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Informational Externalities, Strategic Delay, and the Search for Optimal Policy

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Abstract

This paper examines optimal policy when agents, private investors and a government, can learn about the economy by observing others. Investors can delay investment in order to exploit future information. Importantly, investors ignore the informational value of their actions to others when deciding: this externality results in inefficiently high delay, motivating government intervention. The government searches for the optimal policy, while learning about the economy. Complications arise since investors are aware of any systematic component to policy and may respond perversely to government initiatives. The paper characterizes the optimal government policy and shows that the government achieves its desired outcome.

KEYWORDS: Optimal Policy, Investment, Informational Externalities, Strategic Delay.

JEL CLASSIFICATION: D 83, E 60.

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1 Introduction

When policy makers operate under uncertainty, policy may play a dual role: policy initiatives influence outcomes in the economy and simultaneously generate information for policy makers. This dual role of policy can lead to rich interactions between policy makers and private agents, which complicate the policy making process. This paper studies the effects of interactions between private investors and the government when both seek to learn about the economy by observing the actions of others.

The paper takes as a starting point the model of Chamley & Gale (1994), in which private investors have the option to undertake an irreversible project with uncertain profitability. Each investor receives a private signal about the profitability of investing, and furthermore investors who delay are able to infer the private information of others by observing their actions. Investors who invest quickly provide others with valuable information on which to base subsequent decisions, but these investors cannot internalize the informational value that such a decision has to others. Essentially, there exists an informational externality in the investment process. As a result, too many investors wish to adopt a wait and see strategy, which results in inefficiently low investment.

The paper examines the problem of a policy maker operating in this environment, who can influence investment decisions through an investment subsidy financed by non-distortionary taxation. Like the investors, the policy maker lacks full information, as is likely to be the case when policy makers work in settings where uncertainty is an important feature of the economic environment. The nature of the inefficiency in the economy means that the policy maker will try to increase the speed at which investors invest, but any intervention has to account for the fact that the policy maker does not know the optimal level of investment, and thus does not know what level of investment subsidy to set.

In order to overcome this informational limitation, the policy maker can employ strategies which involve choosing a subsidy, observing the outcome and then re-optimizing the subsidy to reflect any new information. As observed by Caplin and Leahy (1996), however, when policy makers adopt such strategies, they may inadvertently provide rational investors with incentives to respond perversely to policy initiatives. To the extent that investors expect future subsidies to be high, investors have an additional incentive to delay investment.

Caplin and Leahy motivate their paper with the experiences of U.S and German monetary policy makers in the early nineties. The argument is that monetary policy makers at that time, confronted with recession but unsure of the required stimulus and afraid of over-stimulating and inducing inflation, gradually lowered interest rates. These gradual interest rate cuts were not successful at stimulating the economy as agents, aware of the policy makers' 'search' strategy, were tempted to postpone action in hopes of benefitting from the lower interest rates they anticipated in the future.

The central question of this paper is to ask how this sort of strategic behavior by investors affects policy making in uncertain environments where both agents and policy makers learn over time. In this paper, unlike Caplin and Leahy's original treatment where differences in the preferences of investors and the government motivate government action, policy intervention is motivated by the existence of inefficient delay. The paper asks whether the existence of strategic investors renders policy making ineffective or impedes policy makers in implementing fully efficient outcomes or whether policy makers are able to compensate for this effect and achieve efficiency. The paper also examines the effect of this interaction on the optimal level of the investment subsidy.

The paper first examines the social optimum, modelled as the outcome that all investors would agree to were they able to commit to their actions ex-ante. The paper shows that the social optimum involves a increase in the speed of investment relative to the "laissez-faire" outcome. There is still some delay in the 'first best' equilibrium, but this represents the optimal balancing of the costs of delaying with the benefits of acting with superior information.

Recognizing the inefficiency of the "laissez-faire" outcome, the policy maker wishes to subsidize early investment. However, investors know that the policy maker will likely also subsidize investment in the future and thus have an incentive to ignore the initial policy offering. The policy maker recognizes this and in equilibrium is able to set a subsidy which compensates the investors not only for the informational value of early investment, but also for any future subsidies which they forego by investing sooner. The result is that, in equilibrium, the policy maker is able to induce investors to invest sooner that they would do in the absence of intervention.

The policy maker achieves more than a partial reduction in delay. The main result of the paper is that the policy maker is able to induce the socially optimal investment profile as an equilibrium outcome. The policy maker is able to achieve this 'first best' outcome without access to any commitment mechanism.

While the strategic behavior of investors does not prevent the policy maker from inducing the socially optimal investment profile, this does not mean that strategic interactions between investors and the policy maker are unimportant or non-existent. As noted above, the policy maker has to compensate investors for the future subsidies that they forego by investing earlier. This means that the optimal subsidy exceeds the subsidy that would be required were investors to behave more myopically, making policy intervention more expensive than would otherwise be the case. The reason that this does not prevent the policy maker from obtaining the efficient level of delay in this case is that the investment subsidy is financed with non-distortionary taxes. In other cases, the strategic response of investors creates an additional cost to the policy maker of reducing delay.

Models of informational externalities with the endogenous timing of decisions have been applied to a variety of economic problems. Chamley & Gale (1994), Schivardi (2000) argue that strategic resulting from informational externalities may help understand business cycle fluctuations. González (1997) emphasizes the interaction of individual experimentation and learning from others to explain the slowness of economic recoveries. Thimann & Thum (1998) argue that the possibility of learning by observing others induces foreign firms to delay investment in emerging economies, a claim which finds some empirical support in Kinoshita & Mody (2001). Caplin & Leahy (1993), Lee (1998), Chari & Kehoe (2000) and Chamley (2001) emphasize the role of informational externalities and delay in understanding market crashes and financial crises. Despite the interest in these models, and the well known potential for inefficient equilibria, questions of policy making in these environments remain poorly understood. The contribution of this paper is to begin to fill this gap in the literature.

The paper is also related to Smith & Sørensen (1997), who examine optimal policy in the exogenous sequencing, informational externalities framework of Banerjee (1992) and Bikhchandani, Hirshleifer & Welch (1992). Rob (1991) studies a social planner's problem in an economy exhibiting informational externalities, in which firms choose whether or not to enter a market based on observations of previous prices and quantities. Finally Bertocchi & Spagat (1993) and Wieland (2000) study optimal experimentation on the part of monetary policy makers, though not in social learning environments. These papers, however, do not explore the kinds of strategic interactions between policy makers and private agents that are the main focus here.

The paper proceeds as follows: Section 2 presents the benchmark model and discusses the problem of an investor both in the presence and absence of an investment subsidy. Section 3 presents the problem of a benevolent social planner in this environment, and shows that the social optimum involves a reduction in delay relative to the "laissez-faire" outcome. Section 4 examines the possibility for constructive policy in the economy and demonstrates that a policy maker is able to induce the socially optimal profile of investment. Section 5 discusses the results and offers concluding comments.

2 A Social Learning Economy

This section analyzes an economy in which a set of investors have the opportunity to make an irreversible investment in project with uncertain profitability. The model is a version of Chamley & Gale's (1994) model of information revelation and strategic delay, in which the investors obtain information about the profitability of an investment project by observing the actions of other investors. The ability to learn by observing others creates an informational externality: investors do not take into account the value of the information their actions reveal to others when choosing their optimal action. This externality results in excessive delay in equilibrium, as investors wait for others to test the waters before making their own decisions. Investors only interact informationally in this model, meaning that there are no network or congestion effects associated with the investment of others.

The Chamley and Gale model is extended to the case where investors possess idiosyncratic costs of starting a project. Heterogeneity of investors gives rise to a pure strategy equilibrium in which all investors with a low costs invest quickly while those with higher costs prefer to wait. This means that additional information can only be generated by inducing investors whose projects are more likely to be unprofitable to invest. Therefore, a policy maker will wish to intervene in the economy on an ongoing basis, rather than opting for a one-period-only intervention to generate a lot of investment and information at the start of the game. This characteristic of the environment allows for the kinds of strategic interactions between investors and the policy maker identified by Caplin & Leahy (1996) that are the focus of the paper.

2.1 The Model

Consider a game in which there are N investors, where N is given and known to all. Of the N players, n receive a project in which they have the option to invest, where n is a random variable drawn from a known distribution $G_o(n)$, with associated density $g_o(n)$. The realization of n is not observed.

The n projects are randomly assigned amongst the N investors. Conditional on the realization of n, each player has an equal chance, n/N, of receiving a project. A player who does receive a project uses Bayes' Theorem to derive posterior beliefs on n as follows:

$$g(n) = g_o(n) \cdot n[\sum_{n'=0}^{N} g_o(n') \cdot n']^{-1}. \tag{2.1}$$

An investor who receives a project chooses whether or not to invest at any time $t = 1, 2, \ldots, \infty$, subject to the constraints that investment is irreversible and a given investor can invest at most once. investors who do not receive projects do nothing, trivially.

Investment generates revenue $r(n)$, which is common across all projects. Revenue is strictly increasing in n, the unobserved state of the economy.[1] Notice that revenue depends on the number of available projects and not the number of projects that are actually undertaken. This means that the revenue of investing does not depend directly upon the investment decisions of others, and that investors only interact informationally. This assumption allows us to isolate the informational effect without introducing additional complications that arise when network or congestion externalities are present.

An investor with a project must also pay a cost c upon investing in the project. investors have idiosyncratic investment costs, drawn independently from a known distribution $F(c)$, with associated density $f(c)$ and support $[\underline{c}, \overline{c}]$. An investor's cost is private information and is realized after the investor receives a project but before any decisions are made.

Finally, an investor who invests may receive an investment subsidy, s. The subsidy is not conditional on either n or c.

Investors discount the future at rate δ, so that an investor with cost c who invests at date t, given n and s, receives a payoff of $\delta^{t-1}(r(n) + s - c)$. An investor who never invests

[1] For illustration, imagine a firm that is aware of a potential investment project in an emerging economy. The revenue generated by investing in this project will depend on a number of characteristics of the emerging economy (demand, infrastructure, corruption, etc...) about which the firm is imperfectly informed. The firm might reasonably conclude that, in an economy with high demand, good infrastructure, honest politicians and so on, there will be many other opportunities for investment, while in an economy without these characteristics there will be few such opportunities. In other words, the characteristics of the economy that make investment more profitable are correlated with the number of potential projects.

receives a payoff of zero. Agents are risk neutral and choose the timing of any investment to maximize expected profits.

