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Contributory Infringement Rule and Patents

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Abstract

The contributory infringement rule assesses liability to a third party that contributes to the infringement of a patent. Not only are firms that directly infringe liable, those who indirectly contribute are also liable. We investigate how this rule affects the creation of a network of members (e.g., an e-commerce network). We find that the enforcement of indirect liability does not induce more trials in equilibrium. Firms settle out-of-court but, because of the threat of trial, the network size decreases and the social welfare is reduced. Surprisingly, we find that if the compensation paid by the indirect infringers is high, the rule does not benefit the patentholder and may not even give enough R&D incentives ex ante. It is possible to find a direct compensation for the patentholder that is socially preferable.

Keywords: Patents; Network; Infringement


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1 Introduction

Firms can be accused of direct infringement if they illegally use an innovation protected by a patent (copyright or trademark). But they can also be accused of indirect infringement\(^1\) (also called contributory infringement) as soon as they help to sell or promote products or services of a company that infringes upon a patent.\(^2\) Formally, patent law also makes liable someone who actively induces infringement of a patent, or someone who contributes to the infringement by another if he “offers to sell or sells a component of a patented machine, manufacture, combination or composition, or a material or apparatus for use in practicing a patented process.”\(^3\)

In this paper we wonder to what extend Intellectual Property (IP) laws should allow IP owners to sue people who are not directly involved in the infringement, but contribute to it. We therefore investigate whether the contributory infringement rule induces more trials, whether patent owners benefit from its enforcement, and whether this rule is socially desirable.

In the e-commerce world this rule takes on an important dimension because of the network structure of the Internet, and the increasing number of e-commerce patents. Indeed, business-method software is one of the fastest-growing categories of new patents, and software patents account for fifteen percent of all patents (Bessen and Hunt, 2004). Nowadays, patents exist for methods of accepting credit cards over the Internet, for processing orders and transactions of all types, and for alerting consumers of the status of their orders.\(^4\) The network structure of the Internet has an indirect effect on the value of these patents, due to the contributory infringement rule.

To fully capture the potential effects of this rule, let us have a better understanding of the structure of the relationships on the Internet. The existence of the network induces specific relationships between firms: their web sites are cross-linked and they advertise other web sites.

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\(^1\)“Contributory infringement... imposes liability where one person knowingly contributes to the infringing conduct of another” Fonovisa, Inc. v. Cherry Auction, Inc., 76 F.3d at 263 (9th Cir. 1996).

\(^2\)For instance, on March 23, 1999, the on-line auction eBay was sued by Network Engineering Software (NES) for “using third party software packages that infringe the NES’ patent,” Bloomberg news; “NES sues eBay, alleges Patent Infringement,” CNET, news.com.

\(^3\)35 USC section 271 (c).

\(^4\)Patents owned by Amazon.com, Priceline.com, eBay.com, among others.
A given web site of a firm $B$ may use, via hyperlinks or cross promotions, some of the software (or some of the resources) of another web site created by firm $C$. Both firms can possibly contract upon the provision of such software. Consequently, the Internet can be seen as a network of contracts in which firms are committed to other firms, and thus, become liable when the nature of the shared item is altered. Imagine that another firm, say firm $A$, holds a patent on software that firm $B$ uses; that firm $B$ has contracted with firm $C$; and, furthermore, that firm $C$ derives benefits from this software. Is firm $C$ liable if firm $B$ has infringed upon firm $A$’s patent?

In order to realize the magnitude of such an issue, consider the following example.\(^5\) In 2000, Amazon.com had about 250,000 members in its associated program. Each of these members received a payment of between 7 and 15 percent of the profit from any book or video sold to consumers that they refer. Imagine now that another firm that holds a specific patent filed a lawsuit against Amazon.com for infringement. The contributory infringement rule allows the patentholder to sue the 250,000 members in the case of a successful trial, because they contribute to the infringement. They appear as third parties in the infringement, but have their share of responsibility.

Although the previous story is hypothetical, there exists anecdotal evidence that firms plan to make third parties liable in the case of a successful direct infringement trial. British Telecommunications (BT) lost a recent patent infringement lawsuit against Prodigy.\(^6\) The Sargent (hyperlink) patent describes a system in which multiple users, located at remote terminals, can access data stored in a central computer. BT has argued that the Internet infringes the Sargent patent and that Prodigy facilitates infringement by its subscribers by providing them with access to the Internet. BT wanted to have all of the Internet service providers pay a license fee for hosting pages that use hyperlinks (the building blocks on the web). In the case of a successful trial, BT would have sued all Internet service providers. This example gives an idea of the magnitude of such a rule and its potential consequences.

However, we should also note that, so far, we have not witnessed cascading trials with regard to the Internet due to the contributory infringement rule. We attempt to offer a possible


explanation by investigating the effects of the contributory infringement rule on the creation and the size of a network of members. The members are willing to be part of this network only if they can cover the costs of legal lawsuits linked to the possibility of indirect infringement. However, in many situations, disputes over intellectual property rights (IPRs) are not settled in court. In our framework, we allow for the possibility of an out-of-court negotiation in which a license fee can be negotiated between the direct infringer and the patentholder; therefore, trial needs not always be the outcome of the game.

Our main focus is to determine whether and how this rule affects the creation of a network, and whether it is socially harmful. One of our findings is that the enforcement of the contributory infringement rule does not induce more trials in equilibrium. It is, however, harmful to consumers and firms, as it induces a smaller network. Furthermore, the patentholder does not even benefit from high damages paid by the indirect infringers. It is possible to find a direct compensation for the patentholder that is socially preferable.

We propose a three-stage game. At the outset, a business-method innovation (or new software) has been made and patented. An infringer discovers an idea that fully exploits the potential value created by the innovation and decides whether to infringe on it or not. The detection of the infringement is only possible after the membership program has been created and all firms start enjoying returns. In the first stage, the potential infringer creates a network of members. He makes a “take-it-or-leave-it” offer to potential members, and they all accept or refuse it. In the second stage, the infringement is immediately detected by the patentholder, who decides whether to go to court or to settle out-of-court. When a settlement is reached, the infringer pays the patentholder a license fee that is determined as a Nash Bargaining solution.\footnote{One may wonder why the infringer violates the patent and then negotiates a license rather than negotiating a license in the first place, as this would save post-infringement litigation costs to both parties. Here we have in mind a common situation, where the potential infringer adds a “creative component” to the existing patented innovation, but cannot protect it because it is too closely related (Anton and Yao, 1994; 2002). In this case, the infringer may prefer to infringe rather than entering into a negotiation process that would force him to disclose his valuable, but easily copied, business idea.}

In the third period, if a trial occurs, with some probability the patentholder wins the case and then can sue all of the members for contributory infringement if the rule is enforced. We make
the assumption that indirect infringers are aware of their faulty behavior and the risk of trial outcome associated with this behavior.\(^8\)

We show that network size is affected by the contributory infringement rule. We find that its enforcement does not induce more trials in equilibrium, even though it decreases network size, and thus reduces social welfare. If the outcome of the litigation process is a trial, the members of the network anticipate that they risk being sued and need adequate compensation to be part of the membership program. As a result, fewer members join. In the case of a settlement, the threat of trial has the same effect: it reduces the size of the network.\(^9\) High levels of contributory infringement damages should benefit the patentholder. However, the effect of network size reduction outweighs the direct effect of the gain from more damages, and therefore, the patentholder is worse off. Moreover, we find that even if the patentholder can be equally compensated under both regimes (with and without contributory infringement), \textit{ex ante} the regime without is better for society in general. We study the case of a patentholder whose revenue comes from licensing or damages. We show that in such a situation, if society’s goal is to maximize the \textit{ex ante} incentives for R&D of the patentholder, then there exists a level of contributory damage per member that maximizes the expected license fee (or expected amount of damages). The existence and the relevance of such a legal rule can, thus, be justified on that ground.

The paper is organized as follows. Section 2 discusses related literature. In section 3 we introduce the model, and we define the penalty rule adopted in case of trial and the license fee paid by the infringer in case of settlement. Section 4 is devoted to the litigation process and to the analysis of the conditions under which the patentholder prefers to settle out-of-court.

\(^8\)Note that, unlike in the U.S., many countries, such as Japan and Korea, do not require any knowledge of the direct infringement in order to hold the contributory infringer liable (Jong, 2000).

\(^9\)There exists another interpretation of our model that is related to the network structure of the internet and the correlation of infringement risk between the infringer and the members. Indeed, because all commercial sites of the network essentially use and share identical software technology, it is likely that not only the infringer but also the members would directly infringe upon the existing software patent. As such, the patentholder can sue the “main” infringer and, if his patent is being upheld in the court, he can sue or negotiate with the rest of the network for direct infringement. This interpretation is formally identical to the contributory infringement and generates similar results.
section 5 we focus on network creation and on the different contracts offered by the infringer to
the members. Section 6 contains the details of the Nash perfect equilibrium that specifies which
contract is offered and the outcome of the game. In section 7 we investigate the implications of
the contributory infringement rule on the total welfare. We discuss some extensions in section
8. Section 9 concludes and presents our future research agenda.

2 Related Literature

There is an ongoing debate about the existence of software patents and, in particular, it has
been argued that this kind of innovation is not very costly to produce and that licensing is just
a rent-seeking activity. Several studies have pointed out the necessity of tailoring the patent
system to this specific kind of innovation (Mergers, 1999; Shapiro, 2001). Here, we do not
address the problem of whether these patents should be granted; we simply consider that they
exist, as they actually do, and that they can eventually be invalidated in court.

