OPTIMAL WAGE TAXATION WHEN HUMAN CAPITAL AND
EMPLOYMENT ARE ENDOGENOUS

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This paper studies how optimal wage tax conclusions from the classic two-period life cycle model of human capital accumulation are affected by endogenizing the number of taxpaying workers. In the absence of a corrective policy, young individuals underinvest in human capital from a social perspective because tax premiums for transfers to nonworkers are not actuarially adjusted downward for human capital attainment. A combination of wage taxes and wage subsidies can restore proper price signals. Numerical simulations suggest that even modest employment elasticities can be sufficient to substantially impact the magnitudes and even the signs of optimal wage tax rates. (JEL H21, H3, J24)

I. INTRODUCTION

Both equity and efficiency considerations have led to widespread support for government involvement in education and other human capital markets.1 Given the dominant role of human capital as a determinant of national wealth—Davies and Whalley (1991) measure the stock of human capital in the United States to be several times that of physical capital—the consequences of tax policy for human capital accumulation may be at least as important as the consequences for closely related labor supply decisions. Trostel (1993) finds a strong relationship between income taxation and human capital formation. Although the effects of wage taxation on incentives to invest in human capital have been well discussed in the literature, relatively little attention has been given to the optimal tax problem with endogenous human capital.2 Moreover, the relatively few optimal tax analyses that model human capital choices do not model important employment outcomes.

This paper studies how optimal wage tax conclusions from the classic two-period life cycle model of human capital accumulation are affected by endogenizing the number of taxpaying workers. The standard life cycle human capital model presented, for example, in Eaton and Rosen (1980a) assumes that human capital affects only future wages. Analyses based on that model show that lump sum taxation is efficient when future wage rates are deterministic but inefficient when future wage rates are stochastic. In particular, the optimal policy includes a compensated wage tax on uncertain wages because a wage tax reduces the variability of income. Many subsequent studies have provided important extensions to the consumption-smoothing theme of Eaton and Rosen. Hamilton (1987), for example, shows that difficulties in diversifying human capital risk can lead young individuals

1. Efficiency justifications often involve externality aspects of human capital. Haveman and Wolfe (1984) describe a wide array of schooling benefits with external consequences, including enhancements in health status, child quality, and efficiency in consumer choice and job match. Lucas (1988) discusses production externalities. More broadly, educated populations may be more stable, the streets may be safer, and so on. Given the nature of many of these externalities, it is not surprising that there is little agreement about their importance, especially at the relevant margins.


*I thank James Andreoni, Matthew Doyle, Harvey Lapan, and an anonymous referee for providing especially helpful suggestions. I also received valuable comments from Wally Huffman, Bill Johnson, Tom Nechyba, Ed Olsen, Peter Orazem, Holger Sieg, Jon Skinner, Alex Tabarrok, and seminar participants at University of Virginia and Iowa State University. I gratefully acknowledge financial support from the Bankard Fund for Political Economy.

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ABBREVIATION

CPS: Current Population Survey
to underinvest in human capital from society’s perspective. ³

In reality, however, a sizable fraction of adults does not participate in the labor force, and this fraction varies widely with the level of human capital accumulation. Table 1 illustrates the wide variability in U.S. employment rates across schooling levels. ⁴ As discussed by Card (1999) and others, human capital attainment can influence employment outcomes in a variety of ways. In augmenting future wages, greater human capital encourages labor force participation by increasing the opportunity costs of choosing not to work. Skilled workers experience lower unemployment rates, and they are less likely to become disabled. Jobs tailored to skilled workers may also confer additional nonpecuniary benefits. In general, human capital may enhance an individual’s chances of finding and retaining desirable work.

When studying optimal wage taxes, previous analyses have implicitly assumed that the number of taxpayers is exogenous. This paper makes the general point that conclusions about the magnitudes of optimal wage taxes should take into account the effect of the taxes on the number of taxpayers. To keep the analysis tractable, the paper endogenizes employment outcomes in a traditional two-period environment. ⁵ Given the reality of public transfers to nonworkers, this modification is sufficient to overturn the classic result of Eaton and Rosen (1980a) that the optimal wage tax is zero when wage rates are nonstochastic and positive when wage rates are stochastic. In contrast, optimal wage taxes can become negative in both cases when


4. Conservative econometric estimates by the author using data on prime-age respondents in the 2000 Current Population Survey (CPS) suggest that the mean elasticity of employment with respect to years of education is about 0.39, accounting for self-selection. The 95% confidence interval for this elasticity is [0.33, 0.45].

5. It is beyond the scope of this paper to investigate the effects of taxes on long-term economic growth. Fully dynamic models address important sets of issues for long-run growth but require additional sets of assumptions. Kapicka (2006), for example, provides important contributions by extending Mirrlees’ (1971) framework into a dynamic setting, but he does not model employment outcomes, the focus of the current paper.

6. Adverse selection and other well-known market failures generally preclude the provision of private unemployment insurance. The same results in this paper would follow if private unemployment insurance were prevalent as long as insurers face legal or practical difficulties in perfectly adjusting premiums to human capital attainment.

### TABLE 1

<table>
<thead>
<tr>
<th>Years of Schooling</th>
<th>Unemployment Rate (%)</th>
<th>Labor Force Nonparticipation Rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 or more</td>
<td>1.7</td>
<td>17</td>
</tr>
<tr>
<td>13–15</td>
<td>2.7</td>
<td>22</td>
</tr>
<tr>
<td>12</td>
<td>3.5</td>
<td>26</td>
</tr>
<tr>
<td>Less than 12</td>
<td>6.4</td>
<td>41</td>
</tr>
</tbody>
</table>

Source: 2000 CPS.
In the calibrated model, however, net transfers still flow from older workers to younger workers to smooth lifetime consumption. Increased tax credits to the young more than compensate them for their higher wage tax burdens.

The next section presents the theoretical model of labor supply and human capital investment behavior, followed by a characterization of optimal policies in Section III. The endogeneity of employment status is simplistic in that it does not differentiate between various types of nonworkers, but it captures the central idea that the employment probability is increasing in human capital accumulation. Section IV calibrates the model to data from the 2000 CPS, and Section V concludes.

II. INDIVIDUAL BEHAVIOR

The Model

Following Eaton and Rosen (1980a) and Hamilton (1987), the model focuses on a representative young individual whose future earnings depend in part on human capital accumulation. The primary innovation is to show that endogenizing the tax base of employed workers can be sufficient to overturn previous conclusions about the signs and magnitudes of optimal wage tax rates. A young (y) individual allocates Y units of time into labor L_y, leisure L_y, and the accumulation of human capital H. Human capital is taken to represent free schooling or other efforts outside of work to increase future productivity, such as job search. The cost of obtaining human capital is thus measured as foregone first-period earnings and leisure. This treatment is more general than in most previous studies in that there is no restriction that L_y is perfectly inelastic. Human capital increases an individual’s future wage when older (o), w_o(H), such that human capital is productive at a diminishing rate: w_o < 0 and w_o' < 0. Initially, wages rates are assumed to be nonstochastic to isolate the implications of allowing for an endogenous number of workers; the implications of randomizing w_o are discussed later.

After the young complete their human capital investments and become exposed to the adult labor market, they draw an employment match value δ(H, ε) that is increasing in human capital accumulation and a random component ε with known distribution function F and density function f. The match value δ is normalized to zero in the nonworking state and can be thought of as the nonpecuniary aspects of a job relative to the baseline case of not working (e.g., some jobs are more socially desirable than others, some allow for a more flexible schedule, etc.). The role of the match function is to provide a relatively simple mechanism for sorting individuals into the working and nonworking states as a function of human capital accumulation. As discussed further below, individuals who draw sufficiently high values of ε will participate in the workforce and earn labor income; others will be classified as nonworkers and will receive government transfers.