Whether or not a player has a project is not observable, but his actions are. Let $x_t = (x_{1t}, \ldots, x_{Nt})$ denote the decisions made by the N players at date t, where $x_{it} = 1$ if investor i invests at t and $x_{it} = 0$ if investor i does not invest. The history of the game at period t is a sequence of observed investment decisions $h_t = \{x_1, x_2, \ldots, x_{t-1}\}$. Let H_t denote the set of possible histories at time t, where $H_1 = \{\emptyset\}$ is the initial history and $H = \bigcup_{t=1}^{\infty} H_t$ is the set of all histories.

The set of subsidies, $\mathcal{S}$, assigns a subsidy for every possible history. A typical element is $s(h_t)$.

The following sections look for symmetric equilibria, in which an investor's actions depend only on his type and the publicly observed history of the game. In a symmetric equilibrium only the number of investment decisions in each period matter, so we can write a history h_t as a sequence $(k_1, k_2, \ldots, k_{t-1})$, where k_τ is the number of investments observed at τ.

A strategy of an investor is a function $\gamma : H \times C \to [0, 1]$. A probability assessment of an investor is a function $p : H \times N \to [0, 1]$, where $p(n|h_t)$ is the probability that n investors have options conditional on the history h_t.

A perfect Bayesian equilibrium consists of a strategy and a probability assessment such that i) each player's strategy is a best response at every information set and ii) the probability assessments are consistent with Bayes' rule at every information set reached with positive probability.

2.2 Cutoff Rules

Lemma 1 *For a fixed but arbitrary set of subsidies conditioned on the history but not on an investor's cost type, an investor's payoff of investing relative to waiting is decreasing in c.*

Proof See Appendix. ‖

Since investors of differing costs possess the same beliefs and face the same subsidies, investors only play different strategies to the extent that they have different costs. Since investors discount the future, investors with lower costs have more to lose by waiting than investors with higher costs and therefore have more incentive to invest early.

Lemma 1 implies that investors play cutoff rules of the form:

$$\text{invest at t iff } c \leq c^*(h_t; \mathcal{S}),$$

where c is the investor's type and $c^*(h_t; \mathcal{S})$ is some critical value depending on the history and the set of subsidies. Therefore, the equilibrium will take the form of a cutoff cost value in each period and a corresponding realization of the number of projects that are undertaken, k_t. Common knowledge and the fact that h_t and $\mathcal{S}$ are public information implies that all investors can calculate these cutoffs.

2.3 Beliefs and Expected Payoffs

Expected revenue for an investor investing in a project depends on the investors beliefs about n. Using Equation (2.1), it is clear that expected revenue at $t = 0$ is given by:

$$R = \sum_{n=0}^{N} g(n) \cdot r(n), \tag{2.2}$$

and expected profits are therefore $R + s(h_0) - c$.

Expected revenue in subsequent periods depends on the evolution of investors' beliefs on n. Let $R(h_t)$ denote the expected revenue from investing at history h_t. Then,

$$R(h_t) = \sum_{n} p(n|h_t) \cdot r(n), \tag{2.3}$$

where $p(n|h_t)$ represents the investor's beliefs over n at h_t. Un-discounted expected profits from investing at h_t therefore equal $R(h_t) + s(h_t) - c$.

The fact that investors play cutoff rules in equilibrium implies that at a given history, h_t all remaining investors will have costs above the cutoff of the previous period, denoted by $\tilde{c}(h_{t-1})$. Observe that $\tilde{c}(h_{t-1})$ is the lowest possible cost of an investor who has not yet invested at h_t. Then, for a cutoff value $\tilde{c}(h_t)$, such that an investor invests this period if and only if his or her cost c is less that $\tilde{c}(h_t)$,

$$\mathcal{F}(\tilde{c}(h_t)) = \int_{\tilde{c}(h_{t-1})}^{\tilde{c}(h_t)} f(c|\tilde{c}(h_{t-1}))dc = \frac{F(\tilde{c}(h_t)) - F(\tilde{c}(h_{t-1}))}{1 - F(\tilde{c}(h_{t-1}))}, \tag{2.4}$$

is the probability that an investor with an option who has not yet invested invests this period.

If we define $K(h_t)$ as the total number of investments that have been made at h_t, that is $K(h_t) = \sum_{\tau=0}^{t-1} k_\tau$, then the probability of observing k_t investment decisions at h_t, given $\tilde{c}(h_t)$, follows the following binomial distribution:

$$\begin{aligned} p(k_t|n, h_t; \tilde{c}(h_t)) &= \binom{n - K(h_t) - 1}{k_t} [\mathcal{F}_t(\tilde{c}(h_t))]^{k_t} \cdot [1 - \mathcal{F}_t(\tilde{c}(h_t))]^{n-K(h_t)-k_t-1} \\ &= b(k_t|n-1, h_t; \tilde{c}(h_t)). \end{aligned} \tag{2.5}$$

Agents use Bayes' rule to update beliefs, resulting in the following posterior assessment:

$$\begin{aligned} p(n|h_t) &= \frac{b(k_{t-1}|n-1, h_{t-1}; \tilde{c}(h_{t-1})) \cdot p(n|h_{t-1})}{\sum_{n'=0}^{N} b(k_{t-1}|n'-1, h_{t-1}; \tilde{c}(h_{t-1})) \cdot p(n'|h_{t-1})} \\ &= \frac{b(k_{t-1}|n-1, h_{t-1}; \tilde{c}(h_{t-1})) \cdots b(k_1|n-1, h_1; \tilde{c}(h_1)) \cdot g(n)}{\sum_{n'=0}^{N} b(k_{t-1}|n'-1, h_{t-1}; \tilde{c}(h_{t-1})) \cdots b(k_1|n'-1, h_1; \tilde{c}(h_{t-1})) \cdot g(n')} \end{aligned} \tag{2.6}$$

The following Lemma describes the relationship between expected revenue and investment activity.

Lemma 2 *Let,*

$$Pr(k|n) = \binom{n}{k}\mathcal{F}^k(1-\mathcal{F})^{n-k} = b(k|n,\mathcal{F}), \tag{2.7}$$

where,

$$g(n|k) = \frac{b(k|n,\mathcal{F}) \cdot g(n)}{\sum_{n'=0}^{N} b(k|n',\mathcal{F}) \cdot g(n')}. \tag{2.8}$$

If $0 < \mathcal{F} \leq 1$ and $g(\cdot)$ is non-degenerate, then $g(\cdot|k)$ is increasing in k in the sense of first order stochastic dominance. This further implies that $R(k) = \sum_n g(n|k) \cdot r(n)$ is increasing in k.

Proof See Chamley and Gale (1994). ‖

Applying Lemma 2 to Equations (2.3) and (2.7) yields that $R(h_t)$ is increasing in k_{t-1}. This means that, holding the probability that a remaining investor will invest fixed, the observation of a high realization of k_{t-1} means that the expected revenue from an investment is high.

The intuition for this is as follows: it is more likely that there will be a lot of investments being made, given $\mathcal{F}$, when the number of investors with a project, n, is high. The observation that k_t is high therefore leads investors to conclude that higher values of n are more likely. Since $r(n)$ is increasing in n the expected revenue from investing is higher after an observation of a high k_t. Notice that there is only an informational effect, and that k_t does not have a direct effect on the payoff of subsequent investors.

2.4 The Value of Delay

The previous section defined the payoff of an investor who invests at a history h_t. This section examines the expected payoff of an investor who chooses to wait at h_t, but makes an irrevocable decision to invest or not invest in the next period (after observing the realization of k_t). This turns out to be a useful object for calculating the equilibrium cutoff.

Define $W(c, \tilde{c}, h_t; S)$ as the un-discounted payoff to an investor with cost type c who waits one period at h_t before making an irrevocable decision when others play the cutoff $\tilde{c}$ (that is, other investors invest if their costs are less than $\tilde{c}$). Such a player will receive $\mathbf{max}\{R(h_{t+1}) + s(h_{t+1}) - c, 0\}$, where $h_{t+1} = \{h_t, k_t; \tilde{c}\}$, so that that player's un-discounted payoff from waiting is:

$$W(c, \tilde{c}, h_t; S) = \sum_{k_t} p(k_t|h_t, \tilde{c})\mathbf{max}\{R(h_{t+1}) + s(h_{t+1}) - c, 0\}, \tag{2.9}$$

where $p(k_t|h_t, \tilde{c})$ is the probability that k_t people invest at h_t given $\tilde{c}$.

Lemma 3 *For any fixed but arbitrary symmetric Perfect Bayesian Equilibrium where $s(h_t) = 0 \ \forall \ h_t \in H$, the value of delay to an investor indifferent between investing and waiting at any h_t corresponds to the value of waiting at h_t and subsequently making an irrevocable decision to either invest or never invest at h_{t+1}.*

Proof See Appendix. $\|$

The intuition for this result is as follows: An investor who is exactly indifferent between investing and waiting at some period t and decides to not to invest now has the lowest cost type of all remaining investors (by Lemma 1). If this investor does not find it optimal to invest at period $t+1$ then, again by Lemma 1 no one else will find it profitable to invest either. Therefore such an investor has no chance of learning by waiting at $t+1$. If he finds it unprofitable to invest given the information at $t+1$, then he must also find it unprofitable to invest at $t+2$, since his information is the same. By induction, we can see that such an investor will never invest if he does not invest at $t+1$.

Lemma 3 implies that, in the absence of subsidies, the option value of delay can be characterized as follows. Observe that an investor making a decision to invest at h_t or never invest at all would invest if $V(h_t) - c = \sum_n p(n|h_t) \cdot (r(n) - c) \geq 0$. It is immediate from the definition of $W(c, \tilde{c}, h_t; S = 0)$ that $W(c, \tilde{c}, h_t; S = 0) \geq R(h_t)$. It follows that $W(c, \tilde{c}, h_t; S = 0) > R(h_t) - c$ if $R(h_{t+1}) - c < 0$ for some possible values of k_t. Since $R(h_{t+1})$ is increasing in k_t (by Lemma 2), $R(h_{t+1}) - c < 0$ for $k_t = 0$ whenever $W(c, \tilde{c}, h_t; S = 0) > R(h_t) - c$. This further implies that it is optimal for the investor indifferent between waiting and investing at h_t to delay at $t+1$ if $k_t = 0$. These results can be summed up in the following Lemma.