More broadly, the economic literature on IPRs is concerned with the litigation issues of
infringement and the study of how patentholders make their decisions to settle out-of-court or
to sue for damages.10 In a cooperative approach to litigation and settlement, a trial can be
Pareto optimal and the threat of liquidation can be effective in preventing entry (Aoki and Hu,
1999). Related, Crampes and Langinier (2002) adopt a cooperative approach to study a setting
in which monitoring is used as a tool to prevent entry of infringers. In our setting, we also use
a cooperative approach to study the litigation process.

The value of patentholder’s protection depends on the adopted doctrine of damages. Indeed,
in the case of trial, there exist two U.S. damage doctrines, “lost profit” and “unjust enrichment.”
Under some circumstances one doctrine is superior to the other for generating R&D incentives
(Schankerman and Scotchmer, 2001). When the innovator holds a patent on a research tool and
is unable to develop it, he is better off when the doctrine of damages is “unjust enrichment.”11 In

10 A patentholder may decide to award a license to an infringer to avoid litigation on the issue of patent validity
(Meurer, 1989). According to Shapiro (2003), settlements are not always anti-competitive if they leave consumers
as well off as they would have been from litigation.

11 On the same topic, Kaplow and Shavell (1996) and Blair and Cotter (1999) discuss the adequacy of doctrines
of damage in different contexts.
our setup, we do not consider the possibility of injunctions, but we assume that the patentholder is aware of the infringement and then decides to threaten the infringer and eventually bargain over a license.

Settlement negotiations are obviously not limited to IPRs. There is a broad literature on settlement and litigation (see Daughety, 2000, for a survey for “non-terribly technical non-specialists,” and also Daughety and Reinganum, 2005). Recently, emphasis has been put on the problem of information that can be transmitted to subsequent litigants after a first trial occurs. Here, we adopt a very simple viewpoint: once the infringer is found guilty of infringement, so are the third parties.

Our paper is also related to the law and economics literature pertaining to litigation and manufacturer liability in the case of harm to consumers (Spier, 2005; Hay and Spier, 2005). Hay and Spier (2005) show that if consumers (party directly liable in case of damages) have deep pockets, they should be the only ones to be liable. In this case, consumers take the optimal degree of care and they demand optimal safety features. However, in the case of non-solvency, the manufacturer (indirect liable party) should only pay what the direct liable party cannot pay. Even though our setting is different, we find similar results. Indeed, we find that society would be better off if only the direct infringer were to be liable. However, we do not address the issue of solvency because we assume that the benefit to the indirect infringers is a fraction of the direct infringer’s profit.

Along the same lines, Landes and Litchman (2003), wonder how far copyright liability should extend beyond any direct infringement. When third parties can discourage infringement (by making infringement more difficult, for instance), indirect liability can serve as a means to reduce illegal behavior. However, this is not always possible. If there are legitimate uses, the benefits associated with them must be weighed against the harm caused by illegitimate uses. Society may be better off without indirect liability when it is too costly to distinguish legal from illegal uses.

Furthermore, reducing copyright infringement might be done at the price of deterring innovation. In the digital copyright world, Lemley and Reese (2004) wonder how to reduce illegal infringement behavior without reducing innovation.
To the best of our knowledge, our paper is the first in the economic literature to analyze the contributory infringement rule and its consequences. We discuss the importance of this rule within the framework of a commercial network formation within the Internet.

3 The Model

We consider a three-period model with \( m+2 \) players: a patentholder (named firm \( H \)), a potential infringer (firm \( I \)) and \( m \) members of a network. The network of members is created by firm \( I \), as is the case with the Amazon.com example presented in the introduction, and thus, \( m \) is optimally determined by firm \( I \). At the outset, firm \( H \) has a patent on an innovation\(^{12}\) that can be used in an obvious way by firm \( I \),\(^{13}\) which decides whether to infringe or not.\(^{14}\) The infringement decision could be made after (or at the same time as) the creation of the network without altering our results. For expositional purposes we assume that the innovation, the patenting, and the infringement decisions are made at the outset of the game. However, to complete our analysis, in the last section we study the infringement decision.

Timing

1. (Network creation and contracts) In the first period, a network (or membership program) of size \( m \) is created by firm \( I \). He makes a “take-it-or-leave-it” offer to the potential members of the network that consists of the share of profit \( \alpha \) that each member \( i \) will get for \( i = 1, \ldots, m \). The members decide whether to accept or to refuse the offer.

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\(^{12}\) We can also consider that the patenting decision is a variable of decision. However, to simplify, we assume that the parameters of the model are such that it is always worthwhile to patent. A decision not to patent would make the knowledge of the innovation unavailable to the potential infringer, and thus, it would be the end of the game.

\(^{13}\) A patent granted to Amazon.com in 1999, number 5,960,411 entitled “method and system for placing a purchase order via a communication network patent,” is an example of such a method that was already used in-house by many firms.

\(^{14}\) We assume that the infringer cannot infringe if he does not have access to the information contained in the patent files of the patentholder. Note, however, that the infringement that we have in mind can be fairly general. It could be a plain infringement, but it could also represent a situation in which “an imitation” has been developed by the infringer. This imitation may or may not infringe on the original innovation.
2. **(Litigation process)** In the second period, firm $H$ detects the infringement immediately, and observes the size of the network, as well as the share of profit offered to the members. He then decides to litigate, which may result in settlement or trial.\(^{15}\) If a settlement is reached, firms $H$ and $I$ choose the level of license fee (Nash Bargaining solution) that has to be paid by firm $I$ and the commercial network is exploited for two periods, the second and the third.

3. **(Trial and contributory infringement)** In the third period, if firms $H$ and $I$ go to court, with probability $p_H$ firm $H$ wins the trial and the damages are paid according to the outcome of the trial. If the result of the lawsuit is in favor of firm $H$, then he is entitled to sue the $m$ members for contributory infringement if this rule is enforced.\(^{16}\)

**Expected payoffs**

Each firm earns a payoff during the two periods after the creation of the network (i.e., periods two and three). To simplify, we assume that there is no discounting.

Let $\Pi_I(m)$ denote the payoff of the infringer, where $\Pi_I'(m) > 0$, $\Pi_I''(m) < 0$. Each member $i$ gets a fraction $\alpha \in [0, 1/m]$ of it, namely, $\alpha \Pi_I(m)$, and therefore, the gross payoff of the infringer is $(1 - \alpha m) \Pi_I(m)$, which is an inverted U-shaped function where $1 - \alpha m \geq 0$. On one hand, the

\(^{15}\)Crampes and Langinier (2002) consider a model of infringement in which the patentholder must first incur a cost to identify the infringer before deciding whether to go to court, to settle on an agreement or to renounce any pursuit. Here, for the sake of simplicity, we assume that detection is costless. Assuming a cost of detecting infringement would not change the results.

\(^{16}\)We make the assumption that success at the first trial insures success against the members. What is, in fact, crucial for our results is that the first success makes the second success more likely. To establish a contributory liability, there is the requirement that the defendant has “the knowledge of the infringing activity, she (or he) induces, causes, or materially contributes to the infringing activity of another.” The knowledge requirement is satisfied when the defendant knows or has reason to know of the infringing activity (citing Casella v. Morris, 820 F2d 362, 365 11th Cir. 1987). This definition suggests that, for instance, an Online Service Provider should check whether the materials it diffuses are copyrighted. The *Napster* case (i.e., A&M Records, Inc. v. Napster, Inc.), in which major recording companies filed a complaint for contributory infringement against Napster, shows that judges may not want to encourage “willful blindness,” and that Online Service Providers cannot wait for notice that copyrighted plaintiffs detect the (contributory) infringement before stopping any illegal activity. In the *Napster* case, the judges ruled that *Napster* “has a duty to police its system in order to avoid vicarious infringement” (n175 *Id.* at 1096 –1097). For information on the Napster case, see Boldrin and Levine (2003).
members bring consumers and, therefore, increase the profit of the infringer. But on the other hand, for being part of the membership program and bringing consumers to the infringer, the members get a fraction of the profit of the infringer, which reduces his profit. Hence, there is a trade-off between the benefit from having one more member and the cost associated with this member.

If a settlement is reached, the patentholder gets an expected payoff of

\[ G_H = 2\Pi_H - c_H^s + L^{NBS}, \]

where \( \Pi_H \) is the payoff earned by the patentholder in every period, \( c_H^s \) is the cost associated with a settlement (transaction cost of patent licensing) that we normalize to be zero, and \( L^{NBS} \) the total negotiated license fee issued from a Nash Bargaining Solution (that we derive later).

The expected payoff of the infringer is

\[ G_I = 2(1 - \alpha m)\Pi_I(m) - c_I^s - L^{NBS}, \]

where \( c_I^s \) is the cost associated with a settlement paid by firm \( I \). Each member \( i \) gets

\[ G_i = 2(\alpha \Pi_I(m) - \gamma), \]

where \( \gamma \) represents the cost of maintaining the connection to the network. We assume, without loss of generality, that this cost is borne by the members.