Conditional on the previous choice of H and the decision to work, an older individual chooses consumption C_o and leisure time ε_o (with labor hours L_o = Y - ε_o) to maximize weakly separable second-period utility

\[ U^{o}\left[\varphi(C_{o}, \epsilon_{o}), \delta\right] \]

s.t. \( C_{o} = (1 - t_{o})w_{o}(H)(Y - \epsilon_{o}) - \Omega_{o} \),

where \( t_{o} \) is the second-period wage tax rate and \( \Omega_{o} \) is the second-period lump sum tax. Utility is assumed to be increasing in its arguments, twice differentiable, and concave. It will be convenient to work with the indirect utility function \( V^{o}(o_{o}, m_{o}, \delta) \) where \( o_{o} = (1 - t_{o})w_{o}(H) \) and \( m_{o} = -\Omega_{o} \). An older worker’s leisure is assumed to be a normal good with the wage elasticity of labor hours \( \eta = (\partial L_{o}/\partial o_{o})(o_{o}/L_{o}) \) bounded below such that \(-1 < \eta < \infty\). This elasticity assumption rules out labor hour schedules sufficiently backward-bending that tax revenue declines with the wage rate.

7. Most previous studies investigating the human capital decision, including Levhari and Weiss (1974), Eaton and Rosen (1980a), Hamilton (1987), and Brown and Kaufold (1988), have assumed that the cost of human capital is captured by foregone earnings.

8. Card and Krueger (1992) report that an additional year of education raises annual earnings between 5% and 11% on average.

9. The particular normalization has no impact on the model; the match value can be normalized to any constant for the nonworking state.

10. The value of the government transfer B below may be defined to include Ω_o. In this sense, Ω_o is truly lump sum in that it cannot be escaped by not working.
As a departure from the previous literature, an older individual chooses to work if the utility associated with working exceeds that associated with receiving transfer income. The nonemployed supply no labor hours and consume a benefit amount \( B \). Like the tax rates, \( B \) is chosen optimally by the government but is taken parametrically by individuals. The probability of not participating in the labor force is thus given by \( \phi = \Pr\{V^0[\omega_o, m_o, \delta(H, \varepsilon)] - U^0(B, Y, 0) < 0\} \). The critical value \( \varepsilon \) that makes an individual indifferent about participating in the labor force is defined implicitly by \( V^0[\omega_o, m_o, \delta(H, \varepsilon)] - U^0(B, Y, 0) = 0 \). Thus, the probability of not working can be written as \( \phi = \Pr(\varepsilon < \varepsilon) = F(\varepsilon) \). Using the implicit function theorem, the impact of human capital on the probability of not working is given by \( \phi_H = -f(\varepsilon) (\partial \varepsilon / \partial H) = -f(\varepsilon) [\partial V^0 / \partial H] / (V^0 / \partial \varepsilon) < 0 \). Note that human capital increases the probability of employment both by improving the match value and through its effect on the wage rate: greater human capital leads to greater labor earnings which in turn increase the opportunity cost of not working.

In this formulation, older individuals with a sufficiently low match value \( \delta \) do not work. The outcome of not working can be motivated as arising from either a lack of available jobs or preferences for not working. In one interpretation, we can suppose that all individuals are identical and \( \varepsilon \) represents differences in the nonpecuniary characteristics of job offers. In this case, the nonwork state might be called unemployment because some workers are offered only undesirable jobs. An alternative interpretation is that all jobs are identical, and \( \varepsilon \) represents differences in workers (e.g., poor health makes work more onerous). In that interpretation, there is no involuntary unemployment; the jobs are available, but some individuals prefer not to work based on their personal characteristics. The key requirement for the subsequent optimal tax results is that \( \phi_H < 0 \): the size of the employment base of taxpayers is positively associated with human capital accumulation. The qualitative conclusions of the paper do not hinge on the specific reasons for being employed or nonemployed. What matters in the model is that some fraction of individuals end up working, and this fraction is increasing in the amount of human capital accumulation.\(^{12} \)

While young, an individual chooses \( L_y, \ell_y \), and \( H \) to maximize expected utility across the two periods under the assumption that second-period labor hours, if working, will be chosen optimally as a function of \( H \). The government’s policy is chosen and known to individuals before they make any of their decisions. First-period choices are made by solving:

\[
\max_{H, \ell_y} U = U^y(C_y, \ell_y) + \beta \int_{-\infty}^{\infty} [U^o(B, Y, 0) f(\varepsilon) d\varepsilon
\]

\[
+ \beta \int_{-\infty}^{\infty} [V^o((1 - t_o) w_o(H), Y - H - \ell_y) + sH - \Omega_y
\]

s.t. \( C_y = (1 - t_y) w_y(Y - H - \ell_y) + sH - \Omega_y \),

where \( t_y \) and \( \Omega_y \) denote the first-period wage tax and the lump sum tax, respectively, \( s \) denotes a unit subsidy to human capital investment, and \( \beta \) is the subjective time discount factor. Assuming an interior solution, the first-order conditions for optimal investment and leisure time are given by

\[
H : U_H = - (1 - t_y) w_y + sU^o_C
\]

\[
+ \beta \int_{-\infty}^{\infty} [V^o((1 - t_o) w_o(H)
\]

\[
+ V^o_\delta \delta_H f(\varepsilon) d\varepsilon = 0
\]

11. The conclusions in this paper continue to hold if \( B \) is determined by a policy rule \( B = r(w, L) \), where the income replacement rate function \( r \) is restricted such that \( B \) does not rise with \( H \) faster than second-period tax revenue rises with \( H \) as the tax base expands.

12. As a simple alternative to the match function, one could directly specify a nonemployment probability \( \phi(H) \), with \( \phi > 0 \), and abstract away from the particular sorting mechanism driving the work outcome. All the subsequent optimal tax results continue to go through in that case. A more elaborate approach would model the demand side of the labor market in detail and specify an adjudication process for a nonworker’s receipt of government benefits. Such a model, which would be considerably more complicated, would need to specify markets for skilled and unskilled workers where skilled workers are in greater demand by employers.
(2) \(\ell_y : U_t = -(1 - t_y)w_y U_C^t + U_L^t = 0\).

Holding leisure constant, the marginal cost of human capital investment is the marginal value of foregone consumption in the first period; its moral benefit is the marginal discounted expected utility gain associated with improved employment and wage prospects when older. 13

Second-order conditions and comparative statics results used later in the analysis are provided in Appendix A. The representative young individual is assumed to be credit constrained and unable to borrow. This assumption is made mostly for simplicity, but it may also more appropriately describe the choice set than an alternative polar extreme assumption of perfect capital markets. Well-documented information problems, along with the difficulty in providing collateral for human capital investment, make it difficult to obtain loans for such investments. Hamilton’s (1987) model provides important tax insights for the case that a young person can invest in both human capital and a risky physical asset.

 Responses to Policy Changes

Before turning to the government’s problem, it will prove useful to examine individuals’ privately optimal responses to changes in policy. Consider first a policy change involving a small increase in the direct human capital subsidy, accompanied by an increase in the first-period lump sum tax to preserve utility at its initial level (with any tax deficit or surplus retained by the government). The net changes in human capital and leisure following this policy change are given by \(\Delta H^o = \partial H_0/\partial t_0 - w_0 L_0 (\partial H/\partial \Omega_y)\) and \(\Delta L^o = \partial L_0/\partial t_0 - w_0 L_0 (\partial L_0/\partial \Omega_y)\). 15 Responses associated with a compensated increase in the second-period wage tax are given by \(\Delta H^s = \partial H_s/\partial t_s - w_s L_s (\partial H_s/\partial \Omega_y)\) and \(\Delta L^s = \partial L_s/\partial t_s - w_s L_s (\partial L_s/\partial \Omega_y)\). A compensated increase in the second-period wage tax on second-period leisure at the optimal investment level. The following lemma guarantees that \(\Delta H^o, \Delta H^s, \Delta L^o,\) and \(\Delta L^s\) are all positive, while \(\Delta H^s\) is negative under the earlier elasticity assumption that \(\eta > -1\). The proof is provided in Appendix B.

