fertility transition is already under way, people are actively limiting births and investing in the quality of their surviving children. Setting \( \pi = 1 \) and \( \zeta_n = 0 = \zeta_x \) in the first order conditions (9)–(10), and after some rearrangement, we arrive at an equation that implicitly solves for the household's fertility choice,

\[
\Delta(n_t) \equiv \frac{\gamma (\theta - \sigma (1 - \theta))}{n_t} + \lambda \left( \frac{\eta - n_t}{\eta^2} \right) = \phi \psi \left[ \frac{\tau v_t + \delta - 1}{v_t - (\tau v_t + \delta - 1) \phi n_t} \right] \equiv \Omega(n_t), \tag{11}
\]

where \( \psi \) is as defined in Assumption 1(d), and the time-subscript on \( \phi \) has been dropped for the present. Parameter restrictions in Assumption 1 ensure an unique solution to (11) at a positive value of \( n_t \).\(^{19}\) At a first glance, the effects of child mortality and contraception effectiveness be-

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\(^{18}\)We call it so because, in the absence of child quality investment, child quantity is the sole focus of the household’s fertility calculus. In Section 6, we introduce exogenous TFP growth, \( g \). In that setting, what we term “Malthusian” corresponds more to the “post-Malthusian Regime” in Galor (2012).

\(^{19}\)Briefly, Assumptions 1(a) and (d) ensure that the right hand side of this equation is positive. The left hand side \( \Delta(n_t) \) is decreasing in \( n_t \) under Assumption 1(b) and the capacity constraint, while the right hand side \( \Omega(n_t) \) is increasing in \( n_t \) under Assumption 1(a) and (d). Moreover \( \lim_{n \to 0} \Delta(n) = \infty \) while \( \Omega(0) > 0 \). The interior fertility choice is given by the unique intersection point.
come apparent: if either $\phi \to \phi_{\min}$ or $\lambda \to \infty$, then $n \to \eta$. In other words, faced with either very low child survival probabilities or grossly ineffective methods of contraception, households behave as if fertility regulation is outside their control. In such regimes, fertility is left to nature.

How does child survival affect the household’s fertility behavior more generally? Under Assumption 1, when $\phi$ rises, $\Delta(n_t)$ is unaffected while $\Omega(n_t)$ rises. Hence $\partial n / \partial \phi < 0$ at an interior optimum and household fertility necessarily falls with $\phi$. This echoes the standard result in Beckerian models, for instance, Proposition 1 in Doepke (2005). The more relevant metric for population movement, however, is the number of surviving children. Define the latter as $z_t = \phi n_t$ and rewrite (11) as

$$\hat{\Delta}(z_t) = \frac{\gamma \left( \theta - \sigma (1 - \theta) \right)}{z_t} + \lambda \left( \frac{\hat{\eta} - z_t}{\hat{\eta}^2} \right) = \psi \left[ \frac{\tau v_t + \delta - 1}{v_t - (\tau v_t + \delta - 1) z_t} \right] = \hat{\Omega}(z_t)$$

(12)

where $\hat{\eta} = \phi \eta$. Clearly only $\hat{\Delta}(z)$ depends on $\phi$. Since $\hat{\Delta}(z)$ is decreasing in $z$, $\partial z / \partial \phi < 0$ whenever $\partial \hat{\Delta} / \partial \phi < 0$. The latter requires $n$ to be lower than $\eta / 2$ which obtains for income levels satisfying

$$v_t > \frac{(1 - \delta) \phi \eta (2 \psi + \varphi)}{\tau \phi \eta (2 \psi + \varphi) - 2 \varphi} \equiv \bar{v}(\phi)$$

(13)

where $\varphi \equiv 2\gamma[\theta - \sigma (1 - \theta)] + \lambda / 2$. The threshold, $\bar{v}(\phi)$, is decreasing in $\phi$. Thus, an increase in $\phi$ makes it more likely that $z$ falls with $\phi$: households previously at full-income $\bar{v}$ would now find themselves above it. Since we have established that $\partial n / \partial \phi < 0$ at an interior optimum, this means at low income levels ($v_t < \bar{v}$) fertility falls less than proportionately when more children survive.\(^{20}\) Proposition 1 summarizes this discussion and identifies several additional shifter s of household fertility.

**Proposition 1** In a Modern Regime – one with quality investment in children – household fertility, $n$, is decreasing in full income ($v$), child rearing costs ($\tau, \delta$) and the child survival rate ($\phi$), but increasing in the ineffectiveness of the contraception method ($\lambda$). The effect of the child survival probability ($\phi$) on the number of surviving children ($z$) depends on household full income: $z$ rises with $\phi$ for $v < \bar{v}(\phi)$ and falls for $v > \bar{v}(\phi)$ where $\bar{v}(\phi)$ is defined in (13).

To see the effect of full income, note only $\Omega(n_t)$ depends on $v_t$ and positively: since $\Omega$ is upward sloping, an increase in full income lowers the fertility choice. Similarly only $\Delta(n_t)$ in equation (11) depends on $\lambda$ and positively. As one would expect, less efficient methods lead to higher number of childbirths. Similar reasoning shows household fertility falls with $\delta$ and $\tau$.

Next, turn to the consumption-fertility tradeoff. From (7) and (8), it follows $s_t = \beta c_{1t}$ which converts (6) into $(1 + \beta)c_{1t} = v_t (1 - \tau \phi n_t) - \delta \phi n_t - \phi n_t x_t$. For interior $x$, using (10) in the last

\(^{20}\) $\bar{v}$ also depends on $\lambda$ (via $\phi$); indeed, $\bar{v}$ is increasing in $\lambda$ implying, given a $\phi$, the threshold $\bar{v}$ falls as better methods of contraception get adopted, making it easier for households to cross it.
equation, it is easy to check that \( \psi c_{1t} = v_t - (\tau v_t + \delta - 1)\phi n_t \) where \( \psi \equiv [1 + \beta + \gamma \sigma (1 - \theta)] \). Assumption 1(a) guarantees a tradeoff between \( c_{1t} \) and \( \phi n_t \); we'll have more to say on this below.

Finally, the child quantity-quality tradeoff. Given the optimum level of fertility from equation (11), child quality investment is given by

\[
x_t = \frac{\gamma \sigma (1 - \theta)}{1 + \beta + \gamma \sigma (1 - \theta)} \left[ \left( 1 - \tau z_t \right) z_t - \frac{1 + \beta}{\gamma \sigma (1 - \theta)} \right].
\] (14)

Clearly investment in child quality and surviving fertility move in opposite directions. In conjunction with Proposition 1, this implies reductions in child mortality lead to an increase in parental investment in child quality and richer parents invest more in their children than poorer parents. Also, note from Proposition 1 that \( z \) falls when \( \lambda \) falls which means \( x \) rises when \( \lambda \) falls. Investment in child quality rises when parents use more efficient methods of contraception since they are able to regulate fertility at lower cost. *Ceteris paribus*, the heightened focus on child quality becomes more of a rational imperative with either declines in child mortality or improvements in the contraception technology.