Lemma 4 *If $W(c, \tilde{c}, h_t; S = 0) > R(h_t) - c$ there exists $0 < k^* \leq N - K(h_t)$ such that $R(h_{t+1}) - c < 0$ if $k_t < k^*$ and $R(h_{t+1}) - c \geq 0$ if $k_t \geq k^*$. The investor indifferent between waiting and investing at h_t to will only invest at $t+1$ if at least one investor invests at h_t.*

The intuition is that investors only wait in order to observe the actions of others because these actions contain valuable information. Implicitly, this means that the decision of an investor who waits depends non trivially on the information he receives, otherwise the investor would not delay. An observation of no investment in a given period is the worst possible news. An investor who delays and observes the worst possible news concludes that it is not worth investing.

The intuition of Lemma 3, that an investor only waits in order to learn, only learns when other people invest, and can not expect to learn anything if he has the lowest remaining cost type, is only valid when there are no subsidies. If at $t+1$ the investor expects that a future subsidy will be sufficiently great that it compensates for the losses (due to discounting) of waiting, then he will wait. Furthermore, given that such an investor cannot hope to learn by waiting, an expectation of such a future subsidy is the only reason that the investor will wait at $t+1$ and yet invest at some future date. This leads to:

Corollary 1 *An investor indifferent between investing and waiting at h_t finds it optimal to not make a once and for all decision at $t+1$ if and only if he is expecting a sufficiently great subsidy at some date $t+j$ where $j > 0$.*

It is useful to categorize sets of subsidies depending on whether, for a given set of subsidies, indifferent investors ever find it optimal to not make this sort of once and for all decision in the next period. The following Restriction expresses this idea.

Restriction 1 *A set of subsidies, $\tilde{S}$, is said to satisfy Restriction 1 if it is the case that given $\tilde{S}$, at any fixed but arbitrary symmetric Perfect Bayesian Equilibrium, the value of delay to an investor indifferent between investing and waiting at any h_t corresponds to the value of waiting at h_t and subsequently making an irrevocable decision to either invest or never invest at h_{t+1}.*

2.5 The Informativeness of Investment

Recall that for arbitrary cutoff $\tilde{c}$, $\mathcal{F}(\tilde{c})$ measures the probability that an investor with an unknown cost invests this period. In a precise sense, $\mathcal{F}$ measures the amount of information revealed by observing the number of investors who invest. If $\mathcal{F} = 0$ (in which case $\tilde{c} = c_l$) then no information is revealed. If $\mathcal{F} = 1$ (in which case $\tilde{c} = \overline{c}$) then the true value of n.

Let X and X' denote the random number of investors who invest when the probabilities are $\mathcal{F}'$ and $\mathcal{F}$ respectively. It can be shown (see Chamley and Gale (1994), pg. 1070) that when $\mathcal{F}' < \mathcal{F}$, X' is equal to X plus noise, so that observing X is more informative than observing X' in the sense of Blackwell. This implies that as $\mathcal{F}$ increases from zero to one, the amount of information revealed increases. Since $\mathcal{F}$ is increasing in $\tilde{c}$, this means that the amount of information revealed increases as the cutoff, $\tilde{c}$ increases from c_l to $\overline{c}$.

Lemma 5 *$W(c, \tilde{c}', h_t; S) \leq W(c, \tilde{c}, h_t; S)$ whenever $\tilde{c}' < \tilde{c}$. $W(c, \tilde{c}, h_t; S)$ is strictly increasing in $\tilde{c}$ whenever $\mathcal{F}$ is strictly increasing in $\tilde{c}$ and $W(c, \tilde{c}, h_t; S) > R(h_t)$.*

Proof See Appendix. ||

2.6 Equilibrium with no Subsidies

Recall $W(c, \tilde{c}, h_t); S = 0$, the un-discounted value of waiting one period before making a once and for all decision at date $t + 1$ when there are no subsidies. We can rewrite this as follows:

$$\begin{aligned} W(c, \tilde{c}, h_t; S = 0) &= \sum_{k_t=0}^{N-1-K(h_t)} p(k_t|h_t, \tilde{c}) \mathbf{max}\{R(h_{t+1}) - c, 0\} \\ &= \sum_{k_t=0}^{N-1-K(h_t)} \mathbf{max}\left\{\sum_{n=0}^{N} b(k_t|n-1, h_t, \tilde{c}) \cdot p(n|h_t) \cdot [r(n) - c],\ 0\right\}, \quad (2.10) \end{aligned}$$

where $p(n|h_t)$, $b(k_t|n - 1, h_t, \tilde{c})$ and $K(h_t)$ are as previously defined.

The evolution of investment in the economy is as follows.

Proposition 1 *At any h_t in equilibrium one of three things will happen:*
i) $R(h_t) - c^*(h_{t-1}) \leq 0$, in which case $c^*(h_t) = c^*(h_{t-1})$,
ii) $R(h_t) - \overline{c} > \delta W(\overline{c}, \overline{c}, h_t; S = 0)$, so that $c^*(h_t) = \overline{c}$, or

iii) neither i) nor ii) obtains, in which case $c^*(h_{t-1}) < c^*(h_t) < \overline{c}$ and $c^*(h_t)$ is uniquely determined by:

$$R(h_t) - c^* = \delta W(c^*, c^*, h_t; S = 0). \tag{2.11}$$

Proof By Lemma 3 we need only compare $V(h_t) - c$ with $\delta W(c, c, h_t; S = 0)$ to calculate an equilibrium cutoff.

Define $\pi(c, h; S = 0) = \Big(\delta W(c, c, h_t; S = 0) - [R(h_t) - c]\Big)$. By Lemma 3, an equilibrium occurs when $\pi(c, h; S = 0) = 0$. By Lemma 1, $\pi(c, h; S = 0)$ is decreasing in c. Continuity of $\pi(c, h; S = 0)$ implies that there is a c that solves $\pi(c, h; S = 0) = 0$, and the fact that$\pi(c, h; S = 0)$ is decreasing in c implies that this solution is unique. If the value $\hat{c}$ which solves this lies between $c^*(h_{t-1})$ and $\overline{c}$ then it is the unique equilibrium cutoff.

It remains to characterize 'corner' solutions. If $R(h_t) - c^*(h_{t-1}) \leq 0$ then no remaining investor expects to profit by investing, given beliefs at h_t. As a result, no new information will be forthcoming, and the value of delay is zero . Therefore the cutoff will be $c^*(h_t) = c^*(h_{t-1})$.

If $R(h_t) - \overline{c} > \delta W(\overline{c}, \overline{c}, h_t; S = 0)$, then the value of investing today exceeds the value of delay even for the highest cost investor. Notice that this is the case even though, with $\tilde{c} = \overline{c}$, the value of delay is maximized, since all information will be revealed to any investor who delays one period. This implies the cutoff will be $\overline{c}$. $\|$

Note that there is delay in equilibrium. That is, some investors with projects whose expected value is positive at h_t choose not to invest in period t. This can be seen by observing that the value of waiting, $W(c, \tilde{c}, h_t)$, is greater than or equal to zero, and strictly positive if the return to investment is negative for some states of the world which are possible at h_t (recall Lemma 5).

It is also clear that the equilibrium cutoff is determined by investors who do not take into consideration the fact that a decision to invest reveals information that is of value to investors who have yet to make their decision. Individual investors balance the returns to investing immediately against the fact that they may learn if they wait, but are not able to internalize the informational value of their actions. In this sense, there is an informational externality which causes the set of investors who decide to invest in any period to be lower than would be the case if investors could capture all of the returns to investing. It is this externality that justifies policy intervention, since at any given history, a policy maker will wish to reduce delay by encouraging a larger set of investors to invest.

Finally, Lemma 4 implies that the game will end after a finite number of periods. Since the only reason an investor ever chooses to delay is to gain further information, it must be the case that if he learns that the worst possible outcome has occurred, that is if no one invests, he will not invest. Therefore, investment only continues if at least one person invested in the previous period. Since there are a finite number of players, the game must have a finite number of periods.

Note that finiteness depends on the fact that a low number of investors is 'bad news' and a high number of investors represents 'good news'. One can model social learning in such a way that this is not the case; that is, that more activity implies more accurate information, but not necessarily better news. Most of the properties of the equilibrium will be preserved in such cases, but the game need not be finite. Zhao (2001), for example, studies a model in which investors delay strategically in hopes of learning by observing others. In his paper, however, a different structure on the signalling process gives rise to an equilibrium in which investment can stop and restart.

2.7 Equilibrium with Subsidies

We can apply the same logic to the case where investors face an arbitrary set of subsidies $\tilde{\mathcal{S}}$. Assume that the set of subsidies $\tilde{\mathcal{S}}$, whose elements are denoted $\tilde{s}(h_t)$, satisfies Restriction 1. Then the value of delay to an indifferent investor at h_t can be calculated by corresponds to the value of waiting at h_t and subsequently making an irrevocable decision to either invest or never invest at h_{t+1}. In this case, the logic of the previous section applies and the game evolves as follows.

Proposition 2 *At any h_t in equilibrium one of three things will happen:*
i) $R(h_t) + s(h_t) - c^s(h_{t-1}) \leq 0$, in which case $c^s(h_t) = c^s(h_{t-1})$,
ii) $R(h_t) + s(h_t) - \bar{c} > \delta W(\bar{c}, \bar{c}, h_t; \{\tilde{s}(h_{t+1})\})$, so that $c^s(h_t) = \bar{c}$, or
iii) neither i) nor ii) obtains, in which case $c^s(h_{t-1}) < c^s(h_t) < \bar{c}$ and $c^*(h_t)$ is uniquely determined by:

$$R(h_t) + \tilde{s}(h_t) - c^s = \delta W(c^s, c^s, h_t; \{\tilde{s}(h_{t+1})\}). \tag{2.12}$$

Proof Follows from the arguments for Proposition 1. ‖

Note that it is not necessarily the case that there is delay in the equilibrium with subsidies. If subsidies in early periods are high relative to those in subsequent periods, investors will have incentives to invest sooner in order to enjoy the higher subsidies and this may outweigh the desire to delay in order to learn. Alternately, if future subsidies are high relative to the current subsidy then a greater set of investors will desire to wait than would be the case were there no subsidies.