**Lemma.** (a) The investment response to a utility-compensated increase in the human capital subsidy is positive: \(\Delta H^s > 0\). (b) The investment and leisure responses to a compensated increase in the first-period wage tax are also both positive: \(\Delta H^o > 0\) and \(\Delta L^o > 0\). (c) A compensated increase in the second-period wage tax increases leisure \(\Delta L^s > 0\), and it decreases human capital investment, \(\Delta H^s < 0\), given \(\eta > -1\).

Thus, a utility-compensated wage tax on the young stimulates human capital investment and leisure, while a compensated wage tax on older workers discourages their labor effort and discourages investment among the young since it taxes away their future returns. With this lemma in hand, we can now turn to an analysis of optimal policy.

**III. OPTIMAL POLICY**

The government’s objective is to choose a balanced-budget tax-transfer policy that maximizes a representative individual’s expected discounted lifetime utility. The policymaker chooses the policy instruments \(s, t_y, \Omega_y, t_0, \Omega_o, \) and \(B\) to solve

15. Following the reasoning in the previous footnote, the compensation \(\partial \Omega_y/\partial t_s\) necessary to maintain first-period utility at its original level following an increase in \(t_y\) is \(-w_y L_y\).
\[
\max_{s,t_o,\Omega_o} \Psi
\]
\[
= U^\alpha(C_y, \ell_y) + \beta \int U^\alpha(B, \gamma, 0)f(\varepsilon) d\varepsilon
\]
\[
+ \beta \int V^\alpha[(1 - t_o)w_0(H),
\]
\[- \Omega_o, \delta(H, \varepsilon)]f(\varepsilon) d\varepsilon + \lambda \left\{ t_y w_y(\gamma - H - \ell_y)
\]
\[+ \Omega_y - sH - \int Bf(\varepsilon) d\varepsilon / R
\]
\[+ \int T_o d(\varepsilon) d\varepsilon / R - G \right\},
\]
where \(G\) is the government’s required revenue per capita for uses other than \(s\) and \(B\), \(\lambda = -(\partial U / \partial G) > 0\) is the discounted shadow value of decreasing the government’s revenue requirement by a dollar, \(R\) is the government’s available gross rate of return on capital, and \(T_o \equiv t_o w_o[H]L_o + \Omega_o\) is the total tax payment per worker in the second period.

Denote \(\Lambda \equiv -(B + t_o)\phi_H/R + (1 - \phi)t_o \times w_o L_o (1 + \eta)f(\varepsilon) d\varepsilon / R\) the marginal impact of human capital accumulation on the government’s second-period net tax revenue. The first term measures the increase in net revenue arising from the expanding population of taxpaying workers (including declines in benefit payments) where \(\phi_H < 0\) follows from above. The second term measures the increase in wage tax revenues among existing workers, accounting for the change in the wage rate and labor hours.

Writing down the first-order conditions for the government’s policy instruments and combining terms, we can obtain the following requirements for optimal policy:

1. \(\Psi_s^* = -t_y w_y(\Delta H^s + \Delta \ell^s_y) + (\Lambda - s) \Delta H^s = 0\)
2. \(\Psi_t^* = -t_y w_y(\Delta H^t + \Delta \ell^t_y) + (\Lambda - s) \Delta H^t = 0\)

where \(\Delta \ell^t_o \equiv \partial \ell_t / \partial t_o - w_o L_o (\partial \ell_t / \partial \Omega_o)\) and the symbol \(\approx\) means “has the same sign as.” These conditions are derived in Appendix C.

The effect of each policy instrument on social welfare includes direct effects on utility, indirect effects on utility via investment and leisure changes, and consequences for government revenue. The indirect utility effects can be ignored by the Envelope Theorem. In Equations (3)–(5), the direct utility effects have been translated into monetary values. In deriving Equation (3), a small increase in \(s\) has been combined with a negative transfer sufficient to cancel out the direct utility gains associated with increasing \(s\). This leaves only monetary effects that can be assessed using the lemma. The first component, \(-t_y w_y(\Delta H^t + \Delta \ell^t_y)\), represents the loss in first-period wage tax revenue associated with a utility-compensated change in first-period labor hours when \(s\) is increased (recalling that labor hours equal \(\gamma - H - \ell_y\)). The second component, \((\Lambda - s) \Delta H^t\), represents the effect of the induced change in human capital on second-period tax revenues less the cost of the subsidy. At an optimal value of \(s\), the two revenue effects must be equal. Equations (4) and (5) have similar interpretations.

At the privately optimal level of human capital given by Equation (1), an individual is indifferent about attaining an additional unit of human capital. If employment outcomes are exogenous, as implicit in previous models, then \(\phi_H = 0\) and then lump sum taxation \(s^* = t^*_y = \ell^*_o = 0\) is efficient as shown below. If employment is endogenous, however, the social marginal value of human capital is still positive at privately optimal investment levels. Because the tax price of transfers to nonworkers does not vary with an individual’s choice of \(H\), young individuals have no incentive to account for the effects of their investments on the future tax base. With an additional unit of \(H\), the fraction of workers rises by \(-\phi_H\). For each additional worker, the government saves benefit payments \(B\) and gains an extra working taxpayer. Since individuals are not harmed by allocating an extra unit of time to human capital (they are indifferent), the additional tax revenue could be
used to increase the well-being of all individuals above that associated with their privately optimal choices.

Optimal Human Capital Subsidy

As one policy option, the market failure can be corrected using the direct human capital subsidy. In this case, the direct subsidy is set equal to the increase in revenue associated with a small increase in the employment rate and increase in the second-period wage rate evaluated at efficient values. In particular, the value functions in Equations (3)–(5) are all zero when \( s = \Lambda \) and \( t_y = t_o = 0 \), implying that the optimal subsidy \( s^* = \Lambda^* = -\phi_H(B + T_o)/R > 0 \) is positive and should be financed with lump sum taxes. Note that the endogeneity of future wages with respect to human capital does not drive a wedge between private and social returns to human capital under lump sum taxation. The optimality of a direct human capital subsidy depends only on the endogeneity of employment with respect to human capital: if \( \phi_H = 0 \), there is no need to stimulate human capital \( (s^* = 0) \) even when \( w_o^*(H) > 0 \).

The market failure would also disappear if the government could extract from older workers the actuarially fair premium \( \phi(H)/[1 - \phi(H)](B + T_o) \) for transfers to non-workers, a price schedule decreasing in the chosen level of human capital accumulation. This premium represents the net transfer from workers to nonworkers multiplied by the odds ratio of not working. Inclusion of this premium in an older worker’s budget constraint results in \( \Lambda = 0 \) when \( t_o = 0 \), in which case there is no divergence between private and social returns from another unit of human capital investment. The direct human capital subsidy \( s^* \) becomes optimal given the absence of this type of pricing policy when employment is endogenous.