It is instructive to remind ourselves of the standard Beckerian calculus and present a quick contrast with features of the Modern Regime discussed here. Set \( \pi = 0 \) in (11) and solve for a closed-form for \( n \) and \( z \):

\[
\phi, n_t = \left[ \gamma (\theta - \sigma (1 - \theta)) v_t \right] / (\tau v_t + \delta - 1) \left( 1 + \beta + \theta \gamma \right)
\]

Here too \( n \) is falling in \( \phi \) as in the Modern Regime, while \( z \) is invariant to \( \phi \). Simple differentiation reveals \( \delta (\phi n) / \partial v < 0 \) implying both \( n \) and \( z \) fall with \( v \) if \( \delta < 1 \) (Assumption 1(c)). Using the just-derived expression for \( \phi, n_t \) and (10), we derive

\[
x = [\sigma \tau (1 - \theta) v + (\sigma \delta (1 - \theta) - \theta)] / (\theta - \sigma (1 - \theta))
\]

implying that \( x \) is independent of \( \phi \). It is also easy to check that \( \partial x / \partial v > 0 \). The implication is that as incomes rise, investment in child quality rises and fertility falls, the usual Beckerian quality-quantity trade-off. This means transition out of the Malthusian equilibrium in the conventional Becker model is independent of child survival and is purely driven by income growth.

To summarize, in the Modern Regime, households, having adopted deliberate fertility control, respond to a secular increase in child survival by reducing their fertility. However, only one group – households with income higher than a threshold \( \tilde{v} \) – see a decline in the number of surviving children and only these families make investments in the quality of their offspring. The threshold itself falls with \( \phi \) making it more likely for families to join this group over time. In short, inside a modern equilibrium, a persistent decline in child mortality can, in and of itself,
incentivize more and more families to have fewer but higher quality surviving children.

### 4.2 Malthusian regime

Such a regime is characterized by \( x_t = 0 \) and \( n_t < \eta \). It applies to the eve of the fertility transition when fertility is high, people are limiting births using less-effective traditional methods but still not investing much in the quality of their surviving children.\(^{21}\) From (14), it follows

\[
x_t \geq 0 \Leftrightarrow v_t \geq \frac{\phi n_t(v_t)}{1 - \tau \phi n_t(v_t)} \left[ \delta \frac{1}{\delta + \frac{1 + \beta}{\delta \sigma(1 - \theta)}} \right] \equiv f(v_t).
\]

(15)

So positive quality investment \( (x > 0) \) occurs whenever \( v_t < \hat{v} \), an increasing function of \( v_t \), is positive. This happens for \( v_t > \hat{v} \) where the income threshold solves

\[
\hat{v}_t = \frac{\phi n(\hat{v}_t)}{1 - \tau \phi n(\hat{v}_t)} \left[ \delta \frac{1}{\delta + \frac{1 + \beta}{\delta \sigma(1 - \theta)}} \right].
\]

(16)

In short, for \( v_t < \hat{v} \), child quality is absent (Malthusian regime) and when \( v_t \) crosses \( \hat{v} \), the economy enters the Modern regime. From Proposition 1 it follows that for \( v_t < \bar{v} \), the income threshold \( \hat{v} \) rises with \( \zeta \). For \( v_t > \bar{v} \), in contrast, \( \hat{v} \) falls.

Since a closed-form expression for \( \hat{v} \) is unavailable, \( \hat{v} \) cannot be directly compared with \( \bar{v} \) in (13) above. Suppose \( v_t < \hat{v}_t \) so that we are in a Malthusian regime. Substituting \( \zeta_n = 0 \) and \( \zeta_x > 0 \) into the first order conditions yields an equation for optimal fertility choice when the household does not invest in child quality:

\[
\bar{\Delta}(n_t) = \frac{\gamma \theta}{n_t} + \lambda \left( \frac{\eta - n_t}{n_t^2} \right) = \phi(1 + \beta) \left( \frac{\tau v_t + \delta}{v_t(1 - \tau \phi n_t) - \delta \phi n_t} \right) \equiv \bar{\Omega}(n_t).
\]

(17)

We establish Proposition 2.

**Proposition 2** In a Malthusian regime – which obtains when \( v_t < \hat{v} \) – household fertility falls with the child survival rate (\( \phi \)) but rises with full income (\( v \)). The number of surviving children rises with the child survival rate (\( \phi \)) for all income levels satisfying \( v_t < \hat{v} \) as long as \( \tau \phi \eta < 2 \) and \( 2\gamma \theta + \lambda / 2 < (1 + \beta)/(1 - \tau \phi \eta / 2) \).

In contrast, in the Modern regime discussed above, that is when \( v_t > \hat{v} \), the number of surviving children rises with the child survival rate but *only* for the sufficiently poor (\( \hat{v} < v_t < \bar{v} \)).

\(^{21}\)Long (2006) documents that only half of 6-14 year olds were in school in 1851 and that the average child would stay in school for roughly five years. School fees, ranging from 1-8 pence per week, were partly to blame for low school attendance. Opportunity cost in terms of lost wages from child labor was also important. Long argues that, after the 1870s, compulsory schooling laws “dramatically changed the cost benefit analysis”. While that is no doubt true, it is our contention that the socio-economic changes we have described also had a lot of influence on this cost-benefit analysis.
For the rich ($v < v_t > \tilde{v}$), the number of surviving children falls and investment in their quality rises.

Finally note that since our focus is on late nineteenth century England, on the eve of and during the fertility transition, we rely on Assumption 1(d), $\phi_i \in [\phi_{\min}, 1]$, to preclude the possibility that households are at their maximal reproductive capacity in the Malthusian equilibrium.

### 4.3 Fertility transition

A story of the fertility transition is emerging from within the model. Assume households have adopted some deliberate fertility control and contraceptive technology is what it is, the natural kind, and does not change. Now suppose a secular decline in child mortality is underway.

Suppose $\tilde{v} < \hat{v}$ where $\tilde{v}$ is defined in (13) and $\hat{v}$ in (16). Starting from $v_0 < \tilde{v} < \hat{v}$ and $n < \eta$, as $v$ increases, $n$ rises and so does $z$, with $x$ at zero. Households respond to a secular increase in child survival by reducing their fertility. When incomes are sufficiently low ($v < \tilde{v}$), improvements in child mortality raise surviving fertility across the board, even as parents do not invest in their children’s quality. As $v$ increases to above $\tilde{v}$, households start substituting child quantity for quality – $n$ starts to fall and $x$ rises. An increase in $\phi$ lowers both $n$ and $z$.