If a trial is the result of the litigation process, the expected payoff of the patentholder is

\[ d_H = \Pi_H - c_H^t + p_H(\Pi_H + R_H) + (1 - p_H)\Pi_H, \]

where \( c_H^t \) is the cost incurred by firm \( H \) in case of a trial, \( p_H \) the probability that he wins the trial, and \( R_H \) the penalty that the patentholder receives if he wins the case.\(^{17}\) This penalty depends crucially on the existence of the contributory infringement rule. If it is enforced, firm \( H \) will first receive a compensation from the infringer for direct infringement before launching a lawsuit against the members of the network. We assume that once the infringer loses the

\(^{17}\)We make the assumption that each party pays its own legal costs. In the United States, each party bears its own legal costs of trial unless it can be proven that there was a willful infringement (See Meurer, 1989; or Aoki and Hu, 1999).
lawsuit, all of the members are liable, and firm $H$ is entitled to receive compensation from all of them.\footnote{This is a simplifying assumption and a shortcut. In practice, although infringement liability is a necessary condition for contributory infringement, the members will have to be sued for contributory infringement by the patentholder and each member’s liability will still have to be established by the court. Our result would not change qualitatively if we assumed that the chances of winning the trial for contributory infringement were strictly less than one. We discuss a sequential bargaining process in section 7 in which negotiation is possible between the patentholder and the members.} Further, we also assume that it is costless for the patentholder to sue the members.\footnote{In section 7, we relax this assumption and show that our results still hold.}

Hence,

$$R_H = R_{H,I} + \sum_{i=1}^{m} R_{H,i},$$

where $R_{H,I}$ is the penalty paid by firm $I$ to firm $H$ and $R_{H,i}$ the penalty paid by each member $i$ to firm $H$. The expected payoff of firm $I$ is

$$d_I = (1 - \alpha m)\Pi_I(m) - c^t_I + (1 - p_H)(1 - \alpha m)\Pi_I(m) + p_H(\Pi^{t,t}_I - R_{H,I}),$$

and each member $i$ gets

$$d_i = \alpha\Pi_I(m) - \gamma + (1 - p_H)(\alpha\Pi_I(m) - \gamma) + p_H(\alpha\Pi^{t,t}_I - c^t_i - R_{H,i}), \text{ for } i = 1, 2, ..., m.$$
purposes and is directed toward a specific target. The former doctrine states that the infringer is required to pay the profits from infringement back to the patentholder. The latter doctrine compensates the patentholder for the foregone profit due to the infringement. This doctrine of damages is somewhat designed to maintain the patentholder’s incentives to invest in R&D activities. We do not make any specific assumptions with respect to the doctrine of damages used by the court. Rather, we assume that the penalty paid to the patentholder will represent a fraction of the gross payoff of the infringer and, if applicable, of the indirect infringers. Consequently,

\[ R_{H,I} = \beta_I(1 - \alpha m)\Pi_I(m) \text{ and } R_{H,i} = \beta\alpha\Pi_I(m), \]  

(7)

where \( \beta_I, \beta \in [0, 1] \).

This assumption, consistent with Schankerman and Scotchmer (2001), may encompass the two doctrines of damages. For instance, \( \beta_I = \beta = 1 \) corresponds to the “unjust enrichment” doctrine, whereas, presumably, \( \beta_I < 1 \) and \( \beta < 1 \) corresponds to the “lost profit” doctrine.\(^{20}\) Note that substantial supra compensatory sanctions are possible if \( \beta_I > 1 \). Such damages are possible when the plaintiff (i.e., the patentholder) can prove that there was willful infringement, and in this case, the penalty paid may incorporate punitive damages.\(^ {21}\)

A high \( \beta \) stands for a high level of liability of the secondary infringers, whereas \( \beta = 0 \) stands for a situation in which the contributory infringement rule is not enforced. Hence, \( \beta \) measures the level of contributory infringement liability.

**Nash Bargaining solution**

To complete the description of the payoffs, we need to specify the negotiated license fee. In the second stage of the game, if a settlement is reached, the patentholder and the infringer determine the level of the royalty fee that firm \( I \) will pay to firm \( H \). We compute this level as the solution of a Nash Bargaining game,\(^ {22}\) and therefore, the fee \( L \) is the solution of the following

\(^{20}\)If we abstract from R&D and trial costs, our assumption that \( \Pi_{H^{w\pi}}^I = \Pi_H \) would imply that a judge awarding damages according to the lost profit doctrine would set \( \beta_I = \beta = 0 \).

\(^{21}\)The doctrine of “unjust enrichment” was used in the famous case of Kodak versus Polaroid. See, Warshofsky (1994) for a detailed explanation of this case.

\(^{22}\)This is in the same vein as Aoki and Hu (1999), or as Crampes and Langinier (2002).
program

\[ \max_L \left[ 2\Pi_H + L - d_H \right]^\rho \times \left[ 2(1 - \alpha m)\Pi_I(m) - c_I^s - L - d_I \right]^{1-\rho}, \]

where \( \rho \) (respectively, \( 1 - \rho \)) represents the bargaining power of firm \( H \) (respectively, firm \( I \)). The first bracket represents the difference between the profit of settlement and the profit of trial for firm \( H \), and the second bracket represents the difference between the settlement payoff and the trial payoff for firm \( I \). The Nash bargaining two-period license is

\[ L^{NBS}(m, \beta) = p_H(1 - \alpha m)\Pi_I(m)(\rho + \beta_I) + p_H(1 - \rho)\beta \alpha \Pi_I(m)m \]

\[ + \rho(c_I^t + c_H^t - c_I^s) - c_H^t. \]  \( (8) \)

For any \( m \), it is straightforward that when the contributory infringement rule is enforced \((\beta > 0)\) the optimal license fee is bigger than in the absence of such rule, i.e., \( L^{NBS}(m, \beta) > L^{NBS}(m, 0) \). Even though firms settle out-of-court, the threat point is the trial outcome, and therefore, if the rule is enforced, the license fee includes a fraction of the profit of the members that the patentholder can claim in case of infringement. Whether or not the outcome is a trial, the threat of the trial affects the members’ payoffs. Therefore, the license fee increases with the penalty paid by the infringer(s), \( R_H \). It also increases with the probability of winning for the patentholder.

To solve the game, we define the Nash perfect equilibrium. To do so, we go backward and start with the very last decision: the litigation decision of firm \( H \). Then we specify the different contracts that firm \( I \) can offer to the members if he decides to infringe upon the innovation. Among those contracts, firm \( I \) chooses the optimal contract. We then derive the Nash Perfect equilibrium.

4 Litigation Process

The patentholder prefers to settle out-of-court whenever \( G_H \geq d_H \) holds true. This inequality is in fact equivalent to having

\[ 2\Pi_H + 2(1 - \alpha m)\Pi_I(m) - c_I^s \geq d_H + d_I, \]
which indicates that a settlement occurs whenever the sum of the payoffs in case of settlement is bigger than the sum of the threat point payoffs (i.e., the payoffs in case of trial). We can rewrite this inequality as

$$\Delta + \Pi_I(m) - (1 + \beta)\alpha m \Pi_I(m) \geq 0,$$

where $\Delta = (c_I^t + c_H^t - c_s^t)/p_H$. The term in brackets represents the aggregate cost savings from settling out-of-court. We assume that $\Delta > 0$, as it seems natural that the total cost associated with a trial is higher than the total cost associated with a settlement, no matter who bears those costs.

In the absence of the contributory infringement rule ($\beta = 0$), a settlement will always be reached, as $\Delta + (1 - \alpha m)\Pi_I(m) > 0$.

Lemma 1 If the contributory infringement rule is not enforced ($\beta = 0$), the patentholder prefers to settle out-of-court, irrespective of the network size.

However, in the presence of the contributory infringement rule ($\beta > 0$), firm $H$ prefers a settlement only if (9) holds true.

For any given share of profit $\alpha > 0$, and for any given size of network $m > 0$, there exists a value $\tilde{\beta} > 0$, such that $\Delta + \Pi_I(m) - (1 + \beta)\alpha m \Pi_I(m) = 0$, where $\tilde{\beta} = (\Delta + (1 - \alpha m)\Pi_I(m))/\alpha m \Pi_I(m)$. Therefore, for any $\beta \in (0, \tilde{\beta})$, firm $H$ prefers a settlement over a trial, whereas for any $\beta > \tilde{\beta}$, firm $H$ prefers to go to court. We summarize this finding in the following lemma:

Lemma 2 If the contributory infringement rule is enforced ($\beta > 0$),

- for relatively small levels of contributory infringement ($\beta \leq \tilde{\beta}$) the patentholder prefers a settlement, and
- for higher levels of contributory infringement ($\beta > \tilde{\beta}$) the patentholder prefers a trial.

Firm $H$ prefers a trial for greater values of $\beta$ because the benefit from suing indirectly liable members outweighs the cost of the trial (as long as $p_H$ is not too small).

In light of the two previous lemmas, it is straightforward that with contributory infringement, trials should be more likely to occur than without the contributory infringement rule.
5 Network Creation and Contracts

We now turn to the first period decisions of firm I: the optimal size of the network $m$ and the share of the profit $\alpha$ for each member $i$ if infringement occurs.\textsuperscript{23} We analyze these actions given the strategy that the members accept or refuse the offer, and that firm $H$ chooses his settlement decision based on the first period’s actions. Firm I correctly anticipates what the outcome will be, as we assume perfect information. Thus, firm I makes an offer that is conditional upon whether the outcome is a trial or a settlement: $\alpha_t$ or $\alpha_s$. Furthermore, the size of the network also depends on whether the outcome is a trial or a settlement: $m_t$ or $m_s$. We denote $(m, \alpha)$ a contract that offers a share of profit $\alpha$ to each member $i$ where $i = 1, \ldots, m$, where $(m_s, \alpha_s)$ is called contract $S$, and $(m_t, \alpha_t)$, contract $T$.