At the individually optimal level of \( H \) given by Equation (1), the social value of increasing investment another unit is given by

\[
\Psi_H = \lambda(\Lambda - s - t_y w_y) = (\Lambda - s)\Delta \ell^s,
\]

where the second equality holds by Equation (4) when \( t_o \) is chosen optimally. When the direct subsidy is set equal to \( \Lambda \), the marginal impact of human capital attainment on second-period net revenues, we have \( \Psi_H = 0 \): the direct subsidy induces an efficient level of investment. These results are summarized in the following proposition:

**PROPOSITION 1.** If employment is exogenous \( (\phi_H = 0) \), then the optimal human capital subsidy and wage tax rates are zero \( (s^* = t_y^* = t_o^* = 0) \). If employment is endogenous \( (\phi_H < 0) \), then a human capital subsidy \( s^* = -\phi_H(B + T_o)/R > 0 \) induces a socially efficient level of human capital investment, with \( t_y^* = t_o^* = 0 \).

In the standard human capital model, employment is not affected by \( H \); therefore, \( \phi_H = 0 \) and lump sum taxation is efficient even when human capital affects future wages. Under lump sum taxation, it can be seen from the first equality in Equation (6) and the definition of \( \Lambda \) that the young underinvest in human capital relative to socially desired levels if and only if employment is endogenous with respect to human capital (i.e., \( \phi_H < 0 \)). In that case, the social marginal value of investment at individually optimal levels is still positive: \( \Psi_H = -\lambda \phi_H(B + T_o)/R > 0 \). This result is stated as Proposition 2.

**PROPOSITION 2.** Under lump sum taxation, individuals underinvest in human capital relative to socially desired levels if and only if employment is endogenous with respect to human capital attainment.

Hamilton (1987) similarly shows that young individuals may underinvest in human capital from society’s perspective. In his model, underinvestment arises because risk-averse agents are uncertain about the effect of human capital investments on random future wages. In contrast, underinvestment occurs in the present model because a lump sum tax system does not appropriately reward young individuals for making human capital investments that increase their chances of later being employed.

Optimal Wage Taxation

In lieu of a direct human capital subsidy, the policymaker can improve social welfare relative to lump sum taxation through a combination of a compensated wage tax on
This derivation presumes that leisure time is not perfectly inelastic in both periods (otherwise, \( \kappa \) is zero). If leisure is perfectly inelastic in both periods, then an infinite number of \( \{t_y, t_o\} \) combinations can reproduce the efficient policy associated with an optimal human capital subsidy.
decisions are made, a wage tax can be used as an instrument to reduce the variability of future consumption (e.g., Eaton and Rosen 1980a; Hamilton 1987). In the present model, the desirability of a consumption-smoothing wage tax must be weighed against the desirability of a wage subsidy to stimulate human capital. As the magnitude of the optimal second-period wage subsidy declines due to wage uncertainty (eventually becoming a wage tax instead of a wage subsidy for a sufficient amount of risk), the optimal size of the compensated first-period wage tax increases to encourage human capital accumulation. As before, the relative elasticities of leisure across periods play a key role in determining the magnitudes of the optimal tax and subsidy rates.19

Next, the model is calibrated to data for the U.S. population. Optimal wage tax rates on prime-age workers are always substantially smaller than those simulated by Eaton and Rosen (1980c) after allowing employment outcomes to depend on human capital; the signs of their optimal wage tax rates often change signs to become negative.

IV. EMPIRICAL SPECIFICATION

The model is calibrated based on Eaton and Rosen’s (1980c) one-period constant elasticity of substitution specification, augmented to allow for human capital and employment outcomes. Lifetime utility is specified as

\[
U = \{\alpha_y[(1 - t_y)w_y(Y - H - \ell_y) - \Omega_y]\}^{-\mu} + (1 - \alpha_y)\ell_y^{-\gamma/\mu} / \gamma + \beta \int_{-\infty}^{\infty} \{\alpha_oB^{-\mu} \\\n+ (1 - \alpha_o)Y^{-\gamma/\mu} / \gamma\}f(\varepsilon)d\varepsilon \\
+ \beta \int_{-\infty}^{\infty} \{[\alpha_o(1 - t_o)w_o(H)(Y - \ell_o) \\
- \Omega_o]^{-\mu} + (1 - \alpha_o)\ell_o^{-\gamma/\mu} / \gamma \\\n+ \delta_0 \ln H + \varepsilon\}f(\varepsilon)d\varepsilon.
\]

Parameter definitions are provided in Table 2. Information on schooling, employment, labor hours, and wages are taken from the October 2000 CPS. A young person aged 16–25 yr is assumed to have available 16 total hours in a day to divide between schooling, labor, and leisure, while an older person between the ages of 26 and 62 is assumed to divide 16 h in the day between labor and leisure. Thus, total annual available hours is \(Y = 5,840\).

Human capital is measured as the estimated average number of hours devoted to schooling per year while young, including technical training. For each year enrolled, a young person is assumed to devote 160 d/yr to schooling for 8 h a day. Based on retrospective questions in the data, I divided respondents in the older group in the CPS into nine human capital categories ranging from \(H_{\text{min}} = 0\) to \(H_{\text{max}} = 1280\). The minimum value reflects no schooling past the age of 15. The maximum value reflects the average annual number of schooling hours (from age 16 to age 25) required to obtain a graduate degree.20 The mean value is \(H = 607\), corresponding to about a year of college.21 Increasing \(H\) by 128 annual hours represents an extra year of schooling.

The average annual number of hours spent in paid work by the young is \(L_y = 967\), implying an average leisure value \(\ell_y = 4,266\). The representative older worker in the CPS devoted 40.6 h of week to the job, implying mean values \(L_o = 2,030\) and \(\ell_o = 3,810\) for labor and leisure, respectively, after accounting for average vacation time. Based on the data, the before-tax wage rate for the young is set to \(w_y = 8.64\). The effects of human capital accumulation on second-period employment and wages are estimated based on a simultaneous equations discrete-choice econometric model of employment that accounts for self-selection of individuals into the labor force.22 Using these estimates, the relationship between an older person’s wage rate and human capital accumulation is calibrated as \(w_o(H) = \exp[0.358 \ln(H) + 0.315]\). Matching

18. Varian (1980) also showed that taxes on earnings can be welfare improving when insurance markets are missing. Snow and Warren (1990) disentangle the effects of uncertainty about future wages on the level of human capital investment. 19. Results for the wage rate uncertainty case are available upon request.

20. Specifically, (8 h/d \(\times 160\) d/yr \(\times 10\) yr)/(10 yr) = 1,280.

21. On average, older respondents were enrolled in school 4.74 yr after age 15. Average annual schooling from age 16 to age 25 is thus 8 h/d \(\times 160\) d/yr \(\times 4.74\) yr)/(10 yr) = 607.

22. Details are available upon request.
the data at $H = 607$, the base wage rate is given by $w_0 = 13.6$ with an elasticity of 0.358.

Combining income data from the CPS with rules in the 2000 Federal 1040 tax form, a representative younger and older person in the CPS faced marginal income tax rates of $t_y = 0.15$ and $t_o = 0.28$, respectively. The base consumption levels among workers are $C_y = 20.280$ and $C_o = 42.836$, with benefits $B = 7104$ transferred to nonworkers. Based on the data, the benefit schedule is approximated as $B = 0.26w_oL_o$. In experiments involving an uncertain second-period wage rate, an older person is assumed to face either a low wage, $(9/10)w_o(H)$, or a high wage, $(11/10)w_o(H)$, with equal probability. An individual chooses labor hours optimally in the second period after learning the realized wage rate.

The utility parameter $\mu = 0.25$ is calibrated simultaneously with $\alpha_y = 0.28$ and $\alpha_o = 0.50$ such that the optimally chosen number of hours working among the young and old accord with the CPS data, with an average uncompensated wage elasticity equal to 0.136. The value $\gamma = -1.5$ determines the degree of relative risk aversion and is taken from Eaton and Rosen (1980c). The discount factor and interest rate are set at $\beta = 0.377$ and $R = 2.65$, respectively, reflecting a 5% interest rate over the 20-yr period between the mean age in the young period and the mean age in the old period.