It is worth pointing out that the assumption that $\tilde{\mathcal{S}}$ satisfies Restriction 1 rules out situations in which $c^*(h_{t+1}) > c^*(h_t) = c^*(h_{t-1})$. In other words, the assumption guarantees that the marginal investor finds it optimal to compare investing today with delaying exactly one period before making an irrevocable decision. This does not, however, rule out cases where $c^*(h_{t-1}, k_t) > c^*(h_{t-1})$ for $k_t = 0$. In other words, if $\tilde{s}(h_{t-1}, k_t = 0) > 0$ then it is possible that the subsidy induces investment subsequent to an outcome in which there was no investment in a given period. This means that, in the presence of subsidies, the game need not be finite, even if Restriction 1 is satisfied.

3 A Planner's Problem

The objective of this paper is to study the problem faced by a policy maker in the economy of Section 2. In order to gauge the efficacy of such a policy maker, it is useful to first describe a benchmark against which this performance can be measured.

The choice of an efficiency benchmark is not immediately clear. One could compare the "laissez-faire" equilibrium to the case where all information is revealed at the beginning of the game. While this clearly maximize profits for all investors, it is trivially true that a policy maker will never be able to achieve this outcome in the absence of information superior to that of the private investors.

This paper considers a welfare benchmark that respects the informational constraints of the economy outlined in the previous section. That is, where n is not observed and inferences on n can only be made by observing the history of outcomes. In essence, the previously described benchmark measures the combined effect of the uncertainty and the externality while this second benchmark isolates the effect of the informational externality. Adopting the latter as the efficiency benchmark against which to compare the performance of policy is appropriate to the extent that policy makers cannot eliminate the underlying economic uncertainty but can influence the process by which investors learn.

Consider a situation in which all investors gather at the start of the game, prior to the realization of n, and commit to their future actions by means of a binding contract. These contracts specify the action of an investor contingent on his cost type for every possible observed history of the game. It is important that investors commit to their future actions prior to the realization of cost types. This means that they take into account the informational value of early investment to investors whose cost types are such that they make their decisions later.

At this ex-ante stage all investors are identical. Since investors are risk neutral, they agree on the contract which maximizes the ex-ante representative investor's expected payoff. This paper take the solution to the problem faced by a planner who maximizes this expected payoff, subject to the informational constraints inherent in the economy, to be the appropriate welfare benchmark.

This section analyzes aspects of the solution to the problem faced by this planner. Since delay in this economy is a result of an informational externality, it is natural to expect that the socially optimal outcome will involve a reduction in delay. This is shown to be the case.

3.1 Beliefs and Preferences of the Planner

The planner knows the probability distribution over n, $G_o(n)$ and the distribution $F(c)$ from which costs are drawn. The planner forms expectations about the state of the economy by observing the history of the game, and maximizes the sum of discounted expected profits.

The planner observes neither the number of investors with an option, n, nor the realized cost type of any individual investor c. Writing down the planner's objective function therefore requires taking expectations over both n and c, as the planner can only calculate the expected payoff of an investment decision.

Assuming that the planner discounts the future at δ, the same rate as the investors, the planner's objective function is:

$$
\begin{aligned}
V &= \sum_{n=0}^{N} g(n) \int_{\mathcal{C}(h_1)} (r(n)-c) f(c) dc \\
&+ \delta \sum_{k_1=0}^{N-1} p(k_1|h_1) \Bigg\{ \sum_{n=0}^{N} p(n|h_2) \int_{\mathcal{C}(h_2)} (r(n)-c) f(c) dc \\
&+ \delta \sum_{k_2=0}^{N-k_1-1} p(k_2|h_2) \Bigg\{ \sum_{n=0}^{N} p(n|h_3) \int_{\mathcal{C}(h_3)} (r(n)-c) f(c) dc \\
&+ \delta \sum_{k_3=0}^{N-K(h_3)-1} p(k_3|h_3) \Bigg\{ \sum_{n=0}^{N} p(n|h_4) \int_{\mathcal{C}(h_4)} (r(n)-c) f(c) dc \\
&+ \ \ldots \Bigg\} \Bigg\} \Bigg\} \Bigg\}
\end{aligned} \tag{3.13}
$$

where $p(n|h_t)$ is given by Equation 2.7.

The planner's problem is to maximize V by specifying a set of cost types for each possible history, $\mathcal{C}(h_t)$, such that an investor invests at h_t if his or her cost is in $\mathcal{C}(h_t)$ and waits at h_t if his or her cost is not in $\mathcal{C}(h_t)$.

3.2 Cutoff Rules

The planner's problem involves choosing a set $\mathcal{C}(h_t)$ for each possible history such that and investor with a project invests if and only if he has some cost c such that $c \in \mathcal{C}(h_t)$. This section provides a first useful result. Namely that the planner will choose a set of contracts in which investment is made according to cutoff rules.

Lemma 6 *The solution to the planner's problem involves cutoff rules. That is, the set takes the form* $[\underline{c}, \hat{c}(h_t)]$*, so that at any history* h_t *an investor with a project invests iff his or her cost is below some critical value* $\hat{c}(h_t)$*.*

Proof See Appendix. ‖

The intuition for this result is similar to the reason that the investors of Section 2 play cutoff rules. Investors with lower costs have more to lose by delaying than investors with higher costs. The planner does influence information generation through the choice of cutoff, but is optimal for the planner to uncover information in the cheapest way possible, which corresponds to choosing that the lowest cost investors are the first to invest.

Lemma 6 implies that the planner's problem can be re-written as:

$$
\begin{aligned}
V \;=\; & \sum_{n=0}^{N} g(n) \int_{\underline{c}}^{c(h_1)} (r(n)-c) f(c) dc \\
& + \delta \sum_{k_1=0}^{N-1} p(k_1|h_1) \left\{ \sum_{n=0}^{N} p(n|h_2) \int_{c(h_1)}^{c(h_2)} (r(n)-c) f(c) dc \right. \\
& + \delta \sum_{k_2=0}^{N-k_1-1} p(k_2|h_2) \left\{ \sum_{n=0}^{N} p(n|h_3) \int_{c(h_2)}^{c(h_3)} (r(n)-c) f(c) dc \right. \\
& + \left. \left. \ldots \right\} \right\} \Big\}
\end{aligned}
\tag{3.14}
$$

The planner's problem is to choose a set of history dependent cutoffs to maximize V. Note that for each possible history h_t the planner picks a profile of cutoffs for h_{t+1} corresponding to every h_{t+1} (i.e every possible k_t) that can follow h_t. The optimal cutoffs will be denoted $\{c^{s*}(h_t)\}_{h_t \in \mathcal{H}}$.

3.3 Recursiveness

Define $V(h_t)$ as those terms of the planner's objective function which apply after a particular history h_t. That is, $V(h_t)$ is the continuation payoff for the planner at date t conditional on having observed history h_t and is given by:

$$
\begin{aligned}
V(h_t) \;=\; & \sum_{n=0}^{N} p(n|h_t) \int_{c_{t-1}}^{c(h_t)} (r(n)-c) f(c) dc \\
& + \delta \sum_{k_t=0}^{N-K(h_t)-1} p(k_t|h_t) \left\{ \sum_{n=0}^{N} p(n|\{h_t,k_t\}) \int_{c(h_t)}^{c(\{h_t,k_t\})} (r(n)-c) f(c) dc \right. \\
& + \delta \sum_{k_{t+1}=0}^{N-K(\{h_t,k_t\})-1} p(k_{t+1}|\{h_t,k_t\}) \left\{ \sum_{n=0}^{N} p(n|\{h_t,k_t,k_{t+1}\}) \int_{c(\{h_t,k_t\})}^{c(\{h_t,k_t,k_{t+1}\})} (r(n)-c) f(c) dc \right. \\
& + \left. \left. \ldots \right\} \right\} \Big\}
\end{aligned}
\tag{3.15}
$$

Let V_t denote all terms of V up to date t, and $V^c(\hat{h}_t)$ denote terms dating from date t following any history of length t, $\hat{h}_t$ except the particular history h_t.

A strategy for the planner's problem is a set $\sigma = \{c(h_1), \{c(h_2)\}, \ldots\}$ which specifies the cutoff cost for every possible history. Let σ_{h_t} denote the subset of history dependent cutoffs which obtain after the first t periods if the particular history h_t is observed. That is $\sigma_{h_t} = \{c(h_t), \{c(\{h_t, k_t\})\}, \ldots\}$.

An optimal strategy for the problem is a strategy, σ^*, for which $V|_{\sigma^*} \geq V|_\sigma$ for all σ. A strategy which is optimal for the continuation of the problem at a given history h_t is a strategy, $\sigma^*_{h_t}$, for which $V(h_t)|_{\sigma^*_{h_t}} \geq V(h_t)|_{\sigma_{h_t}}$. Using these definitions, we can define a strategy, $\hat{\sigma}$, which involves following the optimal plan, σ^*, unless and until history h_t is reached in which case the remainder of the problem is re-solved, thus resulting in the strategy $\sigma^*_{h_t}$ being played subsequent to the observation of history h_t.

Lemma 7 *Given* V_t, $V(h_t)$, $V^c(\hat{h}_t)$, σ^* *and* $\hat{\sigma}$ *as defined above,* $\hat{\sigma}$ *is optimal for the original problem.*

Lemma 7 states that breaking the problem into pieces and solving the continuation problems recursively results in the same solution as optimizing V with respect to all of its arguments simultaneously. This implies that the ability of the planner to commit the optimal cutoffs ex-ante provides no benefit, and that a planner who chooses optimal cutoffs as the game evolves will be able to achieve the globally optimal allocation. Hence the planner's problem can be expressed in the following, recursive, form:

$$\begin{aligned} V(\{h_{t-1}, k_t\}, c_{t-1}) \quad = \quad & \max_c \sum_{n=0}^{N} p(n|h_t; c_{t-1}) \Big\{ \int_{c_{t-1}}^{c} [r(n) - c] f(c) dc \\ & + \delta \sum_{k_t=0}^{N} p(k_t|h_t; c_{t-1}, c) V(\{h_t, k_t\}, c) \Big\} \end{aligned} \tag{3.16}$$

3.4 Finiteness

Given the previous discussion (on the value of information) this section shows that the optimal solution to the planner's problem involves a cessation of investment in finite time.