Contracts that induce settlement

If firm $I$ wants the outcome to be a settlement, the contract $(m_s, \alpha_s)$ must be such that

- a settlement is preferred by firm $H$, i.e., $G_H \geq d_H$, or equivalently, (9) must be satisfied;
- it insures the participation of each member, i.e., $G_i \geq 0$, as defined by equation (3).

Furthermore, we assume that when the members are indifferent between accepting the contract or refusing it, they always accept. Hence, the participation constraint for each member becomes $G_i = 0$, that we can rewrite as

$$\alpha = \frac{\gamma}{\Pi_I(m)},$$

and thus, (9) becomes

$$\Delta + \Pi_I(m) - (1 + \beta)m\gamma \geq 0.$$

Hence, there exists a value $\bar{m}_s > 0$, such that $\Delta + \Pi_I(m) - (1 + \beta)m\gamma = 0$, and then $m_s$ must satisfy $m_s \leq \bar{m}_s$. Furthermore, the cutoff value $\bar{m}_s$ is a function of $\beta$. Therefore, we denote this

\textsuperscript{23}To be more realistic we should assume that each member gets a different share of the profit, depending on how many consumers he has added. However, to keep the analysis simple, we assume they all get the same share.
as cutoff value $\tilde{m}_s(\beta)$. Let us define $\Sigma_s = \{(m_s, \alpha_s) / m_s \leq \tilde{m}_s(\beta) \text{ and } \alpha_s = \gamma / \Pi_I(m)\}$, the set of contracts $S$.

Among all of the possible contracts $(m_s, \alpha_s) \in \Sigma_s$, firm $I$ chooses the contract that maximizes his expected profit $G_I$ as given by equation (2), where the license fee is defined by (8). Hence, the optimal size of network $m^*_s(\beta)$ is the solution of

$$
\Pi'_I(m) - \Phi_s(\beta)\gamma = 0, \tag{10}
$$

where $\Phi_s(\beta) = [2 - p_H(\rho + \beta_I) + p_H(1 - \rho)\beta] /[2 - p_H(\rho + \beta_I)]$ and must be such that $m^*_s(\beta) \leq \tilde{m}_s(\beta)$. Notice that the optimal solution also depends on $\beta$. We summarize this contract in the following lemma:

**Lemma 3 (Contract $S$)** The optimal contract $(m^*_s(\beta), \alpha_s(\beta))$ is such that

- the size of the network $m^*_s(\beta)$ is the solution of (10),
- and $(m^*_s(\beta), \alpha_s(\beta)) \in \Sigma_s$.

Hence, for a given value of $\beta$, if there exists a unique $m^*_s(\beta)$ that satisfies $m^*_s(\beta) < \tilde{m}_s(\beta)$, there exists a unique $\alpha_s(\beta)$ that defines the share of profit for each member.

**Contracts that induce trial**

On the other hand, if firm $I$ wants a trial to be the outcome, the contract $(m_t, \alpha_t)$ must be such that

- a trial is preferred by firm $H$, i.e., $G_H < d_H$, or equivalently, (9) must not be satisfied;
- and $d_i \geq 0$, to insure the participation of each member.

By the same token, using equations (6) and (7), $d_i = 0$ can be rewritten as

$$
\alpha = \Phi_t(\beta) \frac{\gamma}{\Pi_I(m)},
$$
where $\Phi_t(\beta) = (2 - p_H)/(2 - p_H(1 + \beta))$ and there exists $\tilde{m}_t(\beta)$ such that $\Delta + \Pi_t(m) - (1 + \beta)\Phi_t(\beta)m\gamma = 0$, and then $m_t$ must satisfy $m_t > \tilde{m}_t(\beta)$. Let $\Sigma_t = \{(m_t, \alpha_t) / m_t > \tilde{m}_t(\beta) \text{ and } \alpha_t = \Phi_t(\beta)\gamma/\Pi_t(m)\}$ be the set of contracts $T$.

Among all of the possible contracts $(m_t, \alpha_t) \in \Sigma_t$, firm $I$ chooses the contract that maximizes his expected profit $d_I$ as given by equation (5), where the penalty is defined in (7). The optimal size of network $m^*_t(\beta)$ is the solution of

$$\Pi'_t(m) - \Phi_t(\beta)\gamma = 0,$$

and must be such that $m^*_t(\beta) > \tilde{m}_t(\beta)$.

**Lemma 4 (Contract T)** The optimal contract $(m^*_t(\beta), \alpha_t(\beta))$ is such that

- the size of the network $m^*_t(\beta)$ is the solution of (11),
- and $(m^*_t(\beta), \alpha_t(\beta)) \in \Sigma_t$.

**Share of profit and size of network**

We find that the fraction that each member gets from participating in selling the good is higher when the outcome is a trial rather than a settlement ($\alpha_t > \alpha_s$) for a given number of members $m$. Indeed, members need to be compensated for a potential trial. Furthermore, the larger the size of the network, the smaller the share of profit per member (as $\partial \alpha_j/\partial m < 0$ for $j = s, t$). The share is also affected by the level of contributory infringement in an obvious way when a settlement is reached ($\text{sign}(\partial \alpha_s/\partial m) = -\text{sign}(\partial m/\partial \beta)$). On the other hand, the total number of members that will be in the network is affected by the outcome of the game.

In the absence of the contributory infringement rule ($\beta = 0$), the optimal network size is not influenced by the possibility of indirect infringement, and thus, $m^*_s(0) = m^*_t(0)$. Indeed, the members’ decisions to enter the network are not affected by the threat of a trial.

With contributory infringement ($\beta > 0$), the optimal size of the network is influenced by the possibility of indirect infringement, and depends on the outcome of the game if infringement occurs. The comparison of (10) and (11) is straightforward and leads to the following inequalities

$$m^*_t(\beta) \leq m^*_s(\beta) \leq m^*_s(0) = m^*_t(0),$$

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as $\Phi_s(\beta) \leq \Phi_t(\beta)$, with strict inequality for $\beta > 0$ and equality for $\beta = 0$.

We can posit the following lemma:

**Lemma 5** *The enforcement of the contributory infringement rule decreases the optimal network size, whether the outcome is a settlement or a trial. The network size is smaller in the case of trial.*

When the contributory infringement rule is enforced, firm $I$ decreases the size of his network, even when a settlement is reached. Indeed, when the contributory infringement rule is enforced, the cost of an extra member is, in fact, the marginal license fee that the infringer has to pay to recruit another member. However, in the case of contributory infringement, the marginal license fee entails some of the payment made by the member in case of a successful trial and, as a result, the marginal license fee is higher than without contributory infringement; therefore, fewer members will join the membership program.

**Optimal contracts and level of contributory infringement**

The feasibility of the contracts depends upon the level of contributory infringement. Indeed, contract $T$ as defined in lemma 4 is not feasible for small values of $\beta$. We can state the following result:

**Lemma 6 (Contract feasibility)** *Contract $T$ is not feasible for relatively small levels of contributory infringement (i.e., $\beta < \hat{\beta}_1$); there exists no optimal network size such that firm $I$ can make firm $H$ prefer a trial. On the other hand, for relatively high levels of contributory infringement (i.e., $\beta \geq \hat{\beta}_1$), both contracts are feasible.*

**Proof.** See appendix.

On the other hand, contract $S$ as defined in lemma 3 is always feasible but for certain values of the contributory infringement parameter firm $I$ must reduce his network size compared to the optimal size. This is summarized in the next lemma:

**Lemma 7 (Network size)** *For small levels of contributory infringement (i.e., $\beta < \hat{\beta}_2$), contract $S$ is always optimally chosen (i.e., $m^*_s(\beta)$). For relatively high levels of contributory in-
fringement (i.e., $\beta \geq \beta_2$), when offering contract $S$, firm $I$ is constrained to reduce his network size to $\tilde{m}_s(\beta)$.

\textbf{Proof.} See appendix.

6 Optimal Contract and Equilibrium Litigation Outcome

We now define the Nash perfect equilibrium. This equilibrium is conditional on the occurrence of infringement.

\textbf{Proposition 1} \textit{The Nash perfect equilibrium is such that}

- for $\beta < \beta_2$, firm $I$ offers the optimal contract $S$ that the members accept and the litigation outcome is a settlement;

- for $\beta \geq \beta_2$, firm $I$ offers a constrained contract $S$ (i.e., $\tilde{m}_s(\beta)$) that the members accept and the litigation outcome is a settlement.

\textbf{Proof.} See appendix.

Thus, for low levels of contributory infringement, each of the $m^*_s(\beta)$ members gets a fraction $\alpha_s(\beta)$ of the profit of the infringer, and a settlement is reached in which the infringer pays a license fee to the patentholder. Members are not liable, even though their potential liability impacts on the choice of the license fee. For higher levels of contributory infringement, each of the $\tilde{m}_s(\beta)$ members gets a fraction $\alpha_s(\beta)$ of the profit, and infringer and patentholder settle out-of-court again. Thus, firm $I$ prefers to insure that the outcome will be a settlement rather than a trial.

Following proposition 1 we can posit the following proposition.