Suppose, instead, $\tilde{v} > \hat{v}$. Starting from $v_0 < \tilde{v}$, an increase in $v$ raises $n$ but there is no quality investment. As $v$ increases above $\tilde{v}$, quality investment turns positive. In this regime, an increase in $v$ lowers $n$ and raises $x$. An increase in $\phi$ also lowers $n$ but, again, raises $z$. Eventually as $v$ crosses $\tilde{v}$, an increase in $v$ lowers $n$ and raises $x$ (as before) but now an increase in $\phi$ also lowers $z$ (that is, the drop in $n$ is strong enough). Of course, this is all conditional on the same contraception technology. As $\phi$ or $v$ becomes large enough, the cost of averting $\eta - n$ births is high enough that some people start switching to modern contraception. As enough people do, there is a further sharp drop in $n$ and increase in $x$.

A reduction in $\lambda$, the effectiveness of contraception, amplifies the effect of income on fertility and child quality investment. This implies, for example, that historically income had a weak causal effect on the two and, hence, the fertility transition had to wait for the contraception revolution. In modern day developing societies where knowledge and supply of modern contraception is more prevalent, in contrast, income becomes the binding constraint.

### 4.4 Intertemporal Equilibrium

Turn now to the dynamic equilibrium of this economy. We reinstate the $i$ (household) and $j$ (contraception choice) subscripts and allow aggregate productivity $A$ to grow exogenously at
rate $g$. Assuming competitive factor markets, the equilibrium wage and rental rates are

$$w_t = (1 - \mu) A_t k_t^\mu \quad \text{and} \quad \rho_t = \mu A_t k_t^{\mu-1}$$

(18)

where $k_t \equiv K_t / H_t$. For full depreciation of physical capital, the interest factor is $R_t = \rho_t$. We define a Malthusian and a Modern equilibrium. In a Malthusian (Modern) equilibrium, every household $i$ finds itself in a Malthusian (Modern) regime. In addition, certain market-clearing conditions apply. Given the initial distribution of full income $G_0(v_0^i)$ defined over $[v_{\min}, \infty)$,

**Definition 1 (Malthusian equilibrium)** Every household $i$ is in a Malthusian equilibrium when $v_i^t < \hat{v}_t$ where $\hat{v}_t$ is defined by (16), $\{n_t^i\}_1^\infty$ satisfies (17), $x_i^t = 0$, and Assumption 1 is satisfied, for a given $j$. In this case, $K_t = \int s_{t-1}(v) \, dG_{t-1}(v)$ and $H_t = \int v \int \epsilon \, dG(v) \, dG(\epsilon)$ and (18) holds.

**Definition 2 (Modern equilibrium)** Every household $i$ is in a Modern equilibrium when $v_i^t > \hat{v}_t$ where $\hat{v}_t$ is defined by (16), $\{n_t^i\}_1^\infty$ satisfies (11), $x_i^t > 0$ and given by (14), and Assumption 1 is satisfied, for a given $j$. In this case, $K_t = \int s_{t-1}(v) \, dG_{t-1}(v)$ and $H_t = \int v \int \epsilon \, dG(v) \, dG(\epsilon)$ and (18) holds.

Steady-state fertility is given by a time-invariant long-run distribution of household-specific fertilities, $n_1^i$, characterized by (17) in a Malthusian equilibrium and by (11) in a Modern equilibrium. The total fertility rate ($TFR$) at date $t$ is given by the average fertility rate at that date: $\int n_t^i \, dG(v_t)$. The net fertility rate ($NFR$) at date $t$ is given by the average surviving fertility at that date: $\int \phi_t n_t^i \, dG(v_t)$.

Our definition of the two equilibria precludes the possibility that some households invest in child quality while others do not. That is, in any equilibrium, all households either have $x = 0$ or they all have $x > 0$. In our computational work later, we allow for a “mixed” equilibrium in light of the evidence discussed in footnote 21.\footnote{Another point deserves mention here. Some demographers, for example Szreter (1996) and Garrett et al. (2001), have argued against the idea of characterizing Victorian England as a single block and pointed to the existence of considerable heterogeneity across time, space, and social groups. Our analysis does not rule such heterogeneity out; in fact, we explicitly allow for heterogeneity and are among the few to do so. While the thrust of our analysis is on the experience of England as a whole, it is quite possible that some geographical regions of England or population groups in those regions were in one regime while others, elsewhere, were in another.}

## 5 Fertility Control and the Diffusion of Technologies

So far we have looked at contraception equilibria under three assumptions: the child survival rate is constant, households have access to only one kind of method and labor productivity
is constant. Together these imply the economy eventually gravitates to one of two types of stationary equilibria, high or low fertility.

The salience of these assumptions for the fertility transition will be clearer from Figure 3. Our specification of costly contraception implies non-homothetic preferences. In Figure 3(a), drawn in \((c_t, n_t)\) space after substituting in optimal decisions for quality investment and saving, the income expansion path (IEP) is non-monotonic. The kink in the IEP occurs at a full-income income level where parents are indifferent about whether or not to invest in child quality. Below this income level, increases in income increase family size. Above, the effect of income is negative as richer households substitute towards child quality. Figure 3(b) shows how the IEP responds to a change in the child survival rate \(\phi\) while Figure 3(c) illustrates the response to a change in contraceptive effectiveness \(\lambda\). A lower value of \(\lambda\), by making it easier to implement a given fertility plan, has an effect similar to a higher \(\phi\) on the IEP. The budget lines have been drawn to give an idea how tradeoffs change with income. For a lower value \(\lambda\), the kink occurs at a lower income level (Figure 3(d)): parents start substituting away from child quantity at a lower income level when it is easier for them to afford the cost of contraception.

For more effective contraception methods (lower \(\lambda\)), a given increase in \(e\) reduces child-births by more which, in turn, enables parents to direct resources towards child quality. For a given contraception method, a higher survival rate (higher \(\phi\)) raises the net marginal benefit of fertility control. The benefit from surviving children does not change but the cost of “excessive” fertility rises since parents may end up with too many survivors. These figures illustrate the demand for children falls when household full income or child survival improve sufficiently. The negative relationship between fertility and income or child survival becomes more pronounced at higher income levels and more effective methods of contraception ease the ability of households to achieve their fertility targets (Figure 3(d), dotted line corresponds to a more efficient technology).

As per the historical evidence discussed in Section 2, the transition to a modern fertility regime is accompanied by marital fertility control through the active use of birth control strategies. Obviously households switch to a more efficient and possibly costlier contraception strategy only if perceived returns from it are high. Comparative statics results discussed above point to two potential causes for such a transition, improvements in income and child survival. The transition would be facilitated by households switching towards more effective methods of contraception. Moreover, our model predicts richer households will switch towards modern contraception first, and that they will have smaller families during and after the transition (reverse in pre-transition Malthusian equilibrium). The remainder of this section formalizes this social change as a switch from one (less effective) technology to another (more effective) and the diffusion of that choice through the population over time.
Initially, after child mortality falls or income rises sufficiently, richer households find it advantageous to switch to modern methods of contraception despite high social and informational costs. Even if this decision were to have no social impact, a steady decrease in child mortality alone would make this switch more and more desirable for households down the socioeconomic ladder. Changes in social attitudes accentuate this process in line with the historical evidence. Specifically, the abatement of stigma associated with “unconventional” fertility regulation and the diffusion of knowledge about modern methods of contraception made it advantageous for households from lower socio-economic status to practice fertility regulation sooner than they would have.