Lemma 8 *Suppose that at some history* h_t *the optimal cutoff chosen by the planner is* $c^{s*}(h_t)$. *If at* h_t *the planner observes* $k_t = 0$ *then* $c^{s*}(\{h_t, k_t\}) = c^{s*}(h_t)$. *Furthermore,* $c^{s*}(h_\tau) = c^{s*}(h_t)$ *for all* $\tau > t$.

Proof See Appendix. ‖

Like an investor of Section 2, the planner only delays investment because there is a value to delay: namely that the planner can make a more informed decision about whether an investor with a high cost should invest or not by observing the actions of low cost investors. The only reason to have such an investor delay is if the decision in the future depends non-trivially on the observation.

An observation that $k_t = 0$ is the worst possible news because it suggests that n is low, and the planner's payoff is increasing in n. Therefore, the planner chooses to have an investor with cost $c^{s*}(h_t)$, who is indifferent between investing and waiting at h_t delay at $\{h_t, k_t\}$ if $k_t = 0$ is observed. In other words, the planner will only increase the cutoff at date $t+1$ conditional on having observed $k_t \geq 1$. Since there are a finite number of players, this implies that investment must end in finite time.

3.5 The Value of Information

Consider the planner's problem at h_t given by Equation (3.16). Blackwell's theorem is not directly applicable to this problem because a decision to increase the cutoff today affects beliefs in the future, and also means that an investor with a cost below the new cutoff cannot invest in the future. The dynamic aspects of the problem bear some similarity to the question studied by Demers (1991). The problem also shares features with the problem studied by Sulganik and Zilcha (1997) in that the feasible set of future actions depends on the signal and the information system. This section establishes some results for a modified version of the planner's problem which will be of use later on in characterizing the solution to the actual problem faced by the planner.

Consider a situation in which the process of information generation is decoupled from investment decisions. That is, suppose that at some h_{t-1} the planner does not observe the play in period $t-1$ but instead has the opportunity to conduct the following experiment prior to choosing the period t cutoff: the planner chooses a cost γ, and any investor with a project and a cost below γ is able to announce that he or she has a project without having to actually invest. The planner observes the announcements and then proceeds to choose the investment cutoff for period t conditional on the information revealed.

In essence, the planner can learn prior to period t(through the experiment associated with γ), without inducing agents to actually invest in period $t-1$ (which depends on c_{t-1}). Then, the planner's problem can be written:

$$\begin{aligned}\hat{V}(\{h_{t-1}, k_{t-1}\}, \gamma, c_{t-1}) \quad = \quad & \max_{c} \sum_{n=0}^{N} p(n|h_t;\gamma)\Big\{\int_{c_{t-1}}^{c} [r(n) - c] f(c) dc \\ & + \delta \sum_{k_t=0}^{N} p(k_t|h_t;\gamma, c) V(\{h_t, k_t\}, c)\Big\} \end{aligned} \tag{3.17}$$

Then the expected value of the problem given that the planner will be able to observe the outcome of the period t play is:

$$S(h_{t-1}, \gamma, c_{t-1}) = \sum_{k_1=0}^{N} p(k_{t-1}|h_{t-1};\gamma)\hat{V}(\{h_{t-1}, k_{t-1}\}, \gamma, c_{t-1}) \tag{3.18}$$

The problem is further complicated by the fact that the value of the problem to the planner in subsequent periods depends on information available at the start of the h_{t+1}. Both $p(k_t|h_t;\gamma, c)$ and $V(\{h_t, k_t\}, c)$ could depend on γ, $p(k_t|h_t;\gamma, c)$ directly and $V(\{h_t, k_t\}, c)$ through the effect that γ has on $p(n|h_{t+1})$. It is therefore not immediate how the value of the continuation problem will depend on the "informational cutoff," γ.

Lemma 9 *For a fixed c, $V(\{h_t, k_t\}, c)$ does not depend on γ.*

Proof See Appendix. ‖

Essentially, Lemma 9 states that all that matters to the planner in period $t+1$ is the portion of the distribution of cost types over which the planner has observations, and not the sequence in which these were obtained.

Lemma 10 *$S(h_{t-1}, \gamma', c_{t-1}) \geq S(h_{t-1}, \gamma'', c_{t-1})$ for $\gamma' > \gamma''$. That is, the planner prefers the observation of γ' to γ''. Hence, $\partial S(h_{t-1}, \gamma, c_{t-1})/\partial\gamma \geq 0$ for all c_{t-1}.*

Proof See Appendix. ‖

Given the discussion in Section 2.5, it is clear that given a set of binomial trials there is more information revealed if the probability of investment for an investor with a project increases. If the planner were able to conduct this experiment the optimal decision would be to set the "informational cutoff," γ equal to $\bar{c}$. This would reveal the true value of n at no cost, because the choice of γ does not affect the possibility of future investment.

Furthermore, it is true that γ' is preferred to γ'' whenever $\gamma' > \gamma''$. The key observation is that γ does not affect the information available in the continuation game, which only depends on the next cutoff, c. Therefore, the planner can always ignore information generated by increased γ in the current period without this altering the value of the continuation game. As a result the planner can be no worse off by observing the experiment associated with a higher value of γ.

The solution to the problem with γ' will not in general be the same as the solution to the problem with γ'', in which case the planner will be strictly better off having observed the higher γ. Analogous to Lemma 5 the planner will often be strictly better off observing a higher γ because the planner only delays in order to learn and this is not a profitable strategy if the planner expects to ignore future information.

3.6 The First Best Solution

This section brings together the previous results to characterize the solution to the planners problem. Note that there are three possibilities for a solution to Equation (3.16) at h_t: **i)** a corner solution in which $c^{s*}(h_t) = \bar{c}$, so that all remaining investors invest regardless of their costs, **ii)** an interior solution where $c_t^{s*} \in [c_{t-1}, \bar{c}]$, where any remaining investors will invest if their costs are below the cutoff $c^{s*}(h_t)$, or **iii)** a corner solution in which $c_t^{s*} = c_{t-1}$, where no investor with a cost in the set of remaining possible costs invests. The objective is to compare the solution to the planner's problem at h_t, denoted $c^{s*}(h_t)$, with the solution that would obtain in the "laissez-faire" economy at the same history, denoted $c^*(h_t)$.

Considering first Case **i)**, it is clear that the $c^{s*}(h_t) \geq c^*(h_t)$, since $c^{s*}(h_t) = \bar{c}$ which is the maximum possible that $c^*(h_t)$ can take.

Where there is an interior solution to Equation (3.16), Case **ii)**, the optimal value of $c^{s*}(h_t)$ is that value of c which satisfies the following first order condition:

$$0 = \sum_n p(n|h_t)(r(n) - c)f(c) - \delta \sum_{\tilde{k}} p(k_t|h_t) \sum_n p(n|h_{t+1})(r(n) - c)f(c)$$

$$+\delta \sum_{\tilde{k}} \frac{\partial p(k_t|h_t)}{\partial c} \cdot V(\{h_t, k_t\}, c), \tag{3.19}$$

where the notation $\tilde{k}$ denotes that the summation is over those values of k_t for which $c \geq c_{t-1}$.

The first term in Equation (3.19) is the additional expected profit (or loss) from current investment that will accrue to the planner if the cutoff is raised. The second term is the negative of the option value of delay, and represents a cost of increasing the current cutoff, c. Namely that increasing the cutoff at h_t means that those investors with costs below the new cutoff lose the option of making an investment decision in the future, based on new information. The third term represents the effect of increasing the cutoff c on beliefs. It is this effect that the individual investors of Section 2 ignore but that the planner takes into account.

The third term is equal to $\partial S(h_t, \gamma, c)/\partial \gamma$, which, by Lemma 10, is greater than or equal to zero for all values of c. Therefore the third term is greater than or equal to zero. This is just the result that information is valuable. Although this term is always positive, the planner will not, in general, choose to set $c = \bar{c}$ and obtain maximum information because the cost of doing so, represented by the second term, is that there will be fewer investors around in the future to take advantage of this information.

Note that the first two terms equal the two terms of Equation (2.11) and therefore sum to zero when $c = c^*(h_t)$. By Lemma 1 the sum of these terms is negative when c is greater than $c^*(h_t)$. Since the third term is positive, it follows that any interior solution to the planner's problem involves $c^{s*}(h_t) > c^*(h_t)$.

The remaining possibility, Case **iii)**, is that the solution to the planner's problem involves $c^{s*}(h_t) = c_{t-1}$. In this case it can be shown that the 'laissez-faire" equilibrium cutoff, $c^*(h_t)$, also equals c_{t-1}.

To see this, assume that it is not the case, so that $c^{s*}(h_t) = c_{t-1}$, and $c^*(h_t) > c_{t-1}$. Since $c^*(h_t) > c_{t-1}$,

$$\sum_n p(n|h_t)(r(n) - c_{t-1}) f(c_{t-1}) - \delta \sum_{k_t} p(k_t|h_t) \sum_n p(n|h_{t+1})(r(n) - c_{t-1}) f(c_{t-1}) > 0. \tag{3.20}$$

Since $\partial S(h_{t-1}, \gamma, c_{t-1}) \geq 0$, the right hand side of Equation(3.19) must greater than zero. Therefore, the planner's utility can be increased by increasing $c^{s*}(h_t)$, which contradicts the assumption that $c_t^{s*} = c_{t-1}^*$ is the solution to the planner's problem. Therefore, if $c^{s*}(h_t) = c_{t-1}$ it must be the case that $c^*(h_t) = c_{t-1}$

The following proposition summarizes these results.

Proposition 3 *The planner reduces delay (chooses a higher cutoff) at any history h_t relative to the "laissez-faire" equilibrium cutoff that would obtain given history h_t.*

Observe that there is no guarantee that the true value of n will be revealed under the socially optimum investment strategy. To see this simply note that it is possible for the

	c^s	c_b
Case 1	0.62	0.60
Case 2	1.00	0.78
Case 3	0.18	0.21

Table 1: Optimal vs. "One Period" Cutoffs

true value of n to be high but for all agents with projects to have high costs. In which case, k_1 can be zero and the game will end with no investment despite the fact the investment may well be profitable for some or all investors.