\textbf{Proposition 2} \textit{Whether the contributory infringement rule is enforced or not, a settlement is always achieved. The higher the $\beta$ the smaller the network size.}

As $\beta$ increases, we go from a regime in which a settlement is achieved without a constrained network size to a regime in which a settlement is achieved with a constrained network size. The network size is decreasing with $\beta$. 
Thus, surprisingly the contributory infringement rule does not induce more trials in equilibrium, even though the threat of a trial decreases the network size. This is due to the fact that firm I chooses the contract first, and thus has a first-mover advantage. Indeed, firm I shapes the contract in such a way that firm H always prefers a settlement over a trial. Furthermore, we have made the implicit and plausible assumption that whenever firm H is indifferent between a trial and a settlement, he prefers a settlement. This is consistent with what we observe in reality: there are very few trials for contributory infringement.

7 Welfare Analysis of Contributory Infringement

In order to fully characterize the welfare effect of the contributory infringement rule, we need to specify the demand side more precisely. Let us consider that each consumer can consume one or zero units of the good per period. A consumer has the following utility function:

\[
U = \begin{cases} 
  v - p & \text{if he buys the good at price } p, \\
  0 & \text{otherwise},
\end{cases}
\]

where \( v \) represents the taste parameter of the consumer, and is distributed according to some density \( f(v) \) and cumulative distribution function \( F(v) \). Hence, \( F(v) \) is the fraction of consumers with a taste parameter smaller than \( v \), and the total demand at price \( p \) is therefore

\[
D(p, m) = [1 - F(p)]S(m),
\]

(12)

where \( S(m) \) is the number of consumers, with \( S'(m) > 0 \) and \( S''(m) < 0 \). The larger the network, the larger the consumer base. Furthermore, the number of consumers increases at a decreasing rate. With this specification of the demand, we can define more precisely the profit of firm I,

\[
\Pi_I(m) = (p - C)[1 - F(p)]S(m),
\]

where \( (p - C) \) represents the markup, with \( C \) being the marginal cost of production. To keep the model simple and because we do not consider price setting, we do not really need the characterization of this profit function. However, this function is consistent with our assumption that the profit increases at a decreasing rate.
This specification is, however, interesting in defining the consumers’ surplus,

\[ CS(m) = \int_{p}^{\infty} [1 - F(x)]S(m)dx. \]

It increases at a decreasing rate with the number of members in the network. At the optimal size of the network, \( m_s(\beta) = \min\{\bar{m}_s(\beta), m^*_s(\beta)\} \), the consumers’ surplus is a decreasing function of the level of contributory infringement, as

\[ \frac{\partial CS}{\partial \beta} = \frac{dS}{dm} \frac{dm_s(\beta)}{d\beta} < 0, \]

and thus, the higher the \( \beta \) the lower the consumers’ surplus.

We now define the total social welfare depending on the outcome of the litigation process.

If the outcome of the litigation process is a settlement, the total welfare is

\[ W_s(\beta) = GH + GI + m_sGi + 2CS(m_s), \]

or equivalently

\[ W_s(\beta) = 2\Pi_H - c^*_I + 2(\Pi_I(m_s) - \gamma m_s) + 2CS(m_s). \]

Note that the license fee is just a transfer from firm \( I \) to firm \( H \) and has no impact on the social welfare.

If a trial occurs, the total welfare is

\[ W_t(\beta) = d_H + d_I + m_td_i + 2CS(m_t), \]

which can be rewritten as

\[ W_t(\beta) = (2 - p_H)\Pi_H - (c^*_H + c^*_I) + (2 - p_H)(\Pi_I(m_t) - \gamma m_t) + 2CS(m_t), \]

where, again, the penalty paid is just a transfer from firm \( I \), and each firm \( i \) in case of liability, to firm \( H \). We can thus posit the following proposition.

**Proposition 3 (Welfare Effect)** For a given level of damages, a regime with contributory infringement (i.e., \( \beta > 0 \)) always decreases the total welfare compared to a regime without (i.e., \( \beta = 0 \)). For a given \( \beta \) the total welfare is higher if a settlement occurs, \( W_s(\beta) > W_t(\beta) \).

**Proof.** See appendix. ■

This proposition shows that, once the innovation is patented, the contributory infringement rule has only a negative impact on the total social welfare.
Our analysis of welfare shows that it is increased by simply removing contributory liability. However, we have been silent on who benefits when the degree of contributory liability $\beta$ is altered. Whenever infringement is not prevented, it is of interest to study how the patentholder’s payoff is affected. Indeed, assuming that this payoff is a good proxy for the patentholder’s \textit{ex ante} incentive to invest in R&D, we shall now discuss how sensitive this payoff is to a variation of $\beta$.

**Proposition 4 (R&D Incentives)** The contributory infringement rule is preferred by the patent holder only if $0 < \beta < \bar{\beta}_s$. Moreover, there exists a unique $\beta^*_s$ such that the private incentives for R&D are at a maximum. This level is characterized by $0 < \beta^*_s < \bar{\beta}_s$.

**Proof.** See appendix. ■

This result may seem counterintuitive. Indeed, raising the level of damages that the patentholder can obtain when he wins the trial can make him worse off. This is due to the fact that a change in the value of $\beta$ has essentially two contradictory effects on the patentholder’s payoff. A higher $\beta$ increases the level of damages per member obtained by the patentholder \textit{ex post}, but it also tightens the participation constraint of the members. This results in a smaller infringer’s network, and damages will be collected from fewer members.

The reason for maximizing the amount of the patentholder’s damages is formally identical to setting a monopoly price. A regulator or a judge willing to maximize the license fee would set $\beta = \beta^*_s$, such that the increased benefit of more damages (i.e., a higher price) is equal to the foregone profit of fewer members (i.e., of having fewer buyers).

It is also interesting to note that when $\beta > \bar{\beta}_s$ everyone (except the judge!) would prefer \textit{ex ante} to waive the contributory liability. In such a case, not only is no contributory liability socially optimal (see proposition 3), but it is also Pareto optimal for all of the players. The next corollary completes proposition 4.

**Corollary 1** At the equilibrium, any level of contributory infringement greater than $\beta^*_s$ but smaller than $\bar{\beta}_s$ is suboptimal.

**Proof.** See appendix. ■
The amount of damages received depends on parameter \( \beta \). Our point is that any optimal level of \( \beta \) set by a policy-maker should lie strictly in the interval \((0, \beta^*_{s})\). A policy-maker may want to foster the *ex ante* incentives of the patentholder to do research, and in this case a \( \beta \) close to but smaller than \( \beta^*_{s} \) would be optimal. Conversely, the policy-maker may want to favor network end-users (i.e., consumers), and then would set \( \beta \) close to 0 to maximize the network size. However, setting \( \beta \in (\beta^*_{s}, \bar{\beta}_{s}) \) would decrease the network size and the total amount of damages received by the patentholder.

This result shows formally that tighter intellectual property protection can strictly decrease the incentive to innovate in the framework of a network. It is consistent with the analysis of Bessen and Maskin (2004), who argue that “moderately weak intellectual protection is optimal.” However, they advocate weaker protection because it favors innovation through imitation. Our point is different, since we argue that the network structure helps to “transmit” infringement liability to all of the members, and makes them less inclined to join the network.

The damages or the license awarded to the patentholder are important for judging his incentives to invest *ex ante*. When contributory liability exists, the amount of damages received by the patentholder is described by the parameters \( \beta_I \) and \( \beta \). Thus, the “weight of liability” is split between the infringer and the members. It is interesting to analyze whether one can generate the same amount of damages without contributory liability. The next corollary develops a result along these lines.

**Corollary 2** Consider a given structure \( B \) of direct and contributory infringement damages, with \( \beta_I > 0 \) and \( \beta > 0 \). There exists another structure \( B' \) characterized by \( \beta'_I > 0 \) and \( \beta' = 0 \), such that, if infringement occurs, the level of the license fee is identical to that obtained with structure \( B \). Structure \( B' \) is socially better than structure \( B \).

**Proof.** See appendix. ■

Therefore, once infringement occurs, it is possible to give the same incentive to invest in R&D *ex ante* with and without the contributory infringement rule, and a larger network is obtained *ex ante*. This result rather pleads for minimizing the liability of contributory infringers and increasing the liability of direct infringers when this is possible.
Our results can be compared with those of Hay and Spier (2005). Indeed, they show that, whenever possible, a manufacturer of a product should not be held responsible when a customer, while using the product, harms a third party. More precisely, they show that if the producer of the harmful good is held responsible, then he will choose to distort the quantity of the good offered on the market. A similar effect is at work in our framework. Indeed, if members are held responsible for indirect infringement, they might choose not to participate in the network, thereby reducing the social welfare.

It is worth noting that in all of our previous analyses we assumed the occurrence of infringement. However, as mentioned in the presentation of the model, the potential infringer has to decide whether to infringe or not.

8   Extensions

We now discuss and/or relax some of the assumptions of our model. First, we discuss the occurrence of infringement behavior. Second, we take into account that indirect trials are expensive and show that our main findings still hold. Third, we analyze a situation in which the members have a more active role in the bargaining process. Indeed, we allow them to be involved in a litigation process with the patentholder once the direct infringer has been found guilty of infringement, and we show that our results still hold. Lastly, we discuss contracts between the infringer and the members.

8.1 Infringement and Level of Direct Damages

The decision to infringe is made at the beginning of the game and depends on whether the expected profit of the infringer, as defined by equation (2), is positive. The next result discusses the occurrence of infringing behavior as a function of direct and indirect damages.