Recall the first of the two contraception technologies, \( j \in \{1, 2\} \), is identified as the traditional one, the second as modern. Specifically we assume \( 0 < \lambda_2 < \lambda_1 \) and the fixed costs of adoption satisfy \( 0 = \alpha_{1t} < \alpha_{2t} = \alpha_t \) initially, before technology 2 is being used. Denote household \( i \)'s indirect utility for the choice of the traditional technology 1 as \( U_{i1}^t \) and utility for the modern technology 2 as \( U_{i2}^t \).

While the effectiveness with which contraceptives can avert childbirth underlies the parameter \( \lambda \), we introduce two types of fixed costs to adopting the more effective modern technology. The overall cost \( \alpha_t \) comprises of an information cost \( \alpha_t^I \) and a social cost \( \alpha_t^S \). Both are endogenous and depend on network externalities. Part of the cost of switching to modern contraception in traditional societies is obviously the lack of reliable information about how to regulate fertility and how to avail of the necessary inputs. The availability of such information as well as supply of these inputs depend on how extensively the modern technology is used. Hence we posit that the information cost depends on an inter-temporal network externality non-linearly, \( \alpha_t^I = a_0(1 - p_{t-1})^\alpha_1, a_0, \alpha_1 > 0 \), where \( p_{t-1} \in [0, 1] \) is the proportion of the prior generation who used modern contraception. The more widely used it was, the more accessible the technology is now. The non-linearity will prove useful later in matching the slow initial update of modern methods during the transition.

The second element of the switching cost comes from social behavior. As we discussed in Section 2, conscious fertility control in a traditional society is fraught with social repercussions. We assume someone deviating from the traditional norm attracts social censure. The more prevalent the use of the modern contraception technology is, the less will its adoption be viewed as radical. Suppose once a household chooses either the traditional or the modern contraception technology, the choice is irreversible. In making that choice, households take into account the social, medical and religious disapproval they may encounter from being more progressive. More concretely, suppose each household randomly socializes with another member of his cohort during his youth. The household’s social payoff from this interaction is determined by its and its social partner’s reproductive choices. This social payoff is \( \zeta \geq 0 \), when
both households have adopted the modern technology. Similar to Munshi and Myaux (2006), ξ denotes the intrinsic utility the household receives from the modern contraception technology. Should the social partner be a traditionalist, on the other hand, a modern household receives the payoff \(ξ - ζ\) where \(ζ > 0\) is the social disapproval it attracts for straying away from “traditional principles”. The household’s payoff in the other two cases is normalized to zero.

Household \(i\) anticipates a \(p_t\) proportion of people adopt the modern technology and forms expectations of her net utility from the two technologies. If \(i\) sticks with the traditional technology, this utility is \(V_{it}^{i1} = U_{it}^{i1}\). If \(i\) switches it is \(V_{it}^{i2} = U_{it}^{i2} - \alpha_t\) where

\[
α_t = α_t^l + α_t^s = a_0(1 - p_{t-1})^α_1 + [(1 - p_t)ξ - ζ].
\]

Household \(i\) will switch to the modern technology at \(t\) as long as \(ΔU_t^i \equiv U_{it}^{i2} - U_{it}^{i1} \geq α_t\). Appendix B shows that \(ΔU_t^i\) is increasing in \(v_t^i\). This means, given the expected \(p_t\) and historical \(p_{t-1}\) there is a threshold income level \(v^*_t\) such that individuals above this threshold are better-off switching to the modern technology. Higher the (expected) adoption rate of this technology at \(t\) and higher the past adoption rate, lower is the threshold income level. Given the distribution of full income, a perfect foresight period-\(t\) equilibrium requires

\[
p_t = 1 - G_t \left( v^*_t (p_t) \right).
\]

We incorporate this social dynamics into the fertility choice model of Sections 3–5 and proceed to study their role in the fertility transition.

### 6 Numerical Experiments

In our theory of the Victorian demographic transition, a steep mortality transition and/or income growth due to productivity improvement trigger a contraception revolution, which in turn, ushers in the long transition towards near-replacement fertility. Our model is designed, first and foremost, to offer qualitative insight. But there is value in seeing how well it does on the quantitative margin. The goal is not a full-blown calibration exercise, challenging as it is to quantify preference-based social change, but rather to paint a picture of the English transition with broad brushstrokes to see if social change and changing cost of contraception could have made a difference to the transition.

We start by assigning parameter values so that, on certain margins, the model economy resembles key aspects of England before and after its fertility transition. Before the transition, all households are assumed to use traditional methods of contraception; afterward, they all use modern ones – recall, that includes the withdrawal method. To better understand the time
frame, consider Table 1. Child mortality (probability of death in the first five years of life) shows a discernible downward trend after 1870 as does the total fertility rate. Consequently, we treat 1870 as the onset of the English mortality and fertility transitions.\(^{23}\)

<table>
<thead>
<tr>
<th>Period</th>
<th>Child Mortality</th>
<th>TFR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1845-70</td>
<td>0.28</td>
<td>4.67  (1861)</td>
</tr>
<tr>
<td>1870-95</td>
<td>0.22</td>
<td>4.04  (1891)</td>
</tr>
<tr>
<td>1895-1920</td>
<td>0.15</td>
<td>2.42  (1916)</td>
</tr>
<tr>
<td>1920-1945</td>
<td>0.06</td>
<td>2.04</td>
</tr>
<tr>
<td>1945-70</td>
<td>0.03</td>
<td>2.40</td>
</tr>
</tbody>
</table>

Table 1: Child Mortality and Total Fertility

It is important to note the pre-transition economy we simulate here is not the Malthusian equilibrium of the theoretical model. In the Malthusian equilibrium, household income and fertility rates are positively correlated. English society in the seventeenth and eighteenth centuries exhibited this positive association but, by the early nineteenth century, the rich were having fewer children (Clark, 2005, 2007). Moreover, literacy and school enrollment rates were steadily rising during the nineteenth century (Long, 2006). In the model, if child mortality is not changing, the only way to obtain a post-Malthusian pre-transition society is from income growth. This is where TFP growth becomes important. To resemble the pre-transition economy, we pick parameter values so as to match aggregate child education data in 1870 as well as ensure, for households investing in child quality (the relatively affluent), fertility is negatively related to household full income.