3.7 Optimal Delay

In this section it is shown that it can be socially optimal for investors with negative expected profits given current information ought to invest today because of the informational value of investment. In other cases, it is optimal for investors with a positive expected payoff from investing given current information to delay, in order to take advantage of future information. In other words, it is not possible to make a general statement about the relative size of the option value effect or the informational effect. Depending on the values of the parameters of the model, either effect can dominate. This is shown to be the case by way of numerical examples.

Table one reports the results from solving the planner's problem in an economy where $N = 3$. The optimal first period cutoff is reported and compared to the cost for which expected profits for an investor who invests in the first period are zero under different assumptions about $g_o(n)$, the prior distribution of n, and r(n), the return to investing given n. All calculations are based on the assumptions that $F(c)$ is uniform in the interval $[0, 1]$ and that the common discount factor, δ equals 0.85.

The cutoffs of row 1 are calculated with $g_o(n) = 0.25$ and that $r(n)$ taking on the values $\{0.5, 0.55, 0.6, 0.65\}$. In this case, the fact that the payoffs to investing do not depend strongly on the state of the world means that information plays little role in investors' decisions. The return to investing is similar enough across the various possible states of the world that more or less all investors with projects that profitable in expected value invest in period 1.

Row 2 assumes that payoffs exhibit a greater dependence on the realization of n so that $r(n)$ takes on the values $\{0, 0.1, 0.5, 2\}$. Under these assumptions, the expected return to investing exceeds one so that the expected payoff from investing in period one is greater than zero for investors of all cost types (since $\bar{c} = 1$). However, investors with high costs stand to suffer substantial losses if they invest and the state of the world is poor. As a result, the option value of delaying such investment decisions is high and the social optimum reflects this.

Row 3 maintains the assumption on $r(n)$ from the previous row, but $g_o(n)$ takes on the values $\{0, 0.985, 0.001, 0.014\}$. Because the state will be either very good or very bad, the planner generates very valuable information by increasing the cutoff relative to the myopic case, since if an investment is observed in the first period all remaining investors know that the state of the world is almost certainly very profitable. The optimal cutoff cost is still relatively low, so that if an investor with such a cost invests and the state turns out to be bad, the loss is not great. Contrast this to the previous example where an investor with a very high cost, but positive expected profits from investing, might when the state is bad and suffer a large loss.

4 The Policy Maker's Problem

This section examines the main question of interest: what can a policy maker achieve in the economy of Section 2? While the previous section characterized the socially efficient outcome, it remains to be seen to what extent a policy maker is able to induce efficiency gains in the economy. The policy maker's problem is non-trivial since it is unclear to what extent the strategic interactions identified by Caplin & Leahy affect the policy maker's ability to induce efficient outcomes.

This section shows that the policy maker is able to achieve the first best outcome even in the absence of any commitment technology. The strategic behavior of investors does affect the policy maker's problem, but only affects the level of the subsidy and not the profile of investment.

4.1 The Policy Maker

The policy maker considered is benevolent and shares the same objective function and information as the benevolent planner of the previous section. The policy maker's objective is to maximize the discounted, pre-subsidy, expected profits of the ex-ante representative investor introduced in the previous section.[2]

Like the planner the policy maker observes neither the number of investors with an option, n, nor the realized cost, c, of any individual investor. The policy maker does observe all public information in the economy and therefore, like the planner, has beliefs given by $p(n|h_t)$ as defined by Equation (2.7).

The policy maker's instrument is a state contingent investment subsidy. We assume that the policy maker is able to finance this subsidy by levying non-distortionary, lump-sum taxes elsewhere in the economy. As a result, the policy maker neither benefits nor suffers from the use of subsidies, all else being equal. Therefore the policy maker only has preferences over the use of subsidies to the extent that a given path of subsidies affects the path of investment, and the size of the subsidy does not directly enter the government's objective function.

[2]The results of this section also hold if the policy maker maximizes the expected, aggregate, pre-subsidy, profits of the economy.

Since the policy maker does not observe the realized costs of individual investors, the investment subsidy cannot be conditioned on these costs. As a result, Lemma 1 applies and investors play cutoff rules. Furthermore, the planner must make decisions sequentially as the game evolves, so that the policy maker's objective function is given by Equation 3.16.

The policy maker's problem in each period is to choose an optimal subsidy (denoted $s^*(h_t)$) to maximize Equation 3.16 subject to the constraint that investors are acting in their interests as outlined in Section 2. The policy maker's choice of subsidy will result in an equilibrium cutoff at each history, which will be denoted $c^{p*}(h_t)$.

4.2 Influencing the Cutoff

Unlike the planner of Section 3, the policy maker cannot mandate that some investors invest while others wait, but must try to influence investors' behavior through optimal use of the investment subsidy. This section presents two results concerning the ability of the policy maker to influence the equilibrium cutoff.

Lemma 11 *The equilibrium set of subsidies is such that an investor indifferent between investing and waiting at h_t finds it optimal to make a once and for all decision at $t+1$. That is, the equilibrium set of subsidies satisfies Restriction 1.*

Proof See Appendix. ‖

The implication of Lemma 11 is that it, rules out situations in which $c^{p*}(h_{t+1}) > c^{p*}(h_t) = c^{p*}(h_{t-1})$. This means that the equilibrium cutoffs can be calculated using Equation 2.12.

The intuition behind the result is as follows. Since the planner loses by discounting, the only reason to delay some investment is to take advantage of future information. If $c^{p*}(h_t) = c^{p*}(h_{t-1})$, then the planner has the same information at h_t as at h_{t-1}, so that if it is profitable to induce a higher cutoff at h_t, then it is preferable to the planner to induce this investment at h_{t-1}. If $c^{p*}(h_{t+1}) > c^{p*}(h_t)$, then the profile of subsidies offered at h_{t+1} are such that investors with $c^{p*}(h_{t+1}) \geq c \geq c^{p*}(h_t)$ are induced to invest at h_{t+1}. Since information is the same at h_t and h_{t+1}, if the planner offers these same subsidies at h_t, investors will invest at h_t. As argued above, this is preferable to the planner.

Lemma 12 *Let $\mathcal{S}$ be a set of subsidies satisfying Restriction 1, where $\mathcal{S}_+ = \{s(h_{t+j})\}_{j=1}^{\infty}$, specifies an arbitrary set of subsidies for any continuation game beginning at h_t and $s(h_{t+j}) < \infty \ \forall \ h_{t+j}$. Let $\tilde{c}(h_{t-1})$ denote the lowest possible cost for an agent who has yet to invest at h_t. Then, for every $\hat{c} \geq \tilde{c}(h_{t-1})$, there exists a subsidy, $s(h_t) < \infty$ which supports $\hat{c}$ as an equilibrium cutoff value at h_t.*

Proof See Appendix. ‖

Both the current and future subsidies influence the relative value of investing versus delay at h_t. Holding future subsidies fixed and varying the current subsidy is one way of altering the relative value of investing versus delay. As long the expected value of future subsidies is finite, it is possible to find a finite current subsidy so as to implement any desired cutoff as an equilibrium.

4.3 Equilibrium

This section shows how one can solve for the equilibrium cutoffs and subsidies in the economy.

Since the policy maker and the planner share the same objective function (Equation 3.16), the optimal cutoffs that solve the planner's problem will also maximize the policy maker's objective function. The question is whether or not a policy maker can induce these cutoffs as equilibrium outcomes.

The results of the previous section suggest that as long as the continuation subsidies are finite, the policy maker can induce any desired cutoff as an equilibrium. Recall that Lemma 8 implied that the solution to the planner's problem involves a finite horizon. Furthermore the socially optimal cutoff in the last period (when only one possible investor remains) is equal to the "laissez-faire" equilibrium cutoff.

This implies that if the policy maker implements the socially optimal cutoffs, the game will have a finite horizon, and the optimal subsidy in the last period (when only one potential investor remains) will be zero, because the optimal cutoff is the "laissez-faire" equilibrium. Working backwards, Lemma 12 implies that in the next to last period the policy maker can induce the socially optimal cutoff with a finite subsidy, because the continuation subsidy is finite. Given the recursiveness of the objective function, a reapplication of this argument suggests that the policy maker is able to induce the socially optimal cutoffs at every history.

This argument leads to the main result:

Theorem 1 *The equilibrium cutoffs, $\{c^{p*}(h_t)\}_{h_t \in \mathcal{H}}$ induced by the policy maker in the absence of a commitment technology are identical to the optimal cutoffs $\{c^{s*}(h_t)\}_{h_t \in \mathcal{H}}$ implemented by a social planner.*

Theorem 1 implies that the equilibrium can be calculated by first solving the policy maker's problem for the optimal cutoffs and then using backwards induction to determine the optimal subsidy from the investors' optimality conditions. The optimality of these cutoffs and the argument used to prove Lemma 11 implies that optimal subsidies will satisfy Restriction 1. Therefore, Equation (2.12) can be used to solve for the equilibrium subsidies.

At any history h_t, given that the policy maker wishes to implement the cutoff $c^{s*}(h_t)$, the value of waiting at h_t to an investor with cost c is given by $W(c, c^{s*}(h_t), h_t; \{s^*(h_{t+1})\})$. The optimal subsidy is, therefore, the value $s^*(h_t)$ that solves:

$$V(h_t) + \tilde{s}^*(h_t) - c^{s*}(h_t) = \delta W\Big(c^{s*}(h_t), c^{s*}(h_t), h_t; \{s^*(h_{t+1})\}\Big) \tag{4.21}$$

The expected value of the subsidy at h_{t+1} to the marginal investor (with cost $c^{s*}(h_t)$) is given by $E(s^*(h_{t+1})) = \sum p(k_t|h_t) \cdot s^*(\{h_t, k_t\})$, where the summation is over those values of k_t following which an investor with cost type $c^{s*}(h_t)$ will invest in period $t+1$.

Since the policy maker reduces delay in equilibrium the optimal subsidy must be declining.