**Proposition 5** Infringement occurs when direct and indirect damages \((\beta_1, \beta)\) are such that \(\beta_1 < \overline{\beta}_1(\beta)\), where \(\overline{\beta}_1(\beta)\) is decreasing in \(\beta\).

**Proof.** See appendix. ■

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Not surprisingly, whether infringement occurs or not is simply a matter of how harsh the punishment is of the direct infringer. In other words, setting a high level of $\beta_I$ makes illegal infringement behavior less likely. It is rather intriguing that an increase of the contributory liability has a first-order effect on the infringement decision; that is, an increase of $\beta_I$ has an effect qualitatively equivalent to an increase of $\beta$. This result is due to the settlement outcome. Indeed, if the outcome is a trial, expression (5) shows that increasing $\beta$ would affect the infringer’s decision only through the network size. With a settlement, a higher $\beta$ directly increases the level of the license fee. Therefore, from a public standpoint, $\beta_I$ and $\beta$ cannot be chosen independently as far as infringement decisions are concerned.

However, in our model, the objective of a lawmaker is not necessarily to prevent infringement, as it is, by assumption, not socially desirable. Indeed, as members and end-users benefit from the (infringing) innovation, infringement increases social welfare. The relevance of this assumption can be discussed in a more general context. For instance, when there is a great deal of uncertainty about the future value of an innovation, it may be optimal to let the infringement occur, and then eventually sanction it, once uncertainty has disappeared.\(^{24}\) This “laissez-faire” policy allows for the introduction of valuable innovations that would otherwise not exist if, for instance, a simple injunction was enforced (Lemley and Reese, 2004).\(^{25}\)

In addition to uncertainty, in some cases R&D costs are known to be small, and (too) strong intellectual property rights are not needed, since adequate incentives to innovate are easily obtained. Many innovations related to digital networks, such as patented business-method software, fall into that category (Rivette and Kline, 2000). Therefore, when the infringing innovation generates enough (transferable) surplus to cover the R&D costs of the original innovator,

\(^{24}\)Note that this could be incorporated in our model by introducing some uncertainty on the consumer’s taste; this uncertainty would later be resolved.

\(^{25}\)The “Sony-Betamax” case (i.e., Sony v. Universal City Studios, Inc., 464 U.S. 417 (1984)) illustrates this dilemma well. In 1976, the U.S. Supreme Court ruled that the introduction by Sony of the Betamax VCR did not constitute an act of (contributory) infringement, although it gave users the possibility of recording television programs that were copyrighted. In the spirit of patent law, the Court ruled that, since the VCR was capable of being used for “commercially significant non-infringing” purposes, Sony would not be held responsible for contributory infringement. Retrospectively, judges were arguably right to rule in favor of Sony, since later this innovation would even benefit Universal.
The next proposition extends the findings of corollary 2 when the infringement decision is taken into account.

**Proposition 6** Assuming that infringement is socially desirable and that the structure of damages $B$ satisfies $\beta_I < \beta_I(\beta)$ (i.e., infringement occurs), a benevolent law maker would implement a structure $B'$ where $\beta'_I > 0$ and $\beta' = 0$, as defined in corollary 2. Under this structure, infringement would occur as well.

This result states that minimizing the liability of indirect infringers, while keeping incentives for R&D constant through identical levels of license fees, has no effect on the infringing decision. In fact, doing so is optimal from a social standpoint. The logic for this result goes as follows. Although the negotiation between the infringer and the patentholder is a zero-sum game, the lawmaker strictly increases the surplus to be divided by transferring all of the liability from the members to the infringer.

To understand the total effect of a change from structure $B$ to $B'$, it is worth decomposing it into two effects. First, a decrease of the indirect liability makes the infringer better off as the network grows, but makes the patentholder worse off as the license fee is reduced. This is the indirect liability effect. Second, to keep the R&D incentives constant the direct liability must be increased, which makes the infringer worse off. However, overall, the infringer’s loss from the increase of the direct liability is smaller than the gain from reducing the indirect liability. In other words, one property of our model is that the structure $B'$ always belongs to the set of damages that triggers infringement.

### 8.2 Indirect Trial Cost

We now investigate the impact of the introduction of a patentholder indirect trial cost. Our model assumes that it is costless for the patentholder to launch lawsuits against members once the direct infringer has been found guilty of infringement. However, in reality, since the patentholder would have to prove intent on the part of each contributory infringer, lawsuits might be expensive. Thus, we introduce a trial cost $c^t$ that the patentholder has to incur to launch
a lawsuit against each of the members. Therefore, the total cost is $mc_t$, to be included in the profit function (4) as long as $\beta > 0$. The introduction of these trial costs has an impact on equation (9) that becomes

$$\Delta + \Pi_I(m) - (1 + \beta) \alpha m \Pi_I(m) + \frac{1}{\rho} mc_t \geq 0.$$  

In the rest of the analysis all of the threshold values are affected, but in the same way. For instance, in the case of settlement, instead of having a value $\tilde{m}_s(\beta)$ such that $\Delta + \Pi_I(m) - (1 + \beta)m\gamma = 0$, we now have a value $\tilde{m}'_s(\beta)$ such that $\Delta + \Pi_I(m) - (1 + \beta)m\gamma + mc_t/\rho = 0$, where $\tilde{m}'_s(\beta) > \tilde{m}_s(\beta)$. Similarly, in the trial case, we now have $\tilde{m}'_t(\beta)$ such that $\Delta + \Pi_I(m) - (1 + \beta)\Phi_t(\beta)m\gamma + mc_t/\rho = 0$, where $\tilde{m}'_t(\beta) > \tilde{m}_t(\beta)$. Overall, the comparisons of all of the optimal network sizes are identical, and therefore, we still obtain the same qualitative results. In fact, making trials more expensive for the patentholder increases his incentive to settle out-of-court.

### 8.3 Sequential Bargaining Process

In our model the members have a passive role, as they do not participate in the bargaining process. They are all sued by the patentholder in the case of a successful trial against the infringer. However, it may be the case that the members and the patentholder engage in a litigation process once the patentholder wins against the infringer. We now explore this case and determine how our findings are affected when members are involved in a negotiation process with the patentholder.

In the case of a successful trial against the direct infringer, we explore the litigation process between the patentholder and the indirect infringers. We assume that the patentholder bargains with all of the members simultaneously or with a representative member (since they are all identical, there is no loss of generality). We find that the patentholder and the members prefer to settle out-of-court and the license fee paid by each member is $L_i = p\beta \alpha \Pi_I(m) + \delta_i - c^t_i$, where $p$ is the probability that the patentholder wins the case against the members (that can eventually be equal to $p_H$), $\delta_i = \rho_i(c_i^t + c^t - c_i^s)$, $c_i^s$ is the settlement cost for the members, $c^t$ is the trial cost borne by firm $H$, and $c_i^t$, the trial cost borne by each member.

If a settlement occurs with the direct infringer, the payoffs of the patentholder, the infringer
and each member are therefore

\[ G_H^s = 2\Pi_H + L^s, \]
\[ G_I^s = 2(1 - \alpha m)\Pi_I(m) - L^s - c_I^s, \]
\[ G_i^s = 2(\alpha \Pi_I(m) - \gamma), \]

where the superscript \( s \) stands for settlement with members, and \( L^s \) is the license fee that is paid by the infringer when the threat point is a trial with the direct infringer, potentially followed by a settlement with the members.

In the case of trial, the payoffs are

\[ d_H^s = 2\Pi_H - c_H^t + p_H R_{H,I} + p_H m L_i - p_H m e^s, \]
\[ d_I^s = (2 - p_H)(1 - \alpha m)\Pi_I(m) - c_I^t - p_H R_{H,I}, \]
\[ d_i^s = (2 - p_H)(\alpha \Pi_I(m) - \gamma) - p_H L_i - p_H c_i^s. \]

The license fee between the patentholder and the direct infringer is now

\[ L^s = p_H \Pi_I(m)[(1 - \alpha m)(\rho + \beta_I) + (1 - \rho)p\beta m] + (1 - \rho)p_H m(\delta_i - c_i^t) + \rho(c_I^t - c_I^s) - (1 - \rho)c_H^t. \]

Taking into account these changes in the different payoffs, we can redefine the specific contracts \( S \) and \( T \). We find that the previous results are not qualitatively affected by these changes. More precisely, at the equilibrium, the patentholder has less incentive to settle out-of-court with the direct infringer, as a settlement will still occur with the indirect infringers in the case of a successful trial against the direct infringer. Further, the infringer reduces the optimal size of his network.

### 8.4 Indirect infringement, ownership structure and contracting

In our analysis, we have been silent about the exact links existing between the infringer and the members. The nature of this relationship is likely to impact our model and, in particular, the nature of the contract passed between the infringer and the members. If the members are subsidiary/parent of the infringer then the profit sharing contract that we propose may no longer be (privately) optimal for the infringer. For instance, a contract contingent on the occurrence
of a lawsuit may fare better; it would specify that all of the profits would be transferred to the members if a lawsuit arises or is likely to arise. This contract would essentially render the infringer judgment-proof, leaving the patentholder with an “empty shell.” In our context, any incentive to sue the infringer in the first place would be reduced.

The effectiveness of this “strategic” contract is however sensitive to the strict enforcement of the limited liability principles of common law, which explicitly protects corporation owners from legal suit launched against subsidiary on the basis that the parent corporation does not “operate” the subsidiary. However, the law has broadened the definition of operation and extended liability in many instances (for examples see Boyd and Ingberman, 2003). In our context, extending liability would simply nullify the effect of such a strategic payment scheme, and our results would still hold.