The random component $\varepsilon$ of the employment match value $\delta(H, \varepsilon)$, specified as $\delta \ln H + \varepsilon$, is assumed to be uniformly distributed over the support $[a, b]$. At the base tax rates $t_y = 0.15$ and $t_o = 0.28$, the parameters $\delta_0$, $a$, and $b$ are calibrated simultaneously such that a young individual optimally chooses $H^* = 607$ h of schooling, the ex ante probability of not working in the second period $\phi$ is 0.24, and the elasticity of employment with

### Table 2
Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>$-1.5$</td>
<td>Determines relative risk aversion</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.25</td>
<td>Determines elasticity of substitution between consumption and leisure</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>0.28</td>
<td>Consumption share parameter for young</td>
</tr>
<tr>
<td>$\alpha_o$</td>
<td>0.50</td>
<td>Consumption share parameter for older</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>$0.175 \times 10^{-6}$</td>
<td>Match value parameter</td>
</tr>
<tr>
<td>$[a, b]$</td>
<td>$[0.535 \times 10^{-7}, 0.632 \times 10^{-6}]$</td>
<td>Support for $\varepsilon$ component of match value</td>
</tr>
<tr>
<td>$t_y$</td>
<td>0.15</td>
<td>Base wage tax rate for young</td>
</tr>
<tr>
<td>$t_o$</td>
<td>0.28</td>
<td>Base wage tax rate for older</td>
</tr>
<tr>
<td>$w_y$</td>
<td>8.64</td>
<td>Wage rate when young</td>
</tr>
<tr>
<td>$w_o(H)$</td>
<td>$0.9w_y(H)$ with probability 0.5</td>
<td>Wage rate when older under certainty</td>
</tr>
<tr>
<td>$\hat{w}_o(H)$</td>
<td>$1.1w_y(H)$ with probability 0.5</td>
<td>Wage rate when older under uncertainty</td>
</tr>
<tr>
<td>$Y$</td>
<td>5.840</td>
<td>Total annual available hours</td>
</tr>
<tr>
<td>$H$</td>
<td>607</td>
<td>Base human capital amount</td>
</tr>
<tr>
<td>$L_y$</td>
<td>967</td>
<td>Base labor hours while young</td>
</tr>
<tr>
<td>$L_o$</td>
<td>2.030</td>
<td>Base leisure hours while young</td>
</tr>
<tr>
<td>$\hat{L}_o$</td>
<td>3.810</td>
<td>Base labor hours while older</td>
</tr>
<tr>
<td>$\hat{L}_o$</td>
<td>3.810</td>
<td>Base leisure hours while older</td>
</tr>
<tr>
<td>$C_y$</td>
<td>20.280</td>
<td>Base consumption while young</td>
</tr>
<tr>
<td>$C_o$</td>
<td>42.836</td>
<td>Base consumption while older and working</td>
</tr>
<tr>
<td>$B$</td>
<td>7104</td>
<td>Base transfer amount to nonworkers</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.377</td>
<td>Time discount factor</td>
</tr>
<tr>
<td>$R$</td>
<td>2.65</td>
<td>5 annual rate</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.175</td>
<td>Interest rate</td>
</tr>
</tbody>
</table>

Note: See Section IV for discussion about the parameters.

23. To account for the graduated rate schedule, corresponding tax “refunds” (virtual income) for earnings taxed at lower rates are estimated to be $\tau_y = 1.093$ and $\tau_o = 5.444$ for the young and old, respectively. These values are incorporated in the transfers $\Omega_y$ and $\Omega_o$. Accounting for average state income tax rates in $t_y$ and $t_o$ when calibrating the model had no appreciable effects on the simulated optimal tax results.

24. See Footnote 11. The qualitative conclusions for tax rates are not sensitive to the benefit policy function.
TABLE 3
Optimal Policy under Certain Wages

<table>
<thead>
<tr>
<th>s</th>
<th>$t_y$</th>
<th>$t_o$</th>
<th>$\phi$</th>
<th>$H$</th>
<th>$t_y$</th>
<th>$L_y$</th>
<th>Welfare × 10^5</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Direct human capital subsidy available</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Perfectly inelastic first-period leisure</td>
<td>1.65</td>
<td>0</td>
<td>0</td>
<td>0.225</td>
<td>705</td>
<td>4,266</td>
<td>869</td>
</tr>
<tr>
<td>(b) Perfectly inelastic second-period leisure</td>
<td>1.13</td>
<td>0</td>
<td>0</td>
<td>0.219</td>
<td>755</td>
<td>4,161</td>
<td>924</td>
</tr>
<tr>
<td>(c) Endogenous leisure in both periods</td>
<td>1.64</td>
<td>0</td>
<td>0</td>
<td>0.218</td>
<td>794</td>
<td>4,434</td>
<td>612</td>
</tr>
<tr>
<td>II. Optimal $t_y$, $t_o$ (no direct subsidy)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Perfectly inelastic first-period leisure</td>
<td>—</td>
<td>0.191</td>
<td>0</td>
<td>0.225</td>
<td>705</td>
<td>4,266</td>
<td>869</td>
</tr>
<tr>
<td>(b) Perfectly inelastic second-period leisure</td>
<td>—</td>
<td>0</td>
<td>-0.314</td>
<td>0.219</td>
<td>755</td>
<td>4,161</td>
<td>924</td>
</tr>
<tr>
<td>(c) Endogenous leisure in both periods</td>
<td>—</td>
<td>0.020</td>
<td>-0.093</td>
<td>0.228</td>
<td>697</td>
<td>4,493</td>
<td>650</td>
</tr>
<tr>
<td>III. Constrained $t_y = t_o$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Perfectly inelastic first-period leisure</td>
<td>—</td>
<td>0.075</td>
<td>0.075</td>
<td>0.236</td>
<td>627</td>
<td>4,266</td>
<td>947</td>
</tr>
<tr>
<td>(b) Perfectly inelastic second-period leisure</td>
<td>—</td>
<td>-0.022</td>
<td>-0.022</td>
<td>0.230</td>
<td>681</td>
<td>4,187</td>
<td>972</td>
</tr>
<tr>
<td>(c) Endogenous leisure in both periods</td>
<td>—</td>
<td>-0.018</td>
<td>-0.018</td>
<td>0.231</td>
<td>674</td>
<td>4,471</td>
<td>695</td>
</tr>
</tbody>
</table>

Notes: Frame I presents optimal values for the human capital subsidy. Frame II presents optimal wage tax rates for the young and old in the absence of a direct human capital subsidy. If leisure is perfectly inelastic in either period, then optimal wage taxation induces the same efficient allocation of resources as the human capital subsidy. Frame III presents corresponding results if wage tax rates are constrained to be identical for the young and old.

As indicated in Proposition 1, the optimal human capital subsidy $s$ in Frame I is positive and the optimal wage tax is zero for both younger and older workers ($t_y^* = t_o^* = 0$). The optimal subsidy induces a higher level of human capital accumulation (by an extra three-quarters of a year when leisure is endogenous in both periods) than observed in the data under the status quo tax structure. Frame II presents optimal wage tax rates in the absence of a direct subsidy. When the leisure time of the young is perfectly inelastic, the optimal first-period wage tax is 0.191 and the optimal wage tax on older workers is zero; in contrast, when leisure time of older workers is perfectly inelastic, their optimal wage tax is -0.314 and the young face no wage tax. Corresponding to Proposition 3(a), an individual’s human capital and labor hours choices in Rows (a) and (b) are identical across Frames I and II when leisure time is perfectly inelastic in either period. Recall for that case that the optimal combination of taxes on the young and old induces the same allocation of resources and thus the same social welfare, as induced by the optimally chosen human capital subsidy.