Secondly, in the data, the TFR drops below replacement during the 1930s (effect of World War I) and rises above it soon after World War II (baby boom). Since our model is not tailored to study these temporary shifts, we treat 1945 as the end of the transition; as such, the post-transition TFR is set to replacement, 1.0 in model terms.

### 6.1 Parameter Values

Table 2 lists the parameter values. We assume the modern contraception technology is highly efficient ($\lambda_2 = 0$) – it can costlessly implement a household's fertility target when everyone uses

it – as a simple normalization. One can be more general by calibrating \( \lambda_2 \) to replicate post-transition fertility or some empirical measure of modern contraceptives. But our normalization has the advantage of mimicking the Beckerian model where contraception costs are implicitly set to zero. Moreover, recall that if \( \lambda = 0 \), the expected number of surviving children is independent of \( \phi \). This means, by design, the modern technology alone cannot generate a fertility transition from the child mortality transition.

<table>
<thead>
<tr>
<th>Preference</th>
<th>Fertility</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate ( \beta = 0.37 )</td>
<td>Natural fertility ( \eta = 3.585 )</td>
<td>Schooling prod. ( a = 0.445 )</td>
</tr>
<tr>
<td>Altruism factor ( \gamma = 0.45 )</td>
<td>Time cost ( \tau = 0.09 )</td>
<td>Schooling return ( \sigma = 0.47 )</td>
</tr>
<tr>
<td>Weight on child quantity ( \theta = 0.55 )</td>
<td>Goods cost ( \delta = 0.001 )</td>
<td>Ability ( \theta = 1.35 )</td>
</tr>
<tr>
<td>Survival probability ( \phi = 0.72 )</td>
<td>Efficiency trad. ( \lambda_1 = 1.685 )</td>
<td>Capital's share ( \mu = 0.23 )</td>
</tr>
<tr>
<td></td>
<td>Efficiency modern ( \lambda_2 = 0 )</td>
<td>Initial TFP ( A_0 = 141 )</td>
</tr>
<tr>
<td></td>
<td>Social cost ( \zeta = 0.0099 )</td>
<td>TFP growth rate ( g = 0.0315 )</td>
</tr>
<tr>
<td></td>
<td>Social cost ( \zeta = -0.0356 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fixed cost trad. ( a_0 = 0.0241 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fixed cost modern ( a_1 = 2.28 )</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Parameter Values

Each period of adulthood is taken to be 25 years, childhood to be 15 years. The subjective discount rate \( \beta \) follows from the standard quarterly rate of 0.99 compounded over 25 years. \( \theta \), the weight on child quantity has to exceed 0.5 to satisfy Assumption 1(b); it is set to 0.55. The intensity of warm-glow altruism, \( \gamma \), is picked to match post-transition TFR. Specifically, for \( \lambda = 0 \), equation (11) simplifies to

\[
\phi n_t = \frac{\gamma(\theta - \sigma(1-\theta))}{1 + \beta + \gamma\theta} \left[ \frac{1}{\tau - (1-\delta)/v_t} \right],
\]

where household fertility choice is decreasing in full income, \( v_t \). As \( \phi \rightarrow 1 \) and \( v_t \rightarrow \infty \) (when \( g > 0 \)), household fertility falls to

\[
\frac{1}{\tau} \left( \frac{\gamma(\theta - \sigma(1-\theta))}{1 + \beta + \gamma\theta} \right)
\]

which is independent of household income. Given other parameter values, \( \gamma \) is chosen so that this TFR is at replacement. The pre-transition TFR, on the other hand, is used to pin down the effectiveness of the traditional method, \( \lambda_1 \). Finally the child survival rate is calibrated based on the child mortality rate data in Table 1: 0.28 prior to the transition, zero after the transition is complete.
The natural fecundity $\eta$ depends on biological capacity as well as social constructs such as age at marriage and celibacy. Demographers often rely on data for the Hutterites who were perceived to practice little fertility regulation and had 10 children on average. Woods (2000) argues against blindly adopting this number for English natural fecundity, estimating it to be 7.17 instead. This is the value (3.585 per model agent) used in our simulations. Haveman and Wolf (1995) report the opportunity cost of childrearing to be 15% of parental time. Scaled to the 15 years of childhood, this gives $\tau = 0.09$. The resource cost of children, $\delta$, is a scaling parameter and its value is set as per Doepke (2004). The contraception cost parameters ($\xi, \zeta, \alpha_0, \alpha_1$) pertain to the transition, that is, 1870-1945. We discuss how they are calibrated later.

The ability distribution is parameterized to be Pareto, specifically,

$$g(\varepsilon) = \theta \frac{\varepsilon^{\theta}}{\varepsilon^{\theta+1}}, \quad \theta > 1.$$ 

Normalizing mean ability to unity implies $\xi = (\theta - 1)/\theta$. Since reliable data on lifetime income inequality in nineteenth century England is not available, we cannot directly calibrate this dispersion parameter. In any case our goal is not to explain inequality, only to broadly capture the cross-sectional correlation between income and fertility behavior. Hence we use $\theta, \sigma$ (elasticity of earnings with respect to education) and $\alpha$ (productivity of quality investment) to generate plausible quality investment in children; specifically, at least 60% of children receive education in the pre-transition economy. Lindert (2004) reports 60.9% of children of ages 5-14 were in private and public schools in England & Wales in 1870 (Table 5.1) while, according to Long (2006), 45% of 5 years olds and 60% of 6 year olds were in school in 1851. In practice this target schooling rate is matched mainly by adjusting the parameters $\theta$ and $\sigma$ with $\alpha$ serving as a scaling parameter. The implied $\sigma$ is modest, consistent with values used in the literature.

The value for $\mu$ comes from Clark’s (2007, Table 10.1) estimate of capital’s share in British income during 1960-2000. For the growth rate of TFP, $g$, we rely on Maddison’s (2001, Table A1-c) estimates of British GDP per capita during 1820-1950. The average annual growth rate of per capita income during this period was 1.08% per year. In other words, income per capita increased by a factor of 4 over 1820-1950. We calibrate $g$ to be 3.15% per generation so that we can match this increase in income per capita over 1820-1945. Initial productivity $A_0$ is picked to match the level of pre-transition GDP per capita reported in Maddison (2001).