We also assume that the infringer is unable to fully insure members against any contributory infringement lawsuits. Indeed, one possibility is for the infringer to pay for any network connection costs and commit to indemnify members for intellectual property claims raised against them. With such a commitment, the infringer would internalize the externalities caused by indirect infringement and restore the efficient network size.

However, the “commitment case” relies on assumptions that may not be true in our context. First, the infringer cannot be bankrupted or insolvent after losing the trial. This may be the case if, for instance, the infringer has to pay high punishment damages, that is $\beta > 1$. Second, this contract, like any insurance contract, creates moral hazard frictions whose level must remain low for it to be feasible. Indeed, since members are no longer facing damages for behaving as if they knew about their infringing activity, proving intent will be easier. Another potential problem with this contract is that the parties involved are no longer adversarial and it creates clear incentives for “side contracting” between the patentholder and the members. Of course, the contract could be amended in ways that prevent these devious incentives but it is no longer clear whether these amendments would be distortion-free. As such, we believe that focusing

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26 The firm ORACLE has recently started to offer to indemnify its membership network of LINUX developers against intellectual property infringement claims that they could face using program furnished by ORACLE. Perhaps not surprisingly, this help is conditioned to ORACLE getting “sole control of the defense and any settlement negotiations” and to ORACLE being given access to the “Information, authority, and assistance it
on the case where members’ participation is induced *ex ante* bears some realism.

9 Conclusion

In this paper we consider a network innovation, and we study the effects of the contributory infringement rule on the size of a network of members and on total social welfare. We do not address the problem of whether e-commerce patents should be granted; rather, we investigate the impact of the contributory infringement rule on this particular kind of patent. We show that whether firms settle out-of-court or go to trial, the optimal size of the network is smaller when the contributory infringement rule is enforced. This decreases total social welfare. At the equilibrium firms settle out-of-court, no matter the level of contributory infringement. Furthermore, we show that even if the patentholder can receive the same compensation under both regimes (with and without contributory infringement), the network size is still smaller under the contributory infringement rule. This rule is harmful to society when some infringement is desirable. Therefore, we question the relevance of such a rule in the case of e-commerce patents.

We have made several assumptions concerning the contract between the infringer and the members. First, the contract as defined in this model is not optimal. We could propose a contract in which the infringer just gave $\varepsilon$ to each member. However, here our aim is to capture the effect of indirect infringement, and therefore, we do not restrict ourselves to an optimal contract. Second, we have assumed that the size of the network is determined *ex ante*, and thus, the members of the network cannot decide to quit the program at a certain point. We next plan to investigate what would happen if members were allowed to exit the program after the infringement had occurred. Or alternatively, we plan to see what the effect would be of a renegotiation of the contracts in the second period of the game.

We have not considered, either, what the outcome would be if the patentholder decided to settle out-of-court with each member if a trial occurred.

Although we discuss the license fee received by the patentholder, we do not consider per se needs to defend against or settle the claim.” Although this example does not straightforwardly apply to our setting, our aim is to point out potential important distortions created by these rather stringent conditions (http://www.oracle.com/technologies/linux/ubl-faq.pdf accessed 22/12/2006).
the R&D investment decision. This aspect is motivated by the fact that network innovations such as business methods are known to be costless to produce, and thus our model represents a valid scenario for these innovations. Nevertheless, we believe that a R&D stage could be added easily to our game in order to consider situations in which R&D investments are substantial.

Finally, our model features one infringer and several contributory infringers. Our choice has been made essentially for expositional simplicity. However, we acknowledge that in many situations there is one contributory infringer and several direct infringers. A variant of our model could be built to investigate this issue.
References


Appendix

Structure of the network

We describe an example of the structure of a membership program. The infringer, called firm $I$, has a program with $m$ members. For the sake of simplicity we will call this program the network of the infringer.

![Network structure diagram](image)

Figure 1: Network structure

Each member $i$ has $n$ end-users. There is a possibility of overlapping. An end-user who is connected to member 1 can also be connected to member 2, and so on and so forth. Let $N$ ($>n$) be the total fixed number of end-users that represents the size of the total network.

The infringer chooses to have as many end-users (i.e., potential buyers) as possible. Every time he accepts a new member, he will have at most $n$ new end-users; at most, because among the new customers (i.e., end-users) some may already be connected to another member of the network. Consider the following trivial situation. The day he creates his membership program, the infringer does not have any members, consequently there are no end-users from his program. He accepts a new member (member 1) with $n$ new end-users.\footnote{This is, in fact, equivalent to a sampling without a replacement. Indeed, it is as if the infringer “draws” the} Then he accepts another member

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\footnote{This is, in fact, equivalent to a sampling without a replacement. Indeed, it is as if the infringer “draws” the}
(member 2) with new end-users, as well. Some, if not all, of the end-users of member 2 can also be connected to member 1. Therefore, the number of new\textsuperscript{28} end-users will be smaller or equal to \( n \). Formally, let \( G_i \) be the set of end-users who are not yet connected to a member that belongs to the network of the infringer when member \( i \) joins the program. First, the infringer chooses member 1 with \( n \) end-users. Second, he chooses another member (member 2) with \( n G_2 / N \) new end-users, where \( G_2 = N - n \). The third member will bring fewer new end-users, namely \( n G_3 / N \), where \( G_3 = N - n - n(N - n)/N \), and consequently the number of new end-users that member 3 brings is \( n(N - n)^2 / N^2 \), and so on and so forth. To simplify the notation, let us denote \( A = (N - n)/N \).

The number of end-users increases with the number of members at a decreasing rate (\( S'(m) > 0 \), and \( S''(m) < 0 \)).

**Proof of lemma 6 (Feasibility of contracts)**

The functions \( m_s(\beta), m_t(\beta), m_s^*(\beta) \) and \( m_t^*(\beta) \) are decreasing with \( \beta \). It is easy to check that \( m_s(\beta) \geq m_t(\beta) \) and \( m_s^*(\beta) \geq m_t^*(\beta) \), with strict inequality for \( \beta > 0 \) and equality for \( \beta = 0 \). Furthermore, \( m_s(0) = m_t(0) > m_s^*(0) = m_t^*(0) \). Let \( \beta_1 > 0 \) be the value of \( \beta \) for which \( m_t^*(\beta) = m_t(\beta) \). According to lemma (5), the optimal contract \( T \) is such that \( m_t > m_t(\beta) \). Hence, for values smaller than \( \beta_1 \), \( m_t^*(\beta) < m_t(\beta) \) and, thus, a trial can never be preferred by firm \( H \). However, for values greater than \( \beta_1 \), \( m_t^*(\beta) > m_t(\beta) \) is always satisfied, and the contract is feasible.

**Proof of lemma 7 (Network size)**

Let \( \beta_2 > 0 \) be the value of \( \beta \) for which \( m_s^*(\beta) = m_s(\beta) \). According to lemma (4), the optimal contract \( S \) is such that \( m_s \leq m_s \). Hence, for \( \beta < \beta_2 \), \( m_s^*(\beta) < m_s(\beta) \), and thus, firm \( I \) chooses first member from a set of members. This first member brings \( n \) new end-users; i.e., \( n \) times the proportion of end-users not yet connected to a member ((\( N - 0 \))/\( N \)), which is 1 for the first member.

\textsuperscript{28}“New” meaning end-users not yet connected to the network of the infringer.

\textsuperscript{29}It is easy to check that \( S(m) = n + n \left( \frac{N-n}{N} \right) + n \left( \frac{N-n}{N} \right)^2 + \ldots + n \left( \frac{N-n}{N} \right)^{m-1} \) is equivalent to \( S(m) = n \frac{1 - (\frac{N-n}{N})^m}{1 - (\frac{N-n}{N})} \).
the optimal network size. However, for $\beta \geq \tilde{\beta}_2$, $m^*_s(\beta) > \tilde{m}_s(\beta)$ and, therefore, if firm $I$ wants the outcome to be a settlement, he needs to reduce his network size to $\tilde{m}_s(\beta)$.

**Proof of proposition 1**

According to lemma (7), for $\beta < \tilde{\beta}_1$, only contract $S$ is feasible. If $\tilde{\beta}_2 \leq \tilde{\beta}_1$, for values of $\beta \in [\tilde{\beta}_2, \tilde{\beta}_1]$ the only possible contract is the constrained contract $S$. For $\beta \geq \tilde{\beta}_1$, as both contracts can be offered, firm $I$ will choose the contract that maximizes his profit, i.e., $\max\{d_I, G_I\}$. It is easy to show that, for any

$$\beta \geq \tilde{\beta}_1,$$

the optimal contract is the constrained contract $S$ and when there is not. When the outcome is a trial,

$$\beta \geq \tilde{\beta}_2,$$

the only possible contract is the constrained contract $S$. For $\beta \geq \tilde{\beta}_1$, as both contracts can be offered, firm $I$ will choose the contract that maximizes his profit, i.e., $\max\{d_I, G_I\}$. It is easy to show that, for any $m, G_I > d_I$; Thus, it is true when $G_I$ is determined at the optimal value $m^*_s(\beta)$ and $d_I$ at the optimal value $m^*_t(\beta)$. Thus, if $\tilde{\beta}_2 > \tilde{\beta}_1$, for values of $\beta \in [\tilde{\beta}_1, \tilde{\beta}_2]$ the optimal contract $S$ is offered. If $\tilde{\beta}_2 \leq \tilde{\beta}_1$, for $\beta \geq \tilde{\beta}_1$ the constrained contract $S$ is still preferred to the optimal contract $T$, as $\tilde{m}_s(\beta) > m^*_s(\beta)$, and therefore, $G_I$ evaluated at $\tilde{m}_s(\beta)$ is bigger than $d_I$, even at the optimal value $m^*_t(\beta)$.