Frame II(c) corresponds to Proposition 3(b). When leisure time is elastic in both periods, the optimal policy involves a combination of a wage tax $t_y = 0.020$ on younger workers and a wage subsidy $t_o = -0.093$ for older workers. The optimal policy induces a higher level of human capital accumulation (by an extra three-quarters of a year when leisure is endogenous in both periods) than observed in the data under the status quo tax structure. Frame III presents corresponding results if wage tax rates are constrained to be identical for the young and old.

25. A lower variance increases the magnitude of $\eta^{f,H}$. Recall from Section II that the effect of human capital on employment is given by $-\phi y = -f(\bar{e})(\partial f(\bar{e})/\partial H)$. A narrower support $[a, b]$ increases the density $f(\bar{e})$. 

respect to human capital accumulation $\eta^{f,H}$ is $-0.39$, as estimated from the data. Heuristically, $\delta_0$ pins down a young individual’s choice of $H$, the mean of the distribution of $e$ pins down the employment rate (i.e., the probability that the utility associated with working will exceed that associated with not working), and the variance of the distribution of $e$ pins down the elasticity of employment with respect to $H$. For the base model in which leisure time is endogenous in both periods and $\eta^{f,H} = -0.39$, the parameters are calibrated to be $\delta_0 = 0.175 \times 10^{-6}$, $a = 0.535 \times 10^{-7}$, and $b = 0.632 \times 10^{-6}$.

Table 3 presents empirical results for the key parameters of interest for the baseline case in which wages are nonstochastic. Frame I displays optimal direct human capital subsidy levels, while Frames II and III display optimal wage tax policy in the absence of a direct subsidy. Corresponding to the theoretical results, three cases are compared for each frame: (a) perfectly inelastic first-period leisure, (b) perfectly inelastic second-period leisure, and (c) endogenous leisure in both periods.
workers. The magnitudes of these values are quite small compared with Cases (a) and (b) since the wage tax and wage subsidy in Case (c) distort leisure decisions in both periods. Given a distortion to leisure hours, the optimal magnitudes of the tax rates will be too small to induce the socially optimal level of human capital investment; this results in lower social welfare (and less employment) in Frame II(c) compared with Frame I(c) under an optimal direct subsidy. The optimal marginal rates balance the benefits of stimulating human capital against the costs of distorting the leisure decision.

Net transfers flow from older workers to younger workers (to smooth lifetime consumption) despite the positive wage tax on younger workers ($t_y = 0.020$) and wage subsidy for older workers ($t_o = -0.093$). In fact, compared with the baseline case of lump sum transfers ($t_y = t_o = 0$), the amount of the transfer to the young actually rises after introducing the tax/subsidy combination. A young person’s wage tax bill rises from $0$ to $167$ after imposing the wage tax, but the corresponding first-period credit ($-\Omega_y$) rises from $6,009$ to $6,198$, which is more than enough to compensate for the wage tax burden. An older worker’s total tax bill rises, despite the introduction of the second-period wage subsidy, because the corresponding increase in $\Omega_o$ more than offsets the wage subsidy payment. Nevertheless, the older worker becomes better off after accounting for the combined impact on consumption and leisure. By stimulating human capital and expanding the tax base (the fraction of nonworkers falls from 0.231 to 0.228), the government receives enough extra revenue to raise utility in all states of nature (young, old, working, and nonworking).

If the wage tax rate is constrained to be the same in both periods (Frame III), the optimal rate may be either positive or negative. The optimal rate is negative in Row (c) for elastic leisure in both periods, implying a small common wage subsidy for both younger and older workers. Under this constraint, human capital accumulation falls short of levels associated with the optimal policies in Frames I and II, and social welfare is necessarily lower. When the wage tax on prime-age workers is constrained to equal the calibrated economy’s status quo value of 0.28 (not shown), this disincentive to human capital accumulation necessitates a positive first-period wage tax as an offsetting stimulus to human capital ranging from 0.04 when leisure is elastic in both periods to 0.30 when first-period leisure is assumed to be perfectly inelastic.

Extensive sensitive analysis was conducted for all the parameters in the model. When other parameters were altered in the sensitivity analysis, the parameters $\{\delta_0, a, b\}$ were always recalibrated to maintain $H^* = 607, \phi = 0.24$, and $\eta^{\phi,H} = -0.39$ (or other stated value) at the base tax values. The simulated optimal wage tax values are most sensitive to the assumed employment elasticity $\eta^{\phi,H}$ and the wage elasticity of leisure hours. As part of the sensitivity analysis, the preceding simulations are repeated under different assumptions about the assumed elasticity of employment with respect to human capital attainment. The base case assumption $\eta^{\phi,H} = -0.39$ is replaced with assumptions $\eta^{\phi,H} = -0.20$ and $\eta^{\phi,H} = -0.75$ in appendix tables 3A and 3B available from the author.

Table 4 illustrates the sensitivity of the conclusion of Eaton and Rosen (1980a, 1980b, 1980c) that an efficient tax policy requires positive taxation of uncertain wages. Frame A represents the one-period empirical framework of Eaton and Rosen (1980c) by fixing the level of human capital at its base value of 607. When the employment probability does not depend on human capital, the first-period wage tax is optimally zero and the only role of the second-period wage tax is to balance the social insurance value of smoothed wage income against the welfare loss arising from the labor-leisure distortion.

Even when only 10% of the wage is random, the framework of Eaton and Rosen calls for a 100% tax on wages if leisure time among older workers is perfectly inelastic (Case i) since the wage tax completely eliminates wage risk without distorting incentives. In stark

26. Compared with the reference case, the magnitudes of the optimal direct human capital subsidies $s$ in Frame I are about two-thirds as large under the smaller elasticity and about 1.5 times as large under the larger elasticity. Human capital accumulation levels are about 90% as large under the lower elasticity and 115% as large under the higher elasticity. Similarly, the magnitudes of the optimal wage tax rates tend to be around two-thirds as large under the lower elasticity and around 1.7 times as large under the higher elasticity (with variation across generations and leisure elasticity assumptions).
contrast, the optimal wage tax in Frame B becomes negative at $-0.028$ after accounting for the responsiveness of employment to human capital. If leisure among older workers is endogenous, the framework of Eaton and Rosen calls for a positive wage tax equal to 0.051. When human capital and employment are endogenous, however, the optimal wage tax rate becomes negative at $-0.042$. Additional tables available from the author provide parallel results under different assumptions about the employment elasticity and degree of wage risk. While the sign of the optimal wage tax eventually becomes positive as the degree of wage risk increases, the magnitude of the optimal tax when human capital and employment are allowed to be endogenous typically remains much lower than calculated in the framework of Eaton and Rosen; the social insurance value of high wage taxes must be balanced not only against the leisure distortion among workers but also against the effect on the number of taxpayers.

### V. CONCLUSIONS

This paper studied how optimal wage tax conclusions from the classic two-period life cycle model of human capital accumulation are affected by endogenizing the base of taxpaying workers. In the standard model, the optimal wage tax is zero when wage rates are nonstochastic regardless of the impact of human capital on the future wage rate or the wage elasticity of labor hours. Lump sum transfers are efficient in that case because the gains from human capital investment are fully internalized. When human capital also increases the employment rate and non-workers receive government transfers, however, distortionary wage taxation can become desirable even with nonstochastic wage rates.

In the absence of a corrective policy, young individuals underinvest in human capital from a social perspective because tax premiums for transfers to nonworkers are not actuarially adjusted downward for human capital attainment. Taking government tax and benefit parameters parametrically, young individuals do not consider the impact of their human capital investment on the size of the benefit level to nonworkers or the government’s cost of financing those benefits. At privately optimal levels of human capital accumulation, social welfare is improved at the margin with additional investment because a larger pool of future workers allows the policymaker to provide higher benefits at lower cost.

The policymaker can restore proper price signals by imposing (compensated) taxes on the earnings of young workers and subsidies to the wages of older workers.\(^{27}\) In doing so, the policymaker stimulates human capital by taxing activities that compete for time that could be spent investing in human capital while subsidizing its returns. The optimal wage tax policy balances the gains from stimulating human capital investment against the

\[^{27}\text{If the premiums for transfers to nonworkers were actuarially tailored to human capital choices, then lump sum transfers would be efficient in the model.}\]
costs of distorting leisure decisions. Compared with the case of exogenous employment, numerical simulations suggest that even modest employment elasticities can be sufficient to substantially impact the magnitudes and even the signs of optimal wage tax rates on prime-age workers. The simulated optimal marginal rates are fairly small, however, when leisure is endogenous in both periods.

Appropriate marginal rates in the real economy will, of course, depend on many factors not considered in the model. The main message of the paper is that optimal marginal rates become higher on younger workers, and lower on older workers, than otherwise prescribed once we account for the endogeneity of the employment base with respect to human capital accumulation. Although advocated by a number of economists such as Phelps (1994) and Haveman and Wolfe (2000), the optimality of a wage subsidy has been explicitly ruled out in most optimal tax models. Given the widespread popularity of tax provisions that include a wage subsidy component such as the Earned Income Tax Credit (see, e.g., Scholz 1996), future optimal tax analyses will hopefully pay explicit attention to important interactions between human capital investment and the extensive employment outcome.

**APPENDIX A**

Comparative Statics

The second-order conditions are given by Equations (1) and (2), and

\[
\begin{align*}
\left( H_t^* \quad H_t^* \quad \Pi_t^* \quad \Pi_t^* \right) \\
\left( \ell_t \quad \ell_t \quad \phi_t \quad \phi_t \right) \\
= - \frac{1}{D_t} \left( U_{tt} - U_{tt} \right) \left( U_{tt} \quad U_{tt} \quad U_{tt} \quad U_{tt} \right) 0 0 0,
\end{align*}
\]

where \( U_{tt} = \omega_t U_{tt}^2 - \omega_t U_{tt} + \beta \Pi_{tt} < 0 \)

\[
\begin{align*}
U_t &= \omega_t U_{tt} \quad - 2 \omega_t U_{tt} + U_{tt}^2 < 0 \\
D_t &= \left| \begin{array}{ccc}
U_{tt} & U_{tt} \\
U_{tt} & U_{tt}
\end{array} \right| = U_{tt} U_{tt} - (U_{tt})^2 \\
&= (s - \omega_t)(U_{tt} U_{tt} - (U_{tt})^2) + \beta U_{tt} \Gamma_{tt} > 0,
\end{align*}
\]

where \( \omega_t = (1 - t)w_t \) and \( \Gamma_t \) represents second-period expected utility. With \( U_{tt} < 0 \) and diminishing marginal returns to human capital investment (\( \Gamma_{tt} < 0 \)), Equation (11) holds as long as the usual condition \( U_{tt}^2 U_{tt} - (U_{tt})^2 \geq 0 \) associated with the maximization of \( U_t^2 \) is satisfied at optimal choices of \( C_t \) and \( \ell_t \).

Differentiation of Equations (1) and (2) yields the following system of equations (the \( U_t \) values are the direct effects on the relevant first-order condition, \( H_t \) or \( \ell_t \), associated with the particular variable):

\[
\begin{align*}
\left( H_t^* \quad H_t^* \quad \Pi_t^* \quad \Pi_t^* \right) \\
\left( \ell_t \quad \ell_t \quad \phi_t \quad \phi_t \right) \\
= - \frac{1}{D_t} \left( U_{tt} - U_{tt} \right) \left( U_{tt} \quad U_{tt} \quad U_{tt} \quad U_{tt} \right) 0 0 0,
\end{align*}
\]

where \( U_{tt} = \omega_t U_{tt}^2 - \omega_t U_{tt} + \beta \Pi_{tt} < 0 \)

\[
\begin{align*}
U_t &= \omega_t U_{tt} \quad - 2 \omega_t U_{tt} + U_{tt}^2 < 0 \\
D_t &= \left| \begin{array}{ccc}
U_{tt} & U_{tt} \\
U_{tt} & U_{tt}
\end{array} \right| = U_{tt} U_{tt} - (U_{tt})^2 \\
&= (s - \omega_t)(U_{tt} U_{tt} - (U_{tt})^2) + \beta U_{tt} \Gamma_{tt} > 0,
\end{align*}
\]

where \( \omega_t = (1 - t)w_t \) and \( \Gamma_t \) represents second-period expected utility. With \( U_{tt} < 0 \) and diminishing marginal returns to human capital investment (\( \Gamma_{tt} < 0 \)), Equation (11) holds as long as the usual condition \( U_{tt}^2 U_{tt} - (U_{tt})^2 \geq 0 \) associated with the maximization of \( U_t^2 \) is satisfied at optimal choices of \( C_t \) and \( \ell_t \).

**APPENDIX B**

Proof of the Lemma

(a) Show \( \Delta H_t > 0 \): Using Appendix A, \( \Delta H_t = -U_{tt}^2 U_{tt}/D_t > 0 \).

(b) Show \( \Delta H_t > 0 \), \( \Delta \ell_t > 0 \): To examine Hicksian properties of the demands \( \{C_t, \ell_t, H_t\} \), consider the expenditure minimization problem

\[
\min C_t + \omega_t \ell_t + \omega_t H_t \\
s.t. U_t(C_t, \ell_t) + \beta \Gamma(H_t) = U_t.
\]

The net wage rate \( \omega_t = (1 - t)w_t \) represents the price of leisure and the price of human capital investment. The first-order conditions and utility constraint lead to the system of equations:

\[
\begin{align*}
\omega_t U_{tt}^2 - \omega_t U_{tt} + \beta \Pi_{tt} &= 0, \\
U_{tt}^2 - \beta \Pi_{tt} &= 0, \\
U_t(C_t, \ell_t) + \beta \Gamma(H_t) &= U_t.
\end{align*}
\]

Differentiating with respect to \( \omega_t \) implies:

\[
\begin{bmatrix}
\omega_t U_{tt}^2 - \omega_t U_{tt} + \beta \Pi_{tt} \\
U_{tt}^2 - \beta \Pi_{tt} \\
U_t^2 - \beta \Pi_t \\
H_t
\end{bmatrix}
\begin{bmatrix}
\omega_t \\
\ell_t \\
\phi_t \\
H_t
\end{bmatrix}
= \left[ \begin{array}{c}
0 \\
0 \\
0 \\
0
\end{array} \right].
\]

Using Cramer’s rule, Hicksian responses to changes in the net wage rate are given by:

\[
\left[ C_{\omega_t}, \ell_{\omega_t}, \phi_{\omega_t}, H_{\omega_t} \right] = \left( \begin{array}{c}
|A1| \quad |A2| \quad |A3| \\
\end{array} \right)/|A|,
\]
where
\[
A = \begin{bmatrix}
\omega_x U_{x}^{r} - U_{x}^{r} & \omega_x U_{x}^{r} - U_{x}^{r} & 0 \\
U_{x}^{r} & U_{x}^{r} & -\beta \Gamma_{xh} \\
U_{x}^{r} & U_{x}^{r} & \beta \Gamma_{xh} \\
-\omega_x U_{x}^{r} & \omega_x U_{x}^{r} - U_{x}^{r} & 0 \\
0 & U_{x}^{r} & -\beta \Gamma_{xh} \\
0 & \beta \Gamma_{xh} & 0 \\
\omega_x U_{x}^{r} - U_{x}^{r} & \omega_x U_{x}^{r} - U_{x}^{r} & -U_{x}^{r} \\
U_{x}^{r} & U_{x}^{r} & 0 \\
U_{x}^{r} & U_{x}^{r} & 0
\end{bmatrix},
\]

The denominator in Equation (13) is given by
\[|A| = U_{x}^{r} D_{1},\]
which is positive since \(U_{x}^{r}\) and \(D_{1}\) are positive. Evaluating the remaining determinants,
\[\omega_x U_{x}^{r} - U_{x}^{r} < 0\] and \(\omega_x U_{x}^{r} - U_{x}^{r} > 0\) (consumption rises with the net wage rate) since \(U_{x}^{r} > 0\) and \(\Gamma_{xh} < 0\). Negative own-price Hicksian responses \(H_{x}^{o} = -(U_{x}^{r})^{2}/(U_{x}^{r} + \Gamma_{xh})/|A| < 0\) and \(\Gamma_{xh} < 0\). These inequalities
in turn imply \(\Delta H^o > 0\) and \(\Delta \Gamma^o > 0\) since \(\Delta H^s = \omega_x U_{x}^{r} - U_{x}^{r}\) and \(\Delta \Gamma^s = -\omega_x U_{x}^{r} - \beta \Gamma_{xh}\) using Appendix A.

(c) Show \(\Delta \Gamma^o > 0\): Using the direct utility function
\[U^o((1 - t_o)w_o H(y - t_o) - \Omega_f, \epsilon_o),\]
differentiate the first-order condition for \(t_o\) separately with respect to \(t_o\) and \(\Omega_f\), and then solve for \(\Delta \Gamma^o = (\partial \Omega_{f}/\partial \Omega_{f}) - \omega_o L_o (\partial \Omega_{f}/\partial \Omega_{f}) \times w_o U_{x}^{o} > 0\). Show \(\Delta H^o < 0\): From Appendix A,
\[\Delta H^o = H_{x}^{o} - \omega_o L_o \Delta H^o = -\omega_o \int (U_{x}^{o}) d \Omega_{f} \quad \text{(4)}\]
Using Appendix A,
\[\Delta \Gamma^o = \omega_o \Delta H^o = -\omega_o \int (U_{x}^{o}) d \Omega_{f} \quad \text{(5)}\]
The second term is zero by Roy’s identity. Differentiating this identity with respect to \(\epsilon_o\) and \(\delta_o\), respectively, yields
\[V_{o}^o = (\partial \delta_{o}/\partial \delta_{o}) + L_o V_{o}^o \] and
\[V_{o}^o = L_o V_{o}^o \]
using \(\partial \Omega_{f}/\partial \Omega_{f} = 0\) by the separability of \(U^o\). Then,
\[\Delta H^o = -\omega_o \int (U_{x}^{o}) d \Omega_{f} + \omega_o (\partial \Omega_{f}/\partial \Omega_{f}) \quad \text{(6)}\]
\[\Delta \Gamma^o = \omega_o \Delta H^o - \omega_o \int (U_{x}^{o}) d \Omega_{f} \quad \text{(7)}\]
is derived from the relations in Appendix A. Solving for \(t_o\) implies
\[t_o = \phi_{H}(B + T_o) w_{y} U_{x}^{H} \]
is equivalent to each other and solving for \(t_o\) obtains the stated result.

Appendix D

Derivation of Equations (7) and (8)

Solving for \(t_o\) in Equation (4) and substituting into Equation (5) implies
\[\Psi_{t_o} = \frac{\Lambda \Delta H^o \Delta \Gamma^o - \Delta H^o \Delta \Gamma^o}{\Lambda (1 - \phi) t_o w_o \Delta C^o / (\Delta H^o + \Delta \Gamma^o)/R}
\]
\[= -\Lambda w_o U_{x}^{o} \Delta H^o \times 1 \text{elastic}/U_{x}^{i} \]
\[= (1 - \phi) t_o w_o \Delta C^o / (\Delta H^o + \Delta \Gamma^o)/R = 0,\]
where \(\Delta H^o \Delta \Gamma^o - \Delta H^o \Delta \Gamma^o = 0\), if \(t_o\) is fixed.

28. The signs of \(\omega_x U_{x}^{r} - U_{x}^{r}\) and \(\omega_x U_{x}^{r} + \Gamma_{xh}\) are invariant to monotonic transformations of the utility function. Let \(g\) be a monotonic transformation of \(U\). Then, \(\omega_x U_{x}^{r} + \Gamma_{xh}\) (\(\partial g/\partial \Omega_{f}\)) and \(\omega_x U_{x}^{r} + \Gamma_{xh}\) (\(\partial g/\partial \Omega_{f}\)), where the first term is zero by the first-order condition for leisure. A similar result holds for the sign of \(\omega_x U_{x}^{r} + \Gamma_{xh}\).
APPENDIX E

Proof of Proposition 3

(a) When the human capital subsidy is available, the first-order conditions for \( H \) and \( \ell \) given by Equations (1) and (2) are

\[
(s^* - w^*)U_C^H + \beta \Gamma_H = 0 \quad \text{and} \quad -w^*U_C^H + U_I^H = 0.
\]

respectively, using \( r^*_t = 0 \). In the absence of a direct subsidy, these equations are replaced by

\[
-(1 - r^*_t)w_tU_C^H + \beta \Gamma_H = 0 \quad \text{and} \quad -(1 - r^*_t)w_tU_C^H + U_I^H = 0.
\]

For perfectly inelastic first-period leisure, the optimal first-period wage tax is given by Equation (4) as \( r^*_t = s^*/w^*_t \), where \( s^* = \Lambda \) is the optimal value of the subsidy (were it available). Then, substituting \( s^*/w^*_t \) in place of \( r^*_t \) in the second set of first-order conditions leads to the conditions

\[
(s^* - w^*_t)U_C^H + \beta \Gamma_H = 0 \quad \text{and} \quad (s^* - w^*_t)U_C^H + U_I^H = 0.
\]

Comparing Equation (17) with Equation (18), the human capital first-order conditions are the same for the subsidy and no-subsidy cases, but the first-order conditions for leisure are different. With perfectly inelastic first-period leisure, only the human capital first-order condition is relevant, and the optimal wage tax policy leads to the same allocation of resources as the optimal human capital subsidy policy. A similar result holds when second-period leisure is instead perfectly inelastic. Equations (7) and (8) reveal that \( r^*_t > 0 \) and \( r^*_t = 0 \) when first-period leisure is perfectly inelastic while \( r^*_t < 0 \) when second-period leisure is perfectly inelastic.

(b) When leisure is endogenous in both periods, Equations (7) and (8) imply that \( r^*_t > 0 \) and \( r^*_t < 0 \) (using \( \xi < 0 \) along with \( \Delta \Gamma^H > 0, \Delta H^H < 0, \) and \( \Delta \phi_H > 0 \) from the lemma) as long as \( \phi_H < 0 \). Given \( \zeta > 0 \), we have \( \Lambda > 0 \) by the first equality in Equation (7). This in turn implies \( \Psi_H > 0 \) by Equation (6) since \( \Delta \phi_H > 0 \) by the lemma: at optimal tax rates, another unit of human capital investment improves social welfare.

REFERENCES