Proposition 4 *The optimal subsidy at h_t, $s^*(h_t)$, equals the value of information plus this expected value of the subsidy $E(s^*(h_{t+1}))$. Since the value of information is positive $s^*(h_t) \geq E(s^*(h_{t+1}))$.*

Proof From equation (4.21) we can see:

$$\begin{aligned}
s_t^* =& \delta W\Big(c^{s*}(h_t), c^{s*}(h_t), h_t; \{s^*(h_{t+1})\}\Big) - \Big(V(h_t) - c^{s*}(h_t)\Big) \\
=& \Big[\delta W\Big(c^{s*}(h_t), c^{s*}(h_t), h_t; \{s^*(h_{t+1})\}\Big) - \delta W\Big(c^{s*}(h_t), c^{s*}(h_t), h_t; \{s^*(h_{t+1}) = 0\}\Big)\Big] \\
& + \Big[\delta W\Big(c^{s*}(h_t), c^{s*}(h_t), h_t; \{s^*(h_{t+1}) = 0\}\Big) - \Big(V(h_t) - c^{s*}(h_t)\Big)\Big] \\
=& \sum p(k_t|h_t) s^*(h_{t+1}) + \Big[\delta W\Big(c^{s*}(h_t), c^{s*}(h_t), h_t; \{s^*(h_{t+1}) = 0\}\Big) - \Big(V(h_t) - c^{s*}(h_t)\Big)\Big],
\end{aligned}$$

where the summation is over those values of k_t following which $c^{*s}(\{h_t, k_t\}) \geq c^{*s}(h_t)$.

By inspection of the first order conditions of the planner's problem, the term $[\delta W(c^{s*}(h_t), c^{s*}(h_t), h_t; \{s^*(h_{t+1}) = 0\}) - (V(h_t) - c^{s*}(h_t))]$ is shown to be equal to the value of information, $\partial S(h_{t-1}, \gamma, c_{t-1})/\partial\gamma$. By Lemma 10 this is greater than or equal to zero. Therefore the equilibrium subsidy in at h_t is greater than or equal to the expected value of the period $t+1$ subsidy to the marginal agent. $\|$

Note that, except in the next to last period, the subsidy does not equal the informational value of extra investment.[3] Instead, the optimal subsidy equals the informational value of extra investment plus the expected value of next period's subsidy.

In short, investors understand that there will be future subsidies, and this provides an additional incentive to delay. The policy maker, however, understands the investors' incentives and account for this when choosing the optimal subsidy, by setting a subsidy profile such that early subsidies compensate investors for any future subsidies they forego by investing early.

That the policy maker is able to induce the socially optimal profile of investment as an equilibrium outcome may seem to suggest that strategic interactions between the policy maker and private investors are unimportant or nonexistent. This is not the case. Strategic interactions do play a role in influencing the optimal subsidy. The fact that the optimal subsidy at any given history depends on both the informational value of investment and the expected future subsidy is a result of the strategic play of investors.

In one sense the results here resemble those of Caplin & Leahy (1996), in that the policy maker must act more aggressively to account for the fact that strategic investors are

[3] In the final period, there is no informational externality, so the "laissez-faire" equilibrium cutoff equals the socially optimal cutoff. As a result, the optimal subsidy is zero.

willing to wait for future policy changes. On the other hand, it is clear that, when policy intervention is explicitly motivated by a benevolent policy maker attempting to address inefficiency in the economy, the strategic behavior of investors does not necessarily prevent the policy maker from achieving this objective.

5 Conclusion

This paper has examined the effects of strategic interactions between private investors and a policy maker in an environment in which inefficiency, in the form of an informational externality motivate policy intervention. It was shown that, even in the absence of a commitment technology, the policy maker is able to reduce equilibrium delay and achieve the socially optimal investment profile. The strategic behavior of investors influenced the profile of the optimal investment subsidy, but did not prevent the policy maker from achieving optimal investment.

The assumption that policy makers have access to lump sum taxes plays an important role in generating these results. If it were costly for policy makers to raise revenue, then the policy maker would not achieve the social optimum, as the costs of employing the subsidy would offset some of the benefits. More interestingly, if the funds available to the policy maker came at some cost, the policy maker would have an incentive to convince investors that future subsidies will be low in order to lower the cost of inducing current investment. Thus considering the cost of funds would likely introduce elements of dynamic inconsistency into the policy maker's problem, thus creating a role for commitment mechanisms. Contrasting this with the discussion of Thimann & Thum (1998), who emphasize the role of the IMF in enhancing credibility by increasing the cost of funds made available to policy makers in developing and transition economies, suggests that more work is needed to study the effects of increasing the cost of funds on the ability of policy makers to induce efficient outcomes.

Another interesting issue concerns the assumption that the policy maker has no additional information and influences the economy by intervening in the process by which the investors generate information. In many cases it is likely that policy makers may have information about the economy that is not available to private investors. In such situations that the policy maker's actions will both influence the investors' decisions and signal the policy maker's information, further complicating the problem.

Finally, the analysis could be extended to include the possibility of payoff interdependency amongst investors. These kinds of interdependencies are thought to play an important role in the experience of developing countries. It is reasonable to think that many situations will exhibit both payoff and information interdependency amongst investors, and an understanding of optimal policy in these cases would also be worthwhile.

Appendix

Proof of Lemma 1

Choose an arbitrary cost $\hat{c}$. Consider an investor with type $c < \hat{c}$. If such an investor invests at h_t, he receives $\hat{c} - c$ more than an investor with $\hat{c}$. For any strategy involving delay, given that all investors observe the same information, an investor with cost $c < \hat{c}$ receives $\hat{c} - c$ more than an investor with $\hat{c}$ if and when he finally invests, but this is discounted.

Therefore, relative to an investor with cost $\hat{c}$, the payoff for an investor with cost c to investing at h_t increases by $\hat{c} - c$, while the payoff at any strategy involving delay increases by less than $\hat{c} - c$. Therefore, an investor with $c < \hat{c}$ has more incentive to invest at h_t than an investor with $\hat{c}$.‖

Proof of Lemma 3

If $c^*(\{h_t, k_t\}) > c^*(h_t)$ then it is optimal for the player with type $c^*(h_t)$ to invest at (h_t, k_t) by Lemma 1.

It remains to consider the case where $c^*(\{h_t, k_t\}) \leq c^*(h_t)$, which can be divided into three possible cases:

i) If $R(h_t, k_t) - c(h_t) > 0$ and $c^*(\{h_t, k_t\}) = c^*(h_t)$, then an investor with $c^*(h_t)$ must be planning to invest at some future information set $h_{t+1+\tau}$. Let $h_{t+1+\tau} = (h_t, k_t, 0, \ldots, 0)$ be the first information set after $(\{h_t, k_t\})$ at which $c^*(h) > c^*(h_t)$. The probability of investment is zero between $(\{h_t, k_t\})$ and $h_{t+1+\tau}$ because, by Lemma 1, if the investor with the lowest remaining cost, $c^*(h_t)$, doesn't find it optimal to invest then no-one does. Therefore the information is the same at both information sets so that $R(h_t) - c^*(h_t) = R(\{h_t, k_t\}) - c^*(h_t) > 0$. Discounting makes the investor better off if he invests at $\{h_t, k_t\}$, meaning that $c^*(\{h_t, k_t\}) > c^*(h_t)$.

ii) If $R(\{h_t, k_t\}) - c^*(h_t) < 0$ and $c^*(\{h_t, k_t\}) = c^*(h_t)$, then at $h_{t+1+\tau} = (h_t, k_t, 0 \ldots, 0)$ no new information has been revealed (again by Lemma 1) and $R(h_{t+1+\tau}) - c^*(h_t) < 0$. Therefore it is not optimal to invest at any such information set $h_{t+1+\tau}$, so that it is never optimal to invest following $\{h_t, k_t\}$.

iii) If $V(\{h_t, k_t\}) - c^*(h_t) = 0$ and $c^*(\{h_t, k_t\}) = c^*(h_t)$, then at $h_{t+1+\tau} = (h_t, k_t, 0, \ldots, 0)$ no new information has been revealed and $R(h_{t+1+\tau}) - c^*(h_t) = 0$. Since $r(n) \neq c^*(h_t))$ for all n, there is some uncertainty about $R(h_t) - c^*(h_t)$, that is, it could be positive or negative. If $c^*(h') > c^*(h_t)$, then a player expects a payoff of zero now, but there is some uncertainty about it. Furthermore, because $c^*(h_{t+1+\tau}) > c^*(h_t)$ if the investor waits he can learns about the uncertainty and thereby increase his expected payoff, so a player with $c^*(h_t)$ will strictly prefer to wait at $h_{t+1+\tau}$. This contradicts $c^*(h_{t+1+\tau}) > c^*(h_t)$ and we have that $c^*(h_{t+1+\tau}) = c^*(h_t)$ for all $h_{t+1+\tau}$.‖

Proof of Lemma 5

Given the construction of X and X', it is (weakly) more informative to observe investment when the cutoff is higher. It follows from Blackwell's theorem on the comparison of experiments (see Blackwell (1951), (1953) or Crémer (1982)) that $W(c, \tilde{c}', h_t; S) \leq W(c, \tilde{c}, h_t; S)$ whenever $\tilde{c}' < \tilde{c}$.

Suppose that $\mathcal{F}(\tilde{c}) > \mathcal{F}(\tilde{c}')$ for $\tilde{c}' < \tilde{c}$, then it is strictly more informative to observe investment under $\tilde{c}$. Suppose, in addition, the option value of waiting, $W(c, \tilde{c}, h_t; S) - V(h_t)$ is positive.

From Lemma 4, if the option value of waiting is strictly positive for an investor with cost c, then that investor's decision on whether or not to invest depends non trivially on information received while waiting. That is, for some number $k > 0$, the expected profit from investing is positive if $X = k$ and negative if $X = 0$. When $X' = 0$, both $X = k$ and $X = 0$ may have occurred with positive probability (by the construction of X'). Therefore the value of observing X is strictly greater than the value of observing X'.‖

Proof of Lemma 6

If $\mathcal{C}(h)$ is not of the form $[\underline{c}, \hat{c}(h)]$, then there exist sets $\check{c}'$ and $\check{c}''$, where every element in $\check{c}'$ is less than every element in $\check{c}''$, such that investors with costs $c \in \check{c}'$ delay at h_t ($\check{c}' \subset \mathcal{C}^c(h_t)$), where $\mathcal{C}^c(h_t)$ is the complement of $\mathcal{C}(h_t)$) while investors with costs $c \in \check{c}''$ invest at h_t ($\check{c}' \subset \mathcal{C}(h_t)$) .

The amount of information generated depends only on $\mathcal{F}(\mathcal{C}(h_t))$, the probability that a given investor with an option invests. Construct an alternate set $\hat{\mathcal{C}}(h_t)$ by transferring cost types from $\check{c}''$ out of $\mathcal{C}(h)$ and replacing them with sufficient elements of $\check{c}'$ so that $\mathcal{F}(\mathcal{C}(h_t)) = \mathcal{F}(\hat{\mathcal{C}}(h_t))$. Therefore the information revealed when the policy maker follows strategy $\hat{\mathcal{C}}(h_t)$ is the same as if the policy maker follows $\mathcal{C}(h_t)$.