**Proof of proposition 3**

The derivative of $W_s(\beta)$ with respect to $\beta$ is $(d\Pi_I/dm - \gamma + dS/dm)m_s(\beta)/d\beta$, where $d\Pi_I/dm - \gamma > 0$, as $d\Pi_I/dm - \Phi_s(\beta)\gamma = 0$ at $m^*_s$ and $\Phi_s(\beta) > 1$. Furthermore, as $dm_s(\beta)/d\beta < 0$, then $\partial W_s(\beta)/\partial \beta < 0$. In the same vein, we can show that

$$\frac{\partial W_t(\beta)}{\partial \beta} = (\frac{d\Pi_I}{dm} - \gamma + \frac{dS}{dm})m_s(\beta) - p_H(\frac{d\Pi_I}{dm} - \gamma)m_s(\beta) < 0.$$

It is also easy to verify that $\partial W_t(\beta)/\partial \beta > \partial W_s(\beta)/\partial \beta$.

**Proof of proposition 4 and corollary 1**

Let us denote by $\Delta B^i_H$ for $j = s, t$ the patentholder’s profit difference when there is contributory liability and when there is not. When the outcome is a trial, $\Delta B^i_H = d_H(\beta) - d_H(0)$ which we can rewrite as

$$\Delta B^i_H(\beta) = p_H[\beta_I(\Pi_I(\beta) - \Pi_I(0)) - m^*_t(\beta)\Phi_t(\beta)(\beta_I - \beta)\gamma + \beta_I\alpha_t(0)m^*_t(0)\Pi_I(0)],$$

where $\Pi_I(\beta) = \Pi_I(m^*_t(\beta), \alpha_t(\beta))$ for $\beta \geq 0$.

When $\beta = 0$, i.e., in case of non-contributory infringement, $\Delta B^i_H = 0$.

The derivative of $\Delta B^i_H$ with respect to $\beta$ can be simplified to

$$\frac{\partial \Delta B^i_H}{\partial \beta} = \Phi_t(\beta)p_Hc(m^*_t(\beta)\Phi_t + \beta \frac{dm^*_t(\beta)}{d\beta}),$$

36
where \( dm^*_{t}(\beta)/d\beta < 0 \). For \( \beta = 0 \) the derivative is positive. However, for \( \beta > 0 \) we cannot conclude. Let us study the function \( \Phi_t(\beta) = (2 - p_H)/(2 - p_H(1 + \beta)) \). It is an increasing and convex function with an asymptote at \( \beta_a = (2 - p_H)/p_H \). Indeed, \( \lim_{\beta \to \beta_a} \Phi_t(\beta) = +\infty \). Using that, we can easily derive that \( \lim_{\beta \to \beta_a} \Delta B^t_H = -\infty \). Thus, the function \( \Delta B^t_H(\beta) \) is first increasing, starting from 0, and then will eventually decrease. Thus, starting from 0 and increasing \( \beta \) yields a higher benefit for the patent holder when contributory liability exists. Therefore, there exists a value of \( \beta, \overline{\beta}_t, \) that satisfies \( \overline{\beta}_t > \beta_t^* > 0 \), and such that \( \Delta B^t_H = 0 \) for \( \beta = \overline{\beta}_t \) and \( \Delta B^t_H \) is at a maximum for \( \beta = \beta_t^* \), where \( \beta_t^* = \arg\left(\frac{\partial}{\partial \beta}d_H(\beta) = 0\right) \).

When the outcome is a settlement, the difference in profit is \( \Delta B^s_H = G_H(\beta) - G_H(0) \), which can be reduced to

\[
\Delta B^s_H = L^{NBS}(\beta) - L^{NBS}(0).
\]

When \( \beta = 0, \Delta B^s_H = 0 \). The derivative of \( \Delta B^s_H \) with respect to \( \beta \) is zero, but the derivative of \( L^{NBS}(\beta) \),

\[
\frac{\partial L^{NBS}(\beta)}{\partial \beta} = p_H(\rho + \beta_I)\frac{\partial \Pi_t(\beta)}{\partial \beta} - p_H\gamma \frac{dm^*_t(\beta)}{d\beta} \left( (1 + \beta + (\beta_I - \beta)) + p_H \frac{dm^*_t(\beta)}{d\beta} (1 - \rho) \right).
\]

For \( \beta = 0, \partial L^{NBS}(\beta)/\partial \beta > 0 \). Furthermore, the second order condition

\[
\frac{\partial^2 L^{NBS}(\beta)}{\partial \beta^2} = p_H(\rho + \beta_I)\frac{\partial^2 \Pi_t(\beta)}{\partial \beta^2} - p_H\gamma \frac{d^2 m^*_t(\beta)}{d\beta^2} \left( (1 + \beta + (\beta_I - \beta)) + p_H \frac{d^2 m^*_t(\beta)}{d\beta^2} (1 - \rho) \right)
\]

is negative if we assume that \( d^2 m^*_t(\beta)/d\beta^2 \geq 0 \). This implies that we need to put more structure on the function \( \Pi_t(m) \). Thus, there exists a level of \( \beta > 0, \beta^*_s \) such that \( \Delta B^s_H \) reaches a maximum, as the function is first increasing and concave. There also exists \( \overline{\beta}_s > 0 \) such that \( \Delta B^s_H (\overline{\beta}_s) = 0 \).

**Proof of corollary 2**

The structure \( B' \) (where \( \beta_t^* > 0 \) and \( \beta = 0 \)) is such that the compensation received by the patent holder under both regimes is identical. At the equilibrium, the outcome is a settlement, and therefore, the optimal level of license fee received under the non-contributory infringement rule, \( L^{NBS}(0, \beta_I^*) \), must be equal to the level of the license fee under the contributory infringement rule, \( L^{NBS}(\beta, \beta_I) \). Let us denote \( m^*_t(\beta, \beta_I) \) the optimal network size under the contributory
infringement rule, and $m^*_s(0)$ under non-contributory infringement rule, as the optimal size does not depend on $\beta_I$ anymore. Hence, the value of $\beta'_I$ that satisfies

$$L^{NBS}(0, \beta'_I) = L^{NBS}(\beta, \beta_I)$$

is

$$\beta'_I = \frac{(\Pi_I(m^*_s(\beta, \beta_I)) - \gamma m^*_s(\beta, \beta_I))(\rho + \beta_I) + p_H(1 - \rho)\beta m^*_s(\beta, \beta_I) - \rho}{\Pi_I(m^*_s(0)) - \gamma m^*_s(0)}.$$

**Proof of proposition 5 and corollary 3**

At the equilibrium, the expected payoff of the infringer is $G^*_I(\beta, \beta_I, m^*_s(\beta, \beta_I))$, where $m^*_s(\beta, \beta_I)$ satisfies equation (10). The infringer is indifferent between infringing or not if $G^*_I(\beta, \beta_I, m^*_s(\beta, \beta_I)) = 0$. The derivative of $G^*_I(.)$ with respect to $\beta_I$ is

$$\frac{dG^*_I}{d\beta_I} = \frac{\partial G^*_I}{\partial \beta_I} + \frac{\partial G^*_I}{\partial m^*_s} \frac{\partial m^*_s}{\partial \beta_I}.$$

Using the envelope theorem, only the direct effect matters, and thus,

$$\frac{dG^*_I}{d\beta_I} = -p_H(\Pi_I(m^*_s) - \gamma m^*_s) < 0.$$

Hence, the expected payoff of the infringer is a decreasing function of $\beta_I$. As long as $G^*_I(0, 0, m^*_s(0, 0)) > 0$, there exists a value of $\beta_I$ that we denote $\beta^*_I$ for which $G^*_I(\beta, \beta_I, m^*_s(\beta, \beta_I)) = 0$. Furthermore, this value is a function of $\beta$. The total differentiation of $G^*_I(.)$ gives

$$\frac{dG^*_I}{d\beta_I} = \frac{\partial G^*_I}{\partial \beta_I} d\beta_I + \frac{\partial G^*_I}{\partial \beta} d\beta = 0,$$

and thus,

$$\frac{d\beta_I}{d\beta} = -\frac{\frac{\partial G^*_I}{\partial \beta}}{\frac{\partial G^*_I}{\partial \beta_I}},$$

where $\partial G^*_I/\partial \beta_I < 0$ as defined above and

$$\frac{\partial G^*_I}{\partial \beta} = \frac{\partial G^*_I}{\partial \beta} + \frac{\partial G^*_I}{\partial m^*_s} \frac{\partial m^*_s}{\partial \beta}.$$

Using the envelope theorem,

$$\frac{\partial G^*_I}{\partial \beta} = -\gamma m^*_s p_H(1 - \rho) < 0,$$

we can conclude that

$$\frac{d\beta_I}{d\beta} < 0.$$

Therefore, for a given $\beta_I = \beta^*_I$, there exists a unique $\beta^* = \beta_I^{-1}(\beta^*_I)$.