Now turn to the contraception cost parameters that govern the fertility transition during 1870-1945. Two types of evidence are available for this, neither perfect. The first, direct evidence, comes from surveys on family limitation. The eminent source is the Lewis-Faning (1949) survey of English women which finds that 15% of women married before 1910 used any form of birth control at any time in their married life, 58% of those married during 1920-24 did, while
55% of those married in 1940-47 did. The last number is certain to be biased downwards: the survey was completed in 1946-47 when the 1940-47 marriage cohort would not have completed its target fertility (parity) and, hence, unlikely to have fully adopted contraception. Based on contraception practice rates of the previous cohorts, Lewis-Fanning estimates this to be around 70%. Even that number is biased downwards since contraception was steadily becoming popular with successive cohorts.\footnote{There are other problems. Besides the small sample size – 3281 respondents from different marriage cohorts – reporting bias is to be expected: women would have been reticent to talk about such matters to surveyors or they might not have known much since birth control was largely a male decision before the advent of the pill. Lewis-Faning also reports contraception rates were correlated with socio-economic status: more women from wealthier occupations were relying on it.}

The second type of evidence, indirect, uses an accounting procedure to arrive at an index of marital fertility regulation. Based on Coale and Trussel’s work from the Princeton Fertility Project, this method compares observed marital fertility rates at each age group to their “natural rates” and estimates what the degree (index) of voluntary fertility control across all age groups must have been. Woods (2000, Table 4.2) calculates an England-specific natural fertility schedule and estimates England’s index of family limitation to have gone from 0.013 (1871) to 0.171 (1891) to 0.842 (1922) to 1.036 (1933). In the model, there is a close link between the fertility ratio \( n_i^f/n_i \), number of births to married women relative to the total births to those women if they were subject to maximum fertility – same as the variable \( I_g \) in the Coale-Trussel method – and contraception strategy \( \lambda \): a reduction in the latter through the use of modern contraception reduces the former. That is, the proportion of households regulating fertility using the modern method in the model directly translates into an aggregate-level index of family limitation.

The Coale-Trussel evidence, as extended by Woods (2000), does not face the response problems of the Lewis-Fanning survey. It does, however, have one drawback for us: no distinction is made between households who were starting to adopt methods such as coitus interruptus and those that were moving on to appliance-based methods. To the extent that adoption of coitus interruptus is associated with a better understanding of the process of procreation, its very usage, arguably, made it modern. More importantly, appliances were rare in the late nineteenth century: some, such as condoms and spermicides, had just started being mass produced but were not widely available for ordinary people, others like diaphragms were popularized later – the Lewis-Faning (1949) survey found 31% of married women had ever used a non-appliance method by 1940-47. With these caveats in mind, we proceed to use Woods’ estimates to impute that the proportion of households practicing modern contraception went from 0% (1871) to 17.1% (1891) to 84.3% (1922) to 100% by 1933. Since the English fertility transition starts around 1870, these imply modern contraception – again, this includes the withdrawal method – usage of 17.1% in the first period of transition (by 1895), 84.3% in the second period (by 1920) and
100% in the third period (by 1945).\footnote{A prior version of this paper used the Lewis-Fanning estimates, assuming the actual practice of contraception by 1945 was close to 100%. Key results were similar to what we present here, including the relative importance of child mortality over income growth in explaining the fertility transition.}

To replicate the mortality and fertility transitions in Table 1, we “shock” the model by the mortality transition period $T$ (1870-95) onwards and trace the model’s prediction about the TFR period $T$ onwards. For this fertility transition we need to calibrate the information cost $(\alpha_0, \alpha_1)$ and social cost parameters $(\zeta, \xi)$. We choose these four parameters to best fit four pieces of evidence: the switch towards modern contraception does not start until period $T$ and the adoption rates are 17%, 84.3% and 100% in periods $T$, $T+1$ and $T+2$. Since each generation gets a fresh draw from the ability distribution, we choose these parameter values so that mean adoption rates (across 20 rounds of simulation) match their empirical counterparts.

### 6.2 Simulation Results

Start with the upper panel of Figure 4 that reports the proportion of households adopting the modern contraception method over time. The solid line corresponds to the model’s prediction, the dotted to actual data. The two are very closely aligned, not surprising, given the calibration strategy. The grey band in this and subsequent figures represents a “95% confidence interval”, computed as the sample mean $\pm 1.96 \times$ the sample standard deviation across 20 rounds of simulation. It is shown only to illustrate the model is not overtly sensitive to the exact ability draws. The lower panel of Figure 4 reports the relative importance of information costs, that is, $\frac{\alpha_t^I}{(\alpha_t^S + \alpha_t^I)}$, during the transition. It remains steady and suggests, given the calibration strategy, social costs were slightly more important in accounting for the shift towards modern methods of contraception.

### Fertility Transition

How well does the model’s predicted fertility transition match the evidence? This is reported in Figure 5 for the transition period of $T+1$ to $T+3$ (1870 – 1945): the solid black line is the predicted path of the TFR, the dashed red line is the data. The predicted path is quite close to the data though the fit is less than perfect in the third period of the transition when predicted TFR is 25% higher than replacement and 29% higher than actual.

Recall, however, that the calibration matched replacement TFR when everyone switches to modern contraception and household income goes to infinity. So a better assessment of the model’s predictive power is to compare the simulated TFR during the first two periods of the transition when the calibration targets only contraception adoption rate, not the fertility rates. By this yardstick, the model does quite well: predicted TFR is 6% above actual by the
end of $T+1$ and 33% above actual by the end of $T+2$. This becomes clearer when we consider counterfactuals below.

Two other points about Figure 5 should be noted. First, the transition to replacement does not complete until another two generations in the model. We set the post-1945 TFR in England and Wales, somewhat artificially, to replacement; in reality, replacement was not reached until 1970 (that is, corresponding to period $T+5$ in the model). Moreover, the post-transition model was calibrated under the assumption household full income $v \to \infty$. As such, that the simulated transition takes longer to converge to replacement than in the data is not a serious shortcoming. While the model does well in capturing how the fertility transition unfolded, it is not tailored to explain why English fertility fell below replacement after WWI and then overshot replacement in the decades following WWII (Figure 1).

Note also, by design, the model is calibrated to reproduce the observed TFR in period $T$. In the data, TFR was steadily falling since the early nineteenth century. While the model does predict a steady decline – solely due to income growth – it does not predict as big of a decline as we see in the data. Demographers believe the early nineteenth century decline was driven mainly by the extensive margin which is missing from our model. In addition, the pre- and post-1841 TFR data come from two different sources – see Figure 1 – and the year for which they overlap (1841) shows a large discrepancy between the two sources. Both data sets point towards some decline in the early nineteenth century. The more pronounced decline, though, occurs after 1870.

Hence, on the basis of Figure 4, we conclude that incorporating the costs of birth control during the transition substantially improves the Beckerian model’s explanatory power and the ability of the mortality transition, in particular, to generate a TFR transition.

The role of income growth and contraception

Income too has an effect on fertility in the model: as England became steadily prosperous over the course of the nineteenth and twentieth centuries, this would have lowered the TFR independently of the mortality transition.\footnote{This prediction about income growth is at variance with conventional thinking among demographers, but related to Galor’s work where technological progress improves the valuation of education, causing households to shift from child quantity to quality.} How relevant was this margin for the English transition? Did the contraception revolution play a pivotal role?

Consider two counterfactual scenarios. To isolate the relevance of income growth, we maintain $\phi$ at its 1870 value of 0.72 and simulate the time path of the TFR, allowing contraception adoption to change from the Coale-Trussel-Woods estimates. A second counterfactual asks whether the switch to more efficient methods of contraception and their falling costs account
for much of the transition. For this we set \( \alpha_0 \) sufficiently high so that no household switches to the modern method during the periods \( T + 1 \) to \( T + 3 \). These scenarios are presented in Figure 6. Since we do not recalibrate the model in either scenario to match the observed adoption of modern contraception, for comparability we present a modified version of the benchmark case where mortality alone changes exogenously while TFP growth is held to zero from 1870 onwards (that is, whatever income growth occurs comes from the endogenous accumulation of human and physical capital).

The solid orange line in Figure 6 (solid black and dashed red lines represent the same series as in Figure 5) presents this modified benchmark case – evidently the lack of TFP growth does not drastically alter the model’s predictive power. The blue dotted line represents the path of the TFR when child mortality does not change, while the green dash-dotted line corresponds to the no-switch scenario. In these two alternative scenarios, even five generations after the fertility transition started, the TFR remains higher than 3. By the third period of the transition, the predicted TFR is more than twice the replacement level and 70% higher than the baseline prediction. By period \( T + 1 \), predicted TFR is 14% above actual, by \( T + 2 \) it is 88% above actual. In other words, while income growth has a significant impact on fertility behavior, its impact on the English fertility transition seems less so if the only channel is exogenous productivity growth.

It is not surprising when switching costs are too high (dash-dotted line), the TFR decline is also relatively small compared to what we observe during the historical transition. Here, however, the model predicts a faster TFR transition relative to the income-only scenario, indicating again the relative importance of the child mortality transition. Fertility remains 74% (38%) higher than replacement (baseline prediction) by \( T + 3 \) in this case. By period \( T + 1 \), predicted TFR is 10% above actual, by \( T + 2 \) it is 71% above actual.

**Net fertility**

Finally turn to Figure 7 which shows predicted and actual net fertility during 1841-1945. A central prediction of our model is that a decline in child mortality also lowers the NFR. The dashed line at the bottom represents the data, the NRR2 series reported in Figure 1. The solid black line represents the predicted path of NFR from the model, computed as \( \phi n \) assuming boys and girls faced the same survival rate which is consistent with the evidence (Hinde, 2003, Table 12.1). Since the model ignores mortality for women during their reproductive years, we overestimate net fertility. But the general trend predicted for 1845–1945 closely follows the data. This is not so for the two counterfactual scenarios. For the constant mortality scenario (blue dotted line), we have seen from Figure 6 that productivity-based income growth alone does not lower the TFR by much and we see here the predicted NFR barely moves. For the scenario where households
do not switch to modern contraception (green dash-dotted line), the predicted NFR behaves similarly and even shows a slight increase: evidently mortality change and income growth together without the accompanying social change are not enough to substantially lower the TFR to compensate for the higher child survival rate.

We recognize there is uncertainty and debate regarding social change and contraception behavior in late-eighteenth and early nineteenth century England. With that in mind, we view these simulation results as strongly suggestive: contraception costs and the social change that accompanied the shift towards fertility regulation were likely to have been important elements of the English fertility transition. Contraception alone could not have triggered the transition. In our broad brushstrokes view, it was the catalyst, and a powerful one at that, in the change unleashed by underlying changes to mortality and income.

It is also worth reiterating that these simulation results should not be taken as efforts to refute alternative theories, for example, the effect of income through the gender wage gap and occupational choice (Galor and Weil, 1996, Mookherjee et al., 2012) or the effect of child mortality under survival uncertainty (Kalemli-Ozcan, 2008). Neither is the success of the mortality or contraception channels in the English context indicative of their relative importance in other transitions. It does show, however, that in many cases ignoring large and changing costs of contraception can lead us to under-appreciate the proximate causes of the fertility transition. More pointedly, since mortality affects net fertility only in the presence of contraception in the model, one may erroneously discount the importance of the mortality transition if one ignores the associated contraception calculus.

7 Implications for Recent Transitions

Demographic transitions are underway even to this day with almost every nation witnessing deep declines in child mortality and marital fertility. World population growth has declined rapidly since the 1960s; the world TFR has fallen from 5.0 births per woman during her lifetime in the early 1950s to 2.5 births in 2010, a remarkable decline of 50%. Women in developing countries – sub-Saharan Africa is an exception – transitioned from having around six births during their lifetime to having closer to two births. Does our work have anything to say about these modern transitions?

In a recent study using Demographic and Health Surveys (DHS) from 65 countries in different years between 1988 and 2005, Canning et al (2013) find the following. In high-fertility countries, those in the early stages of the transition, desired fertility may exceed actual fertility, so that broader socioeconomic changes that affect desired fertility have little impact on observed
fertility. Interestingly, they also find that effects of social learning about the fertility calculus of others, and social spillovers more generally, is strongest in the early stages. To the extent early stages of modern transitions look like their counterparts in historical transitions, the Canning et al (2013) study lends credence to the timelessness of the main mechanisms underlying fertility change in our model.

And what of the importance of the contraception revolution in modern transitions? Surely, access to and information regarding effective means of birth control are not stumbling blocks in the lives of modern couples? Researchers have documented the widespread, secular increase in the use of contraception, due in part to vastly improved access to contraceptives. Economic demographers, however, remain skeptical of their relevance for modern fertility transitions notwithstanding Richard Easterlin’s earlier work. Their skepticism has been reinforced by Pritchett’s (1994) influential study that 90% of cross-country differences in the level of the TFR is explained by differences in women’s reported desired fertility alone. In other words, the means to achieve that desired fertility, contraception for instance, plays a minor role. This conclusion has been recently questioned by Lam (2011) in his presidential address to the annual meeting of the Population Association of America. Lam uses data from 185 DHS surveys for 74 countries, a considerably larger set of surveys than Pritchett used. He looks at changes in desired and actual TFR. While desired fertility fell by an average of 0.038 births per year, the TFR fell by 0.060 births: 47% of the average decline in the TFR came from better ways to achieve fertility targets. Our theory can rationalize these seemingly contradictory findings. Conditional on a contraception method, all changes in the TFR come from changes in desired fertility (due to income growth and better child survival). Over time, changes in the availability and adoption of alternative, more efficient, methods account for an increasingly significant part of TFR declines. The onset of the fertility decline, though, stems from factors other than the availability of contraceptives.

A closer look at country-specific evidence uncovers interesting differences. The arrival of female family planning programs caused an outcry in Bangladesh while in India and Kenya the resistance was more internal – fear of side effects, social acceptance and disapproving husbands. Not only was there little public outcry in Thailand and Taiwan, in contrast, women were enthusiastic adopters of modern contraceptives, in some cases traveling far to avail of them (Cleland, 2001). Why contraception is so controversial in some countries but not in others is itself a fascinating question. Studies connecting family planning to completed fertility, likewise, provide mixed evidence. Miller’s (2009) study of Columbia’s well-established family planning program 27Desired fertility counts only births of women who say they want another birth, with an adjustment for incomplete childbearing. It is not based on ex post statements about children being wanted. Note also that in our model, as in the standard Becker model, all fertility is desired.
concludes that its contribution to fertility decline was minor though it did encourage education and labor force participation among young women. Cochrane and Guilkey (1995), on the other hand, show that access to family planning methods was a significant cause of Tunisia’s fertility decline. Ethiopia’s family planning programs have been credited with reducing completed fertility by one child among uneducated women (Pörtner et al., 2011) while Bailey’s (2012) work shows that, even in the post-transition US, the expansion of family planning programs may have lowered childbearing among poor women by as much as 29%.

An advantage of our modified Becker model is that it provides a common analytical framework to understand fertility change across societies. Whether or not and how much contraception accounts for fertility change may be entirely country-specific. And the proximate determinants of that change may also differ from region to region. It bears repeating, however, that in our theory contraception by itself cannot do much – factors that lower the demand for children are what initiate the change that then, potentially, benefits from the knowledge and availability of contraception.

8 Conclusion

In much of Europe, dramatic declines in both mortality and fertility started in the late nineteenth century, a phenomenon that came to be known as the demographic transition. In most instances, mortality decline preceded fertility decline. We argue, the former incentivized the latter. In our theoretical model, as child mortality declines, the unfettered procreation process produces “too many” surviving children. At that point, families start to adopt modern contraception as a way to economize on costly children and free up resources for their upkeep and quality. Richer households are the first to regulate marital fertility and, in doing so, kindle a process of indirect diffusion that gradually lowers fertility across socio-economic groups. In our story, mortality decline is the principal instigator that necessitates a culture of conscious contraception which, in turn, becomes the facilitator of fertility reductions.

But that is only part of the story. The latter part of the nineteenth century is also a period of rapid social, economic and cultural change brought on, in part, by impressive increases in living standards and improvements in nutrition and health. Surely, these too shaped fertility demand and acted as important catalysts of reproductive change. Our theory is not unreceptive to these alternative explanations; it allows for the possibility that rapid income growth, even in the absence of precipitous mortality declines or massive uptake of modern contraception may bring about a fertility transition.

Since our aim has been to single out social influence for fertility choice and its scope in the historical English transition, this work should not be taken as establishing the salience of mor-
tality over income for that transition. Whether or not mortality was *the* determinant of English fertility change remains an open, quantitative, issue. To answer that one needs to create more space for income growth to matter; Jones and Schoonbroodt (2010), for example, offer one avenue that relies on relatively substitutable child quality and quantity in household preferences. We leave it to future work such a thorough exploration and more directed applications of our theory to twentieth and twenty-first century transitions.
Appendix

A  Second Order Condition

Since the budget constraint can become non-convex with quality and quantity, we check second order conditions. To that end, use the optimality conditions for $s$ and $x$ to express utility as a function of $n$ alone (ignoring non-relevant terms):

$$V(n_t) \equiv [1 + \beta + \gamma(1-\theta)] \ln \left[ \nu_t - (\tau \nu_t + \delta - 1)\phi n_t \right] + \gamma(2\theta - 1) \ln n_t - \frac{\lambda}{2} \left( \frac{\eta - n_t}{\eta} \right)^2.$$  

Optimal $n$ solves

$$V'(n_t) = -[1 + \beta + \gamma(1-\theta)] \frac{(\tau \nu_t + \delta - 1)\phi}{\nu_t - (\tau \nu_t + \delta - 1)\phi n_t} + \frac{\gamma(2\theta - 1)}{n_t} + \frac{\lambda}{\eta} \left( \frac{\eta - n_t}{\eta} \right) = 0.$$  

It follows that

$$V''(n_t) = -[1 + \beta + \gamma(1-\theta)] \frac{((\tau \nu_t + \delta - 1)\phi)^2}{(\nu_t - (\tau \nu_t + \delta - 1)\phi n_t)^2} - \frac{\gamma(2\theta - 1)}{(n_t)^2} - \frac{\lambda}{\eta} < 0 \text{ if } \theta > 1/2.$$  

B  $\Delta U$ is increasing in $\nu$

For a given fertility choice, $n$, suppose optimal consumption and child quality decisions are $c_1(n), c_2(n)$ and $x(n)$. Doing so and substituting these into the utility function allows us to express the household's objective function in terms of $n$ alone. Obviously, given a $j$ technology, $\partial U/\partial n = 0$ at the optimum. We have already established that $\partial n/\partial \nu < 0$ whenever child quality investment is positive, which is the case we consider in Section 7. Now we need to show $\partial \Delta U/\partial \nu > 0$. Since $0 \leq \lambda_2 < \lambda_1$, it is sufficient to prove as $\lambda$ rises, $U(\nu)$ becomes flatter. That is, we need to show $\partial^2 U/\partial \lambda \partial \nu = \partial^2 U/\partial \nu \partial \lambda < 0$ since $U$ is continuous in $\lambda$ and $\nu$.

From the indirect utility function,

$$\frac{\partial U}{\partial \lambda} = -\left( \frac{\eta - n}{\eta} \right)^2 + \frac{\partial U}{\partial n} \frac{\partial n}{\partial \lambda} = -\left( \frac{\eta - n}{\eta} \right)^2 < 0$$

since, at the optimum, $\partial U/\partial n = 0$. Hence

$$\frac{\partial}{\partial \nu} \left( \frac{\partial U}{\partial \lambda} \right) = \frac{\partial}{\partial \nu} \left[ -\left( \frac{\eta - n}{\eta} \right)^2 \right] = \frac{2}{\eta} \left( \frac{\eta - n}{\eta} \right) \frac{\partial n}{\partial \nu}$$

is negative since $n \leq \eta$ while $\partial n/\partial \nu < 0$. 

References


3. Becker, Gary (1960), “An Economic Analysis of Fertility”, in Demographic and Economic Change in Developed Countries, NBER.


52. Seccombe, Wally (1990), “Starting to Stop: Working-Class Fertility Decline in Britain”, *Past & Present*, No. 126 (Feb), 151-188.


Figure 1 Mortality and Fertility in England & Wales, 1770-1970

Source:


CMR from [www.mortality.org](http://www.mortality.org), probability of death between ages 0-5 in England & Wales.
Figure 2 Contraception Technologies: Modern (gray) & Traditional (black)
(a) Income Expansion Path

(b) Effect of higher $\phi$ on the IEP

(c) Effect of lower $\lambda$ on the IEP

(d) Effectiveness of birth control ($\lambda$) lowers fertility at all income levels

Figure 3 Fertility and Income
Figure 4: The use and cost of modern contraception.
Figure 5 Simulated Fertility Transition

Figure 6 TFR under Alternative Scenarios
Figure 7 NFR under Alternative Scenarios