The construction of $\hat{\mathcal{C}}(h_t)$ involves making lower cost investors invest earlier rather than later, and offsetting this by making high cost investors invest later rather than earlier. Since, by Lemma 1, the relative expected payoff of a given c from investing immediately versus delaying is decreasing in c, and the information revealed is identical under both strategies, the planner is better off under $\hat{\mathcal{C}}(h_t)$.

Reapplying this argument eventually yields $\hat{\mathcal{C}}(h_t)$ takes the form $[\underline{c}, \hat{c}(h_t)]$.$\|$

Proof of Lemma 7

Given V_t, $V(h_t)$, $V^c(\hat{h}_t)$, σ^* and $\hat{\sigma}$ as defined in the text, the lemma states that $\hat{\sigma}$ is optimal for the original problem, V. Proceed by assuming that this is not the case, so that $V|_{\sigma^*} > V|_{\hat{\sigma}}$, and looking for a contradiction. Note that at h_t, c_{t-1} is already determined, so the payoffs under the two strategies can be decomposed as follows:

$$V|_{\sigma^*} = V_t|_{\sigma^*} + \sum_{\hat{h}_t \neq h_t} p(\hat{h}_t)V^c(\hat{h}_t)|_{\sigma^*} + p(h_t)V(h_t)|_{\sigma^*} \tag{5.22}$$

and

$$V|_{\hat{\sigma}} = V_t|_{\hat{\sigma}} + \sum_{\hat{h}_t \neq h_t} p(\hat{h}_t)V^c(\hat{h}_t)|_{\hat{\sigma}} + p(h_t)V(h_t)|_{\hat{\sigma}}. \tag{5.23}$$

The probability of observing any history h_t does not depend on the cutoffs that are chosen subsequent to period t (i.e. does not depend on σ_{h_t}). Since $V_t|_{\sigma^*}$ and $V_t|_{\hat{\sigma}}$ are identical by construction, and $V^c(h_t)|_{\sigma^*}$ and $V^c(\hat{h}_t)|_{\hat{\sigma}}$ are also identical by construction, for $V|_{\sigma^*} > V|_{\hat{\sigma}}$ to hold it must be the case that $V(h_t)|_{\sigma^*} > V(h_t)|_{\hat{\sigma}}$. This contradicts the fact that $\sigma^*_{h_t}$ is optimal for the continuation problem $V(h_t)$, which means that the assumption that $\hat{\sigma}$ is not optimal for the whole problem is incorrect. $\|$

Proof of Lemma 8

By discounting, the planner loses from delay. Therefore the planner only delays investment for investors with cost types such that the decision to invest depends non-trivially on the observation of k. The planner's profit (for such an investor) at any h_t, conditional on knowing n, is an increasing function of n given by:

$$\int_{c_{t-1}}^{c} [r(n) - c]f(c)dc + \delta \sum_{k_t=0}^{N-1-K(h_t)} p(k_t|h_t; c_{t-1}, c)V(\{h_t, k_t\}, c) \tag{5.24}$$

Therefore Lemma 2 can be applied to see that the planner's expected payoff is increasing in k_{t-1}. This implies that the indifferent investor will not invest in period t subsequent to observing $k_{t-1} = 0$.

By Lemma 6, no other agent invests in period t following $k_{t-1} = 0$. Therefore, in the next period, information is the same. Since it was optimal not to invest immediately following $k_{t-1} = 0$, it must therefore also be optimal not to invest the subsequent period. By induction, the indifferent investor never invests following an observation of $k_{t-1} = 0$. $\|$

Proof of Lemma 9

First observe that γ only affects the future game through its affect on beliefs. Suppose that, instead of observing both k_t and k_{t-1} (drawn from the experiments associated first with γ and then with c respectively) independently, the planner only observed the final cutoff c and the aggregate variable $\hat{k} = k_t + k_{t+1}$. Then the planner would have beliefs:

$$p(n|\{h_t, k_t + k_{t+1} = \hat{k}\}, c) = \frac{b(k_t + k_{t+1} = \hat{k}|n-1, h_t) \cdot b(k_{t-1}|n-1, h_{t-1}) \cdots g(n)}{\sum_{n'=0}^{N} b(k_t + k_{t+1} = \hat{k}|n-1, h_t) \cdot b(k_{t-1}|n'-1, h_t) \cdots g(n')} \quad (5.25)$$

which, using Equation (2.5) and manipulating, can be written:

$$p(n|\{h_t, \hat{k}\}, c) = \frac{\left(\frac{[n-1-K(h_t)]!}{(n-1-K(h_t)-\hat{k})!}\right)\left(\frac{(1-\mathcal{F}(c))^{n-1-K(h_t)-\hat{k}}}{(1-\mathcal{F}(c_{t-1}))^{n-1-K(h_t)}}\right) b(k_{t-1}|n-1, h_t) \cdots g(n)}{\sum_{n'=0}^{N}\left(\frac{[n-1-K(h_t)]!}{(n-1-K(h_t)-\hat{k})!}\right)\left(\frac{(1-\mathcal{F}(c))^{n-1-K(h_t)-\hat{k}}}{(1-\mathcal{F}(c_{t-1}))^{n-1-K(h_t)}}\right) b(k_{t-1}|n'-1, h_t) \cdots g(n')}. \quad (5.26)$$

Using Equation (2.5) to substitute in the expressions for the $b(k_t|\cdot)$ terms into equation (2.7), and cancelling terms gives that the planner's beliefs following the observation of k_t and k_{t+1}, $p(n|\{h_t, k_t, k_{t+1}\})$, equals the right hand side of Equation (5.26). In other words, what matters is the portion of the cost distribution that has been observed by period $t+1$ and not whether this was observed in one or two steps. ‖

Proof of Lemma 10

Following the discussion of Section 2.5, the experiment associated with γ' is more informative in the sense of Blackwell than the experiment associated with γ'' when $\gamma' > \gamma''$. That is, the experiment associated with γ'' is equal to the experiment associated with γ' plus noise. It follows from this, along with Lemma 9 that the observation of the experiment associated with γ' is more valuable to the planner.

To see this, assume that the planner observes a realization of k_t associated with γ' and subjects it to further randomization to get the experiment associated with γ''. This implies that the probability distribution of k_t following this new, compound experiment is identical to the probability distribution following a realization of k_t of the experiment associated with γ''. Hence the planner can pick the optimal c profile associated with the original experiment and, since Lemma 9 implies that the value of the continuation game does not depend on the intermediate steps, this yields the same value to the planner as the observation of the experiment associated with γ''.

In words, the planner can ignore the new information and be no worse off. Since the solution to the problem following an observation of the experiment associated with γ'' will not in general be optimal for the problem following an observation of the experiment associated with γ', the expected value of the problem of the planner is larger when the planner observes the more informative experiment. ‖

Proof of Lemma 11

Suppose at $\{h_t, k_t\}$ future subsidies $\{s(h_{t+1+j})\}_{j=0}^{\infty}$ are such that investors whose costs are such that they are indifferent between investing and waiting at h_t do not invest at $\{h_t, k_t\}$ but wish to invest at some future information set $h_\tau = (h_t, k_t, 0, \ldots, 0)$.

That is, at h_τ, the indifferent investor facing a set of future subsidies $\{s_{\tau+j}\}_{j=0}^{\infty}$ chooses to invest. Since the probability of investment is zero between $\{h_t, k_t\}$ and h_τ, by Lemma 1, no information is revealed, to either the investor or the policy maker between $\{h_t, k_t\}$ and h_τ.

This implies that the policy maker's expected payoff from inducing investment at h_τ is the same as the policy maker's expected payoff at $t+1$, but discounted. Since the policy maker induces the indifferent investor to invest at h_τ, the expected payoff to the policy maker from such an investment must be positive. Therefore, the expected payoff at $\{h_t, k_t\}$ must also be positive.

Since information is the same at $\{h_t, k_t\}$ and h_τ, the policy maker is able induce the investor to invest at $\{h_t, k_t\}$ rather than at h_τ by setting $s_{t+j} = s_{\tau+j}$ for all $j = \{0, 1, \ldots\}$. Since the policy maker and the investors face the exact same situation at (h_t, k_t) as a h_τ. If $s_{\tau+j}$ induces the indifferent investor to invest at h_τ, then those same continuation subsidies at $\{h_t, k_t\}$ would induce the indifferent investor to invest at $\{h_t, k_t\}$. Since the policy maker loses from discounting, the policy maker is better off by inducing the investor to invest at $\{h_t, k_t\}$. $\|$

Proof of Lemma 12

Take some arbitrary value $\tilde{c}$. Given h_t and $\mathcal{S}_+$, $\tilde{c}$ implies some fixed set of continuation subsidies $\tilde{\mathcal{S}}_+(h_t, c)$, where the possibility that the future subsidies vary with the cutoff, $\tilde{c}$, is allowed for. Given $\hat{c}$, h_t, and $\tilde{\mathcal{S}}_+$ it is possible to calculate the value of the continuation game for an investor with type $\hat{c}$ when others play the cutoffs $\tilde{c}$. This can be written as $W(\hat{c}, \tilde{c}, h_t; \{s(h_{t+1}; \tilde{c})\})$.

Type $\hat{c}$ will be an equilibrium at h_t if:

$$V(h_t) - \hat{c} + \hat{s} = \delta W(\hat{c}, \hat{c}, h_t; \{s(h_{t+1}; \hat{c})\}). \tag{5.27}$$

If we replace with $\hat{s} = \delta W(\hat{c}, \hat{c}, h_t; \{s(h_{t+1}; \hat{c})\}) - (V(h_t) - \hat{c})$ the element of $\tilde{\mathcal{S}}$ denoted $s(h_t)$, $\hat{c}$ will be supported as the equilibrium cutoff. Since $\hat{c}$, $V(h_t)$, and $W(\hat{c}, \hat{c}, h_t; \{s(h_{t+1}; \hat{c})\})$ are all finite, real numbers, such a value of $\hat{s}$ always exists. Furthermore, $\hat{s}$ is finite. $\|$